J. P. Rigaud
Oct. 10. 1828

INTEGRAL TABLES,

OR

A COLLECTION

OF

INTEGRAL FORMULÆ.

By MEYER HIRSCH.

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ADVERTISEMENT.

THIS work, from the great variety and elegance of the results which it exhibits, and their practical as well as theoretical utility, having long attracted, even under the disguise of a foreign garb, the notice of British mathematicians, it is presumed the new dress in which it now appears, will enhance, rather than depress its value, in their estimation.

Regarding the Integral Tables not only as a complete praxis for the novice, but as a treasury of results (in value little short of Tables of Logarithms) for the accomplished mathematician, no pains have been spared to render them perfectly accurate. Should this object have not been wholly attained, any inaccuracies which may be detected, it is hoped, for the benefit of science, will not be withheld. Any communications to the publishers on this subject will be thankfully received, and indeed are earnestly requested, as, from the nature of *stereotyping*, they may be made wholly conducive to the attainment of perfect accuracy.

Some few liberties have been taken with the text, but such only as must be approved of by all competent judges.

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for	arc sin X	+ const.	put	— arc cos X	+ const.
...	arc cos X	+ const.	......	— arc sin X	+ const.
...	arc tang X	+ const.	......	— arc cot X	+ const.
...	arc cot X	+ const.	......	— arc tang X	+ const.
...	arc sec X	+ const.	......	— arc cosec X	+ const.
...	arc cosec X	+ const.	......	— arc sec X	+ const.
...	log $-X$	+ const.	......	log X	+ const
...	log $\sqrt{-X}$	+ const.	......	log $\sqrt{X}$	+ const.

(6.) By aid of these Tables there may also be found innumerable integrals, merely by compounding; this is best shown by an example.

Required the integral of

$$\mathrm{d}Z = \frac{(3x^{12} - 2x^9 - 5x^6 + 2x^4 + 9)\,\mathrm{d}x}{x^6 (3 - 2x^2)^{\frac{7}{2}}}$$

If, for brevity, we put $3 - 2x^2 = X$, then

$$Z = 3\int\frac{x^6\mathrm{d}x}{X^{\frac{7}{2}}} - 2\int\frac{x^3\mathrm{d}x}{X^{\frac{7}{2}}} - 5\int\frac{\mathrm{d}x}{X^{\frac{7}{2}}} + 2\int\frac{\mathrm{d}x}{x^2 X^{\frac{7}{2}}} + 9\int\frac{\mathrm{d}x}{x^6 X^{\frac{7}{2}}}$$

If these integrals be sought for in the Tables for

$$\int\frac{x^m\mathrm{d}x}{(a + bx^2)^{\frac{7}{2}}},\quad \int\frac{\mathrm{d}x}{(x^m (a+bx^2)^{\frac{7}{2}}},$$

(see pp. 127, 128), we find ($a=3$, $b=-2$)

$$3\int\frac{x^6\mathrm{d}x}{X^{\frac{7}{2}}} = \left(\frac{23x^5}{10} - \frac{21x^3}{4} + \frac{27x}{8}\right)\frac{1}{X^2\sqrt{X}} - \frac{3}{8}\int\frac{\mathrm{d}x}{\sqrt{X}}$$

$$-2\int\frac{x^3\mathrm{d}x}{X^{\frac{7}{2}}} = \left(-\frac{x^2}{3} + \frac{1}{5}\right)\frac{1}{X^2\sqrt{X}}$$

$$-5\int\frac{\mathrm{d}x}{X^{\frac{7}{2}}} = -5\int\frac{\mathrm{d}x}{X^{\frac{7}{2}}}$$

$$+2\int\frac{\mathrm{d}x}{x^2 X^{\frac{7}{2}}} = -\frac{2}{3x}\cdot\frac{1}{X^2\sqrt{X}} + 8\int\frac{\mathrm{d}x}{X^{\frac{7}{2}}}$$

$$+9\int\frac{\mathrm{d}x}{x^6 X^{\frac{7}{2}}} = \left(-\frac{3}{5x^5} - \frac{4}{3x^3} - \frac{64}{9x}\right)\frac{1}{X^2\sqrt{X}} + \frac{256}{3}\int\frac{\mathrm{d}x}{X^{\frac{7}{2}}}$$

hence

$$Z=\left(\frac{23x^5}{10}-\frac{21x^3}{4}-\frac{x^2}{3}+\frac{27x}{8}+\frac{1}{5}-\frac{70}{9x}-\frac{4}{3x^3}-\frac{3}{5x^5}\right)\frac{1}{X^2\sqrt{X}}$$
$$-\frac{3}{8}\int\frac{dx}{\sqrt{X}}+\frac{265}{9}\int\frac{dx}{X^{\frac{7}{2}}}$$

But

$$\int\frac{dx}{X^{\frac{7}{2}}}=\left(\frac{32x^5}{405}-\frac{8x^3}{27}+\frac{x}{3}\right)\frac{1}{X^3\sqrt{X}},\quad \int\frac{dx}{\sqrt{X}}=\frac{1}{\sqrt{2}}\text{arc sin } x\sqrt{\frac{2}{3}};$$

whence, these values being substituted, by proper reduction, we get

$$Z=\left\{\begin{array}{l}\dfrac{33727x^5}{7290}-\dfrac{13583x^3}{972}-\dfrac{x^2}{3}+\dfrac{2849x}{216}\\ \quad+\dfrac{1}{5}-\dfrac{70}{9x}-\dfrac{4}{3x^3}-\dfrac{3}{5x^5}\end{array}\right\}\frac{1}{X^2\sqrt{X}}$$
$$-\frac{3}{8\sqrt{2}}\text{ arc sin } x\sqrt{\frac{2}{3}}+\text{const.}$$

(7) Should an integral not be found immediately in these Tables, it may, however, be always obtained, when the general case is possible. It will readily admit of being reduced to one or other of the preceding forms. Thus, for instance, an integral is found not only in the form $\int\frac{x^{\frac{2n+1}{2}}dx}{(a+bx)^{\frac{3}{2}}}$, but also in that of $\int\frac{x^{n+2}dx}{(ax+bx^2)^{\frac{3}{2}}}$, derivable from the former by multiplying both numerator and denominator by $x^{\frac{3}{2}}$.

A BRIEF EXPOSITION

OF THE

METHODS FOR ANALYZING

FRACTIONAL RATIONAL FUNCTIONS

INTO

PARTIAL FRACTIONS,

WITH THE

NECESSARY EXPLANATIONS AND EXAMPLES.

ANALYSIS OF FRACTIONAL FUNCTIONS.

LET $\frac{U}{V}$ be the fraction to be analyzed; U, V being whole rational functions of the forms

$$U = Ax^m + Bx^{m-1} + Cx^{m-2} + Dx^{m-3} + \&c.$$
$$V = x^\mu + \alpha x^{\mu-1} + \beta x^{\mu-2} + \gamma x^{\mu-3} + \&c.$$

and $m < \mu$; the coefficients positive, negative, or zero. We assume the possibility of decomposing the denominator V into real factors of the forms

$$\text{I. } x + a, \qquad \text{II. } (x + a)^n$$
$$\text{III. } x^2 + ax + b, \qquad \text{IV. } (x^2 + ax + b)^n$$

It is required to analyze the fraction $\frac{U}{V}$ *into such fractions as shall have denominators of these forms.*

Since each of these factors requires a distinct mode of treatment, there will arise these four different cases, for which the methods with the necessary explanations and examples we proceed to give.

First Case.

Let the denominator V have one factor only of the form $(x + a)$. Let

$$V = (x + a)\,Q$$

then Q is a known whole and rational function, for $Q = \frac{V}{x + a}$. Make

$$\frac{U}{V} = \frac{A}{x + a} + \frac{P}{Q}.$$

The A, which is still unknown, is a constant quantity. The P, which is also unknown, denotes a whole rational function of x. A and P are determined by the following methods.

First Method.

Let $x + a = 0$, then $x = -a$. Substitute this value of x in U, Q, and let the constant quantities into which they are thus transformed, be denoted by U', Q'. Then we have

$$A = \frac{U'}{Q'}$$

Having thus obtained A, P may be derived from the form

$$P = \frac{U - AQ}{x + a},$$

by actual division; the form $U - AQ$ being always divisible by $x + a$ without a remainder.

Second Method.

Let $dV = Zdx$, Z being a known function of x; also let U', Z' be the values of U, Z when for x, $-a$ is substituted; then

$$A = \frac{U'}{Z'}.$$

The function P is found by the first method.

Remarks.

(1.) When x itself is a factor of the denominator V, in order to get the constant quantities U', Q', Z', from the functions U, Q, Z, we must put o for x.

(2). If V, besides $x + a$, contain other factors of the same kind, $x+a'$, $x+a''$, $x+a'''$, &c.; then from each of them we can form a distinct partial fraction, and the numerators of these fractions, viz. A', A'', A''', &c. may be found in the same way as that for the factor A.

(3). If V be composed of such real factors as $x+a$, $x+a'$, &c.; then $\frac{U}{V}$ may be analyzed into partial fractions of the form $\frac{A}{x+a}$, whose sum $=\frac{U}{V}$.

(4). These methods, however, are practicable only when the factors are different from one another; for otherwise, Q' as well as Z' corresponding to that factor which occurs more than once, would $=0$, and hence by both methods $A=\frac{U'}{0}=\infty$.

Example.

Let $\frac{U}{V}=\frac{2x+3}{x^3+x^2-2x}=\frac{2x+3}{(x-1)(x+2)x}$ be the fraction to be analysed, then

$$U=2x+3;\ V=x^3+x^2-2x=(x-1)(x+2)x.$$

For the first factor $x-1$, $Q=(x+2)x$, and $x-1=0$, gives $x=1$. Also, when this value is substituted, $U'=5$, $Q'=3$, and hence, by the first method, $A=\frac{U'}{Q'}=\frac{5}{3}$. Also $dV=(3x^2+2x-2)dx$ and $Z=3x^2+2x-2$; therefore $Z'=3$, and hence by the second method $A=\frac{U'}{Z'}=\frac{5}{3}$ as before. The partial fraction for the factor $x-1$ is then $\frac{\frac{5}{3}}{x-1}=\frac{5}{3.(x-1)}$.

For the factor $x+2$, we have $Q=(x-1)x$, and $x+2=0$ gives $x=-2$. Hence, when -2 is substituted for x, $U'=-1$, $Q'=6$, and the first method gives $A=\frac{U'}{Q'}=-\frac{1}{6}$. Moreover, $Z=3x^2+2x-2$; hence, when -2 is put for x, $Z'=6$, and by the second method, $A=\frac{U'}{Z'}=-\frac{1}{6}$, as before. Hence the partial fraction is $-\frac{1}{6(x+2)}$.

For the factor x, $Q=(x-1)(x+2)$, and therefore, when

$x=0$, $U'=3$, $Q'=-2$; and by the first method $A = \frac{U'}{Q'} = -\frac{3}{2}$. Moreover $Z = 3x^2 + 2x - 2$, and for $x=0$, $Z' = -2$; hence, by the second method, $A = \frac{U'}{Z'} = -\frac{3}{2}$, as before. The partial fraction is therefore $-\frac{3}{2x}$.

From all these it follows that

$$\frac{U}{Z} = \frac{5}{3(x-1)} - \frac{1}{6(x+2)} - \frac{3}{2x}.$$

SECOND CASE.

The denominator V of the fraction $\frac{U}{V}$, containing the factor $x + a$ more than once, let $V = (x+a)^n Q$; hence Q is a whole rational function of x. Let

$$\frac{U}{V} = \frac{A}{(x+a)^n} + \frac{A'}{(x+a)^{n-1}} + \frac{A''}{(x+a)^{n-2}} + \ldots\ldots\ldots\ldots$$
$$+ \frac{A^{(n-2)}}{(x+a)^2} + \frac{A^{(n-1)'}}{x+a} + \frac{P}{Q}.$$

It is required to determine the constant numerators A, A', A''.

First Method.

If Q', U', U'_1, U'_2, U'_3, &c. denote the values of the functions Q, U, U_1, U_2, U_3, &c. when for x we put $-a$, or $x + a = 0$; then for determining A, A', A'', A''', &c., we have the following forms:

(1) $A = \frac{U'}{Q'}$	(2) $\frac{U - AQ}{x+a} = U_1$
(3) $A' = \frac{U'_1}{Q'}$	(4) $\frac{U_1 - A'Q}{x+a} = U_2$
(5) $A'' = \frac{U'_2}{Q'}$	(6) $\frac{U_2 - A''Q}{x+a} = U_3$
(7) $A''' = \frac{U'_3}{Q'}$	(8) $\frac{U_3 - A'''Q}{x+a} = U_4$
&c.	&c.

In the first place, $-a = x$ is put both in U and Q; whence we have U', Q'. Hence, by form (1) we get A; and substituting this value of A in (2), and actually dividing by the denominator, we have U_1 a whole function. In this, if for x be put $-a$, we obtain U'_1, and thence, by means of form (3) A'. If this value be substituted in form (4), and division by $x+a$ actually performed, we obtain for U_2 a whole function, and, putting $-a$ for x, U'_2; which by aid of (5) gives A''. Substituting this value in (6), we get U_3, and consequently the constant quantity U'_3, and form (7) gives the value of A'''. This operation is continued until all the numerators A, A', A'' ...... $A^{(n-1)'}$ are determined.

If $A^{(n-1)'}$ be found, then the numerator P of the fraction $\frac{P}{Q}$ may be found. For then from $A^{(n-1)'}$ we obtain, by the above method, U_n; and we have $P = U_n$.

Second Method.

We have

$$A = \frac{U}{Q}$$

$$A' = \frac{1}{1 \, . \, dx} \, d \, . \, \frac{U}{Q}$$

$$A'' = \frac{1}{1 \, . \, 2 \, . \, dx^2} \, d^2 \, . \, \frac{U}{Q}$$

$$A''' = \frac{1}{1 \, . \, 2 \, . \, 3 \, . \, dx^3} \, d^3 \, . \, \frac{U}{Q}$$

$$A^{iv} = \frac{1}{1 \, . \, 2 \, . \, 3 \, . \, 4 \, dx^4} \, d^4 \, . \, \frac{U}{Q}$$

and in general

$$A^{m'} = \frac{1}{1 \, . \, 2 \, . \, 3 \, ... \, m \, . \, dx^m} \, d^m \, . \, \frac{U}{Q}$$

when in the obtained results $-a$ is put for x.

In order, therefore, to determine $A', A'' \ldots\ldots A^{(n-1)'}$ we must differentiate the function $\frac{U}{Q}$, $n-1$ times successively, divide the resulting differentials in order by $1 \cdot dx$, $1 \cdot 2\, dx^2$, &c. and in the $n-1$ obtained results put $-a$ for x.

Example.

Let the fraction to be analyzed be

$$\frac{U}{V}=\frac{3x^2+x-2}{(x-1)^3\,(x^2+1)}$$

and assume

$$\frac{U}{V}=\frac{A}{(x-1)^3}+\frac{A'}{(x-1)^2}+\frac{A''}{x-1}+\frac{P}{Q}$$

in which A, A', A'', and P, are to be found.

Calculation by the First Method.

Let $U=3x^2+x-2$, $Q=x^2+1$; also make $x-1=0$, or $x=1$. Hence $U'=2$, $Q'=2$; consequently by formula (1), $A=\frac{U'}{Q'}=1$.

By formula (2) we have

$$U_1=\frac{U-1\,.\,Q}{x-1}=2x+3;$$

therefore $U'_1=5$, and by formula (3), $A'=\frac{U'_1}{Q'}=\frac{5}{2}$.

Hence we obtain by formula (4)

$$U_2=\frac{U_1-\frac{5}{2}Q}{x-1}=-\frac{5}{2}x-\frac{1}{2}$$

therefore $U'_2=-3$, and by formula (5), $A''=\frac{U'_2}{Q'}=-\frac{3}{2}$.

To determine P, by formula (6), we have

$$U_3 = \frac{U_2 + \frac{3}{2}Q}{x-1} = \tfrac{3}{2}x - 1\,;$$

and hence $P = U_3 = \frac{3}{2}x - 1$

We have, therefore,

$$\frac{U}{V} = \frac{1}{(x-1)^3} + \frac{5}{2(x-1)^2} - \frac{3}{2(x-1)} + \frac{3x-2}{2(x^2+1)}.$$

Calculation by the Second Method.

Here, we have

$$\frac{U}{Q} = \frac{3x^2+x-2}{x^2+1}$$

$$\frac{1}{1\,.\,dx}\,d.\frac{U}{Q} = \frac{-x^2+10x+1}{(x^2+1)^2}$$

$$\frac{1}{1\,.\,2\,.\,dx^2}\,d^2.\frac{U}{Q} = \frac{(x^2+1)^2(10-2x)+(x^2-10x-1)(x^2+1)4x}{2(x^2+1)^4}.$$

If in the second members of the equations, we put $x = 1$, we then obtain for A, A', A'', the same values as before.

Third Case.

Let it be assumed that the denominator V of the fraction $\frac{U}{V}$ has the trinomial factor x^2+ax+b, so that $V=(x^2+ax+b)\,Q$, and Q be a whole function. Let it be further assumed, that the trinomial x^2+ax+b does not admit of being reduced to two real factors of the form $x+\alpha$, which is the case when the roots of $x^2+ax+b=0$ are imaginary, or a^2-4b is negative. Required to analyze the fraction $\frac{U}{V}$ into two others, so that

$$\frac{U}{V} = \frac{A+Bx}{x^2+ax+b} + \frac{P}{Q};$$

A, B, being constant quantities, and P a whole function of x.

First Method.

According to the supposition, the equation $x^2+ax+b=0$, has two imaginary roots; these roots are of the form $h \pm k\sqrt{-1}$. Let the form of the function be

$$U-(A+Bx)\,Q=Y;$$

if now throughout the function Y, we put $h+k\sqrt{-1}$ for x, the result will be of the form $M+N\sqrt{-1}$, and M, N, will be two constants containing A, B, at present unknown. Next making

$$M=0,\ N=0;$$

by the solution of these equations we obtain A and B.

Hence the function P is immediately obtained, from

$$P=\frac{U-(A+Bx)\,Q}{x^2+ax+b}$$

by putting for A, B, their values, and actually performing the division.

Second Method.

Substitute in the function Y, $\frac{1}{2k\sqrt{-1}}\cdot\frac{dV}{dx}$ for Q, and then proceed as in the first method.

N. B. The second method is preferable in certain cases which shall be pointed out in due time.

Example.

Let the fraction to be decomposed, be

$$\frac{U}{V}=\frac{2x+1}{(x^2+2x+5)(x^2+x+1)(x^2+1)}.$$

Here, for the factor x^2+2x+5, we have, $Q=(x^2+x+1)(x^2+1)$. Since $U=2x+1$, we have

$$Y=2x+1-(A+Bx)(x^2+x+1)(x^2+1).$$

The equation $x^2+2x+5=0$, gives $x=-1+2\sqrt{-1}$.
If this value be substituted in Y, we obtain

$$Y=-1+4\sqrt{-1}-(A-B+2B\sqrt{-1})(-2+16\sqrt{-1})$$
$$=2A+30B-1+(20B-16A+4)\sqrt{-1};$$

whence $M=2A+30B-1$, $N=20B-16A+4$. We have, therefore, the equations:

$$2A+30B-1=0,\ 20B-16A+4=0,$$

and they give $A=\frac{7}{26}$, $B=\frac{1}{65}$.

For the Factor x^2+x+1, we have $Q=(x^2+2x+5)(x^2+1)$, and hence

$$Y=2x+1-(A+Bx)(x^2+2x+5)(x^2+1).$$

The equation $x^2+x+1=0$, gives $x=-\frac{1}{2}+\frac{\sqrt{3}}{2}\sqrt{-1}$.

By the substitution of this value, we obtain

$$Y=\sqrt{3}\,.\sqrt{-1}-(A-\tfrac{1}{2}B+\frac{B\sqrt{3}}{2}\sqrt{-1})(\tfrac{5}{2}-\tfrac{3}{2}\sqrt{3}\,.\sqrt{-1})$$

$$=-\tfrac{5}{2}A-B+(\tfrac{3}{2}A-2B+1)\sqrt{3}\,.\sqrt{-1}$$

wherefore $M=-\frac{5}{2}A-B$, $N=(\frac{3}{2}A-2B+1)\sqrt{3}$. Hence we have the equations

$$\tfrac{5}{2}A+B=0,\ \tfrac{3}{2}A-2B+1=0$$

which give $A=-\frac{2}{13}$, $B=\frac{5}{13}$.

For the third factor x^2+1, we have $Q=(x^2+2x+5)x(x^2+x+1)$; then

$$Y=2x+1-(A+Bx)(x^2+2x+5)(x^2+x+1).$$

The equation $x^2+1=0$, gives $x=\sqrt{-1}$. Which value being substituted, gives

$$Y=1+2\sqrt{-1}-(A+B\sqrt{-1})(-2+4\sqrt{-1})$$

$$=2A+4B+1+(2B-4A+2)\sqrt{1}$$

wherefore $M=2A+4B+1$, $N=2B-4A+2$. Hence the two equations

$$2A+4B+1=0,\ 2B-4A+2=0,$$

which give $A=\frac{3}{10}$, $B=-\frac{2}{5}$.

We have, therefore,

$$\frac{U}{V}=\frac{\frac{7}{26}+\frac{1}{65}x}{x^2+2x+5}+\frac{-\frac{2}{13}+\frac{5}{13}x}{x^2+x+1}+\frac{\frac{3}{10}-\frac{2}{5}x}{x^2+1}.$$

Fourth Case.

The factor x^2+ax+b is contained in the denominator V of the fraction $\frac{U}{V}$ more than once, so that $V=(x^2+ax+b)^n Q$ gives Q a whole function of x. This fraction will always admit of being analyzed, in the same manner as in the second case, so that

$$\frac{U}{V} = \frac{A+Bx}{(x^2+ax+b)^n} + \frac{A'+B'x}{(x^2+ax+b)^{n-1}} + \dots\dots\dots$$

$$\dots\dots + \frac{A^{(n-1)'} + B^{(n-1)'}x}{x^2+ax+b} + \frac{P}{Q};$$

and then we have only to determine A, B, A', B', &c.

Method.

Form successively the functions U_1, U_2, U_3, &c. after the following scheme:

$$(1.)\ U_1 = \frac{U-(A+Bx)Q}{x^2+ax+b}$$

$$(2.)\ U_2 = \frac{U_1-(A'+B'x)Q}{x^2+ax+b}$$

$$(3.)\ U_3 = \frac{U_2-(A''+B''x)Q}{x^2+ax+b}$$

$$(4.)\ U_4 = \frac{U_3-(A'''+B'''x)Q}{x^2+ax+b}$$

&c.

and determine from U, U_1, U_2, U_3, U_4, &c. the constants A, B; A', B'; A'', B''; A''', B'''; A^{iv}, B^{iv}, &c. wholly by the first method of the Third Case.

First, the constants A, B, are found exactly as in the third Case. Substitute now their values in the form (1), and execute the division by x^2+ax+b; we then get U_1 a whole function. Proceed now with U_1 as before with U, and thence determine the constants A', B'; which being substituted in form (2), and the division actually performed, give U_2 a whole function. And proceeding in this manner the constants A'', B'', may be determined. We continue the process until all the constants A, B; A', B'; ...... $A^{(n-1)'}$, $B^{(n-1)'}$, are found.

If we wish also to determine P_1 we seek from among the known quantities $A^{(n-1)'}$, $B^{(n-1)'}$, the function U_n; we then have $P = U_n$.

Example.

Let the fraction to be analyzed be

$$\frac{U}{V} = \frac{2x^5+7x^2-4x}{(x^2+1)^3\,(2x^4-5)}$$

Here $U = 2x^5 + 7x^2 - 4x$, $Q = 2x^4 - 5$; therefore,

$$Y = 2x^5 + 7x^2 - 4x - (A + Bx)\,(2x^4 - 5).$$

If we put $x^2 + 1 = 0$, or $x = \sqrt{-1}$, and substitute this value of x, the function Y transforms into

$$-7 - 2\sqrt{-1} + 3(A + B\sqrt{-1})$$

Hence we have the equations $3A - 7 = 0$, $3B - 2 = 0$, which give $A = \frac{7}{3}$, $B = \frac{2}{3}$.

Substituting these values in the form (1), we have

$$U_1 = \frac{2x^5 + 7x^2 - 4x - (\frac{7}{3} + \frac{2}{3}x)\,(2x^4 - 5)}{x^2 + 1}$$

$$= \tfrac{1}{3}(2x^3 - 14x^2 - 2x + 35).$$

We now treat U_1, as before we did U, and we get

$$Y = \tfrac{1}{3}(2x^3 - 14x^2 - 2x + 35) - (A' + B'x)\,(2x^4 - 5)$$

and this function when $x = \sqrt{-1}$, transforms to

$$\tfrac{49}{3} - \tfrac{4}{3}\sqrt{-1} + 3(A' + B'\sqrt{-1})$$

$$\text{or } 3A' + \tfrac{49}{3} + (3B' - \tfrac{4}{3})\sqrt{-1}.$$

Hence we have the equations, $3A' + \frac{49}{3} = 0$, $3B' - \frac{4}{3} = 0$, which give $A' = -\frac{49}{9}$, $B' = \frac{4}{9}$.

Hence again by means of form (2), we find

$$U_2 = \frac{\frac{1}{3}(2x^3 - 14x^2 - 2x + 35) - (-\frac{49}{9} + \frac{4}{9}x)\,(2x^4 - 5)}{x^2 + 1}$$

$$= \tfrac{1}{9}(-8x^3 + 98x^2 + 14x - 140)$$

consequently here

$$Y = \tfrac{1}{9}(-8x^3 + 98x^2 + 14x - 140) - (A'' + B''x)\,(2x^4 - 5)$$

and when $\sqrt{-1}$ is put for x,

$$Y = -\tfrac{238}{9} + \tfrac{22}{9}\sqrt{-1} + 3(A'' + B''\sqrt{-1});$$

consequently $3A'' - \frac{238}{9} = 0,\ 3B'' + \frac{22}{9} = 0$;
whence $A'' = \frac{238}{27}$. $B'' = -\frac{22}{27}$.

From A'' and B'' we finally obtain:

$$U_3 = \frac{\frac{1}{9}(-8x^3 + 98x^2 + 14x - 140) - (\frac{238}{27} - \frac{22}{27}x)(2x^4 - 5)}{x^2 + 1}$$

$$= \frac{1}{27}(44x^3 - 476x^2 - 68x + 770)$$

which is the function of P.

Therefore

$$\frac{U}{V} = \frac{7 + 2x}{3(x^2 + 1)^3} + \frac{-49 + 4x}{9(x^2 + 1)^2} + \frac{238 - 22x}{27(x^2 + 1)}$$
$$+ \frac{44x^3 - 476x^2 - 68x + 770}{27(2x^4 - 5)}.$$

FORMULÆ OF REDUCTION.

FROM a close examination of the different methods employed to integrate a proposed differential Function, we discover that they lead us,

(1.) To the knowledge of the *Elementary Integrals*, and consequently of such integrals as either actually appear in the most simple form, or as may be so considered; as, for instance, $\int x^m dx$, $\int \frac{dx}{x}$, $\int \frac{dx}{1+x^2}$, $\int \frac{dx}{\sqrt{1+x^4}}$, &c.

(2.) To the method of reducing a proposed integral to one or other of the *Elementary Integrals*.

The reduction of a proposed integral to another may be otherwise effected:

(a.) By analyzing the given differential; and, therefore, by partial integration.

(b.) By the introduction of a new variable; and, therefore, by substitution.

(c.) By the application of certain formulæ of involution, whence, without the aid of substitution or any other means, a proposed Integral may be reduced to one more simple; and, this again, to one yet more simple, and so on. These Formulæ, ought exclusively, to be termed, Formulæ of Reduction.

(d.) By the application of some, or all of these methods, at once.

The General Formula of Reduction is

$$\int XY dx = X \int Y dx - \int dX \int Y dx$$

where X, Y, denote two arbitrary algebraic, or transcendental functions of x. From this formula, by means of certain

artifices, we may derive all the Formulæ of Reduction for algebraic functions, which are here given.

The use of these Formulæ extends even to the very comprehensive Integral $\int x^{m-1} dx\, X^p$ $(X = a + bx^n + cx^{2n} + dx^{3n} + \&c.)$ where m, n, p, may be any possible numbers whatever, positive or negative, whole or fractional. Tab. I, gives them for $a+bx^n$; Tab. II, for $a+bx^n+cx^{2n}$; Tab. III, for $a+bx^n+cx^{2n}+dx^{3n}$; Tab. IV, for the polynomial. Hence we are enabled to augment or diminish the exponents m and p, until we arrive at such integrals, as, by proper substitutions, are reducible to Elementary Integrals; which then must either be completely integrated or expressed by series.

These Formulæ, are, moreover, applicable so long only as the denominators of the fractional co-efficients do not vanish; which happens, for instance, in form. I. and V. Tab. I, when $m=0$, and in form III and IV Tab. I; when $m+np=0$.

INTEGRAL TABLES

OF

RATIONAL DIFFERENTIALS.

THE Integrals of Rational Differentials, may, on reviewing the operations by which they were calculated, be conveniently arranged in the three following classes:

(1.) Integrals of Xdx, where X is either already a finite series of the form $a+bx^i+cx^k+\&c.$, or such that by the expansion of the binomial and polynomial powers and their products, contained therein, it is immediately reducible to that form.

(2.) Integrals of Xdx, where X is not of that form.

(3.) Integrals of Xdx, where X consists of two parts, Ydx,

Zdx, of which one belongs to the first class, the other to the second class.

The third class having nothing distinct from the others needs no particular notice; for it we have $\int X dx = \int Y dx + \int Z dx$.

First Class of Integrals.

(1.) When k denotes the arbitrary constant, m being positive or negative, we have

$$\int ax^m dx = \frac{ax^{m+1}}{m+1} + k.$$

The case where $m = -1$, is an exception; for then we have

$$\int ax^{-1}dx = \int \frac{adx}{x} = a \log x + k = a \log x + a \log k$$
$$= a \log kx = \log k^a x^a = \log kx^a.$$

(2.) Hence

$$\int (a + bx + cx^2 + \&c.)\, dx = ax + \frac{1}{2}bx^2 + \frac{1}{3}cx^3 + \&c.$$

$$\int \left(\frac{a}{x} + \frac{b}{x^2} + \frac{c}{x^3} + \&c.\right) dx = a \log x - \frac{b}{x} - \frac{c}{2x^2} - \&c.$$

$$\int (ax^i + bx^k + cx^l + \&c.)\, dx = \frac{ax^{i+1}}{i+1} + \frac{bx^{k+1}}{k+1} + \frac{cx^{l+1}}{l+1} + \&c.$$

$$\int \left(\frac{a}{x^i} + \frac{b}{x^k} + \frac{c}{x^l} + \&c.\right) dx = -\frac{a}{(i-1)x^{i-1}} - \frac{b}{(k-1)x^{k-1}} - \frac{c}{(l-1)x^{l-1}} - \&c.$$

(3.) To this class belong also the Integrals $\int X^m dx$, $\int X^m Y^n dx$, $\int X^m Y^n Z^p dx$, &c. $\int \frac{X^m dx}{x^k}$, $\int \frac{X^m Y^n dx}{x^k}$, &c. where X, Y, Z, &c. are functions of the form $a + bx^i + cx^k +$ &c. and m, n, p, &c. whole positive numbers; here the powers X^m, Y^n, Z^p, &c. being expanded by the Binomial Polynomial Theorems will be transformed into a finite series, the terms of which have the form of ax^m, and the product of any number of them will, therefore, be thus reducible. Thus, for example,

$$\int (a + bx)^2 dx = a^2x + abx^2 + \frac{1}{3}b^2x^3$$

$$\int (ax + bx^2)^3 dx = \frac{1}{4}a^3x^4 + \frac{3}{5}a^2bx^5 + \frac{1}{2}ab^2x^6 + \frac{1}{7}b^3x^7$$

$$\int\left(ax^2+\frac{b}{x^3}\right)^2 dx=\frac{1}{5}a^2x^5+2ab\log x-\frac{b^2}{5x^5}$$

$$\int\left(a+bx+\frac{c}{x}\right)^2 dx=a^2x+abx^2+2ac\log x+\frac{1}{3}b^2x^3+2bcx-\frac{c^2}{x}$$

$$\int(a^2+x^2)(a+x)^2x^3\,dx=\frac{1}{4}a^4x^4+\frac{2}{5}a^3x^5+\frac{1}{3}a^2x^6+\frac{2}{7}ax^7+\frac{1}{8}x^8$$

$$\int\frac{(a-x)^2(a+x)dx}{x^h}=-\frac{a^3}{(h-1)x^{h-1}}+\frac{a}{(h-3)x^{h-3}}+\frac{a^2}{(h-2)x^{h-2}}-\frac{1}{(h-4)x^{h-4}}$$

Since Integrals of this class are so easily found, no tables are necessary for that purpose.

SECOND CLASS OF INTEGRALS

To this class belong those Integrals which are comprehended by the general form $\int\frac{Ax^m+Bx^{m-1}+Cx^{m-2}+\&c.}{ax^k+bx^{k-1}+cx^{k-2}+\&c.}dx$, or when the numerator and denominator are divided by a, by

$$\frac{1}{a}\int\frac{A'x^m+B'x^{m-1}+C'x^{m-2}+\&c.}{x^k+b'x^{k-1}+c'x^{k-2}+\&c.}dx.$$

Since, when $m>k$, or $m=k$, by actual division we can decompose the above form into a whole function Y, and a fractional one $\frac{U}{V}$ in which the highest exponent in U is $<$ than that in V (or k), m may be supposed $<k$. Thus, $\int Y\,dx$ being easily found, it is necessary merely to find $\int\frac{U}{V}dx$. The method of effecting this is shown above, and also in all elementary books. As, however, the calculations for each integral would be exceedingly troublesome, it has been deemed useful to construct tables containing the formulæ, reduced to the most simple terms, for all integrals which are likely to occur in practice. The more general forms, together with the Formulæ of Reduction, are placed at the end, because no better place could be assigned them.

TABLES

OF

FORMULÆ OF REDUCTION FOR THE INTEGRAL

$$\int x^{m-1} dx \, (a + bx^n + cx^{2n} + dx^{3n} + \&c.)^p.$$

TABLE I.

Formulæ of Reduction for the Integral

$$\int x^{m-1}\,\mathrm{d}x\,(a+bx^n)^p$$

$$a+bx^n=X$$

I.

$$\int x^{m-1}\,\mathrm{d}x X^p=\frac{x^m X^p}{m}-\frac{pnb}{m}\int x^{m+n-1}\,\mathrm{d}x X^{p-1}$$

II.

$$\int x^{m-1}\,\mathrm{d}x X^p=\frac{x^{m-n} X^{p+1}}{(p+1)nb}-\frac{m-n}{(p+1)nb}\int x^{m-n-1}\,\mathrm{d}x X^{p+1}$$

III.

$$\int x^{m-1}\,\mathrm{d}x X^p=\frac{x^{m-n} X^{p+1}}{(m+np)\,b}-\frac{(m-n)\,a}{(m+np)\,b}\int x^{m-n-1}\,\mathrm{d}x X^p$$

IV.

$$\int x^{m-1}\,\mathrm{d}x X^p=\frac{x^m X^p}{m+np}+\frac{pna}{m+np}\int x^{m-1}\,\mathrm{d}x X^{p-1}$$

V.

$$\int x^{m-1}\,\mathrm{d}x X^p=\frac{x^m X^{p+1}}{ma}-\frac{(m+n+np)\,b}{ma}\int x^{m+n-1}\,\mathrm{d}x X^p$$

VI.

$$\int x^{m-1}\,\mathrm{d}x X^p=-\frac{x^m X^{p+1}}{(p+1)\,na}+\frac{m+n+np}{(p+1)\,na}\int x^{m-1}\,\mathrm{d}x X^{p+1}$$

TABLE I.

Formulæ of Reduction for the Integral

$$\int x^{m-1}\,dx\,(a+bx^n)^p$$

$$a + bx^n = X$$

VII.

$$\int x^{m-1}\,dx X^p =$$

$$\left\{\begin{matrix} Ax^{m-n} - Bx^{m-2n} + Cx^{m-3n} - Dx^{m-4n} + \&c. \\ \pm Kx^{m-(i-1)n} \mp Lx^{m-in} \end{matrix}\right\} X^{p+1}$$

$$\pm L\,(m-in)\,a \int x^{m-in-1}\,dx X^p$$

$$A = \frac{1}{(m+np)\,b},\quad B = \frac{(m-n)\,a}{(m-n+np)\,b}\,A,\quad C = \frac{(m-2n)\,a}{(m-2n+np)\,b}\,B.$$

$$D = \frac{(m-3n)\,a}{(m-3n+np)\,b}\,C,\quad E = \frac{(m-4n)\,a}{(m-4n+np)\,b}\,D, \&c.$$

$$L = \frac{[m-(i-1)\,n]\,a}{[m-(i-1)\,n+np]\,b}\,K$$

VIII.

$$\int x^{m-1}\,dx X^p =$$

$$\left\{\begin{matrix} AX^p + BX^{p-1} + CX^{p-2} + DX^{p-3} + EX^{p-4} + \&c. \\ + KX^{p-i+2} + LX^{p-i+1} \end{matrix}\right\} x^m$$

$$+ L\,(p-i+1)\,na \int x^{m-1}\,dx X^{p-i}$$

$$A = \frac{1}{m+np},\quad B = \frac{pna}{m-n+np}\,A,\quad C = \frac{(p-1)\,na}{m-2n+np}\,B,$$

$$D = \frac{(p-2)\,na}{m-3n+np}\,C,\quad E = \frac{(p-3)\,na}{m-4n+np}\,D, \&c.$$

$$L = \frac{(p-i+2)\,na}{m-(i-1)\,n+np}\,K.$$

TABLE I.

Formulæ of Reduction for the Integral

$$\int x^{m-1}\,dx\,(a+bx^n)^p$$

$$a+bx^n = X$$

IX.

$$\int x^{m-1}\,dx X^p =$$

$$\left\{ \begin{array}{c} Ax^m - Bx^{m+n} + Cx^{m+2n} - Dx^{m+3n} + Ex^{m+4n} - \&c. \\ \pm Kx^{m+(i-2)n} \mp Lx^{m+(i-1)n} \end{array} \right\} X^{p+1}$$

$$\pm L\,(m+in+np)\,b\int x^{m+in-1}\,dx X^p$$

$$A = \frac{1}{ma},\ B = \frac{(m+n+np)\,b}{(m+n)\,a}\,A,\ C = \frac{(m+2n+np)\,b}{(m+2n)\,a}\,B,$$

$$D = \frac{(m+3n+np)b}{(m+3n)\,a}\,C,\ E = \frac{(m+4n+np)\,b}{(m+4n)\,a}\,D,\ \&c.$$

$$L = \frac{[m+(i-1)\,n+np]\,b}{[m+(i-1)\,n]\,a}\,K.$$

X.

$$\int x^{m-1}\,dx X^p =$$

$$-\left\{ \begin{array}{c} AX^{p+1} + BX^{p+2} + CX^{p+3} + DX^{p+4} + EX^{p+5} + \&c. \\ +KX^{p+i-1} + LX^{p+i} \end{array} \right\} x^m$$

$$+L\,(m+in+np)\int x^{m-1}\,dx X^{p+i}$$

$$A = \frac{1}{(p+1)\,na},\ B = \frac{m+n+np}{(p+2)\,na}\,A,\ C = \frac{m+2n+np}{(p+3)\,na}\,B,$$

$$D = \frac{m+3n+np}{(p+4)\,na}\,C,\ E = \frac{m+4n+np}{(p+5)\,na}\,D,\ \&c.$$

$$L = \frac{m+(i-1)\,n+np}{(p+i)\,na}\,K.$$

TABLE II.

Formulæ of Reduction for the Integral

$$\int x^{m-1}\,\mathrm{d}x\,(a+bx^n+cx^{2n})^p$$

$$a + bx^n + cx^{2n} = X$$

I.

$$\int x^{m-1}\,\mathrm{d}x X^p =$$
$$\frac{x^m X^p}{m} - \frac{pnb}{m}\int x^{m+n-1}\,\mathrm{d}x X^{p-1}$$
$$- \frac{2pnc}{m}\int x^{m+2n-1}\,\mathrm{d}x X^{p-1}$$

II.

$$\int x^{m-1}\,\mathrm{d}x X^p =$$
$$\frac{x^{m-2n}X^{p+1}}{(m+2pn)\,c} - \frac{(m-2n)\,a}{(m+2pn)\,c}\int x^{m-2n-1}\,\mathrm{d}x X^p$$
$$- \frac{(m-n+pn)\,b}{(m+2pn)\,c}\int x^{m-n-1}\,\mathrm{d}x X^p$$

III.

$$\int x^{m-1}\,\mathrm{d}x X^p =$$
$$\frac{x^m X^p}{m+2pn} + \frac{2pna}{m+2pn}\int x^{m-1}\,\mathrm{d}x X^{p-1}$$
$$+ \frac{pnb}{m+2pn}\int x^{m+n-1}\,\mathrm{d}x X^{p-1}$$

TABLE II.

Formulæ of Reduction for the Integral

$$\int x^{m-1}\,dx\,(a+bx^n+cx^{2n})^p$$

$$a+bx^n+cx^{2n}=X$$

IV.

$$\int x^{m-1}\,dx X^p = \frac{x^m X^{p+1}}{ma} - \frac{(m+n+pn)\,b}{ma}\int x^{m+n-1}\,dx X^p - \frac{(m+2n+2pn)\,c}{ma}\int x^{m+2n-1}\,dx X^p$$

V.

$$\int x^{m-1}\,dx X^p = \frac{Ax^m + Bx^{m+n}}{K} X^{p+1} + \frac{1}{K}\int (Cx^{m-1} + Dx^{m+n-1})\,dx X^{p+1}$$

$A = 2ac - b^2$
$B = -bc$
$C = n(p+1)(b^2-4ac) - m(2ac-b^2)$
$D = (2pn+3n+m)\,bc$
$K = (p+1)(b^2-4ac)\,na$

TABLE III.

Formulæ of Reduction for the Integral

$$\int x^{m-1}\,dx\,(a+bx^n+cx^{2n}+dx^{3n})^p$$

$$a+bx^n+cx^{2n}+dx^{3n}=X$$

I.

$$\int x^{m-1}\,dx X^p =$$

$$\frac{x^m X^p}{m} - \frac{pnb}{m}\int x^{m+n-1}\,dx X^{p-1}$$

$$-\frac{2pnc}{m}\int x^{m+2n-1}\,dx X^{p-1} - \frac{3pnd}{m}\int x^{m+3n-1}\,dx X^{p-1}$$

II.

$$\int x^{m-1}\,dx X^p =$$

$$\frac{x^{m-3n}X^{p+1}}{(m+3pn)\,d} - \frac{(m-3n)\,a}{(m+3pn)\,d}\int x^{m-3n-1}\,dx X^p$$

$$-\frac{(m-2n+pn)\,b}{(m+3pn)\,d}\int x^{m-2n-1}\,dx X^p - \frac{(m-n+2pn)\,c}{(m+3pn)\,d}\int x^{m-n-1}\,dx X^p$$

III.

$$\int x^{m-1}\,dx X^p =$$

$$\frac{x^m X^p}{m+3pn} + \frac{3pna}{m+3pn}\int x^{m-1}\,dx X^{p-1}$$

$$+\frac{2pnb}{m+3pn}\int x^{m+n-1}\,dx X^{p-1} + \frac{pnc}{m+3pn}\int x^{m+2n-1}\,dx X^{p-1}$$

TABLE III.

Formulæ of Reduction for the Integral

$$\int x^{m-1}\,dx\,(a+bx^n+cx^{2n}+dx^{3n})^p$$

$$a+bx^n+cx^{2n}+dx^{3n}=X$$

IV.

$$\int x^{m-1}\,dx X^p =$$

$$\frac{x^m X^{p+1}}{ma}-\frac{(m+n+pn)\,b}{ma}\int x^{m+n-1}\,dx X^p$$

$$-\frac{(m+2n+2pn)\,c}{ma}\int x^{m+2n-1}\,dx X^p-\frac{(m+3n+3pn)\,d}{ma}\int x^{m+3n-1}\,dx X^p$$

V.

$$\int x^{m-1}\,dx X^p =$$

$$(Ax^m+Bx^{m+n}+Cx^{m+2n})\,X^{p+1}+$$

$$\int(Dx^{m-1}+Ex^{m+n-1}+Fx^{m+2n-1})\,X^{p+1}\,dx$$

the coefficients A, B, C, are given in the three equations

$$dA-3adB+acC=\frac{-bd}{(p+1)\,na}$$

$$(bc-3ad)\,A-2acB+2abC=\frac{ad-bc}{(p+1)\,na}$$

$$b^2-2ac)\,A-abB+3a^2C=\frac{ac-b^2}{(p+1)\,na}$$

and from those coefficients we obtain

$$D=\frac{1}{a}-mA$$

$$E=-\frac{(p+1)\,nb}{a}A-(m+n)\,B-\frac{b}{a^2}$$

$$F=-(m+5n+3pn)\,C.$$

TABLE IV.

Formulæ of Reduction for the Integral

$$\int x^{m-1}\mathrm{d}x\,(a+bx^n+cx^{2n}+\ldots\ldots+tx^{kn})^p$$

Abbreviation

$$a+bx^n+cx^{2n}+dx^{3n}+\&\text{c.}+sx^{(k-1)n}+tx^{kn}=X$$

$\int x^{m+n-1}\mathrm{d}xX^{p-1} = S'$	$\int x^{m-n-1}\mathrm{d}xX^p = S'_1$
$\int x^{m+2n-1}\mathrm{d}xX^{p-1} = S''$	$\int x^{m-2n-1}\mathrm{d}xX^p = S''_1$
$\int x^{m+3n-1}\mathrm{d}xX^{p-1} = S'''$	$\int x^{m-3n-1}\mathrm{d}xX^p = S'''_1$
$\int x^{m+4n-1}\mathrm{d}xX^{p-1} = S''''$	$\int x^{m-4n-1}\mathrm{d}xX^p = S''''_1$
. . . .	. . . .
$\int x^{m+kn-1}\mathrm{d}xX^{p+1} = S^{k'}$	$\int x^{m-kn-1}\mathrm{d}xX^p = S^{k'}_1$

$\int x^{m-1}\mathrm{d}xX^{p-1} = S_2$	$\int^{m+n-1}\mathrm{d}xX^p = S'_3$
$\int x^{m+n-1}\mathrm{d}xX^{p-1} = S'_2$	$\int x^{m+2n-1}\mathrm{d}xX^p = S''_3$
$\int x^{m+2n-1}\mathrm{d}xX^{p-1} = S''_2$	$\int x^{m+3n-1}\mathrm{d}xX^p = S'''_3$
$\int x^{m+3n-1}\mathrm{d}xX^{p-1} = S'''_2$	$\int x^{m+4n-1}\mathrm{d}xX^p = S''''_3$
. . . .	. . . .
$\int x^{m+kn-1}\mathrm{d}xX^{p-1} = S^{k'}_2$	$\int x^{m+kn-1}\mathrm{d}xX^p = S^{k'}_3$

TABLE IV.

Formulæ of Reduction for the Integral

$$\int x^{m-1}\,\mathrm{d}x\,(a+bx^{n}+cx^{2n})+\ldots\ldots+tx^{kn})^{p}$$

I.

$$m\int x^{m-1}\,\mathrm{d}x X^{p}=$$

$$x^{m}X^{p}-pnbS'-2pncS''-3pndS'''-4pneS''''$$
$$-5pnfS'''''-\text{\&c.}-kpntS^{k'}$$

II.

$$(m+kpn)\,t\int x^{m-1}\,\mathrm{d}x X^{p}=$$

$$x^{m-kn}X^{p+1}-(m-kn)\,aS_{1}^{k'}-[m-(k-1)\,n+pn]\,bS_{1}^{(k-1)'}$$
$$-[m-(k-2)\,n+2pn]\,cS_{1}^{(k-2)'}-[m-(k-3)\,n+3pn]\,dS_{1}^{(k-3)'}$$
$$-\text{\&c.}-[m-2n+(k-2)\,pn]\,sS_{1}''-[m-n+(k-1)\,pn]\,tS_{1}'.$$

III.

$$(m+kpn)\int x^{m-1}\,\mathrm{d}x X^{p}=$$

$$x^{m}X^{p}+kpnaS_{2}+(k-1)\,pnbS_{2}'+(k-2)\,pncS_{2}''$$
$$+(k-3)\,pndS_{2}'''+\text{\&c.}+pnsS_{2}^{(k-1)'}.$$

TABLE IV.

Formulæ of Reduction for the Integral

$$\int x^{m-1}\,\mathrm{d}x\,(a+bx^n+cx^{2n}+\dots\dots+tx^{kn})^p$$

IV.

$$ma\int x^{m-1}\,\mathrm{d}x X^p =$$

$$x^m X^{p+1} - (m+n+pn)\,bS_1 - (m+2n+2pn)\,cS_1'$$

$$-(m+3n+3pn)\,dS_1'' - (m+4n+4pn)\,eS_1'''$$

$$-\ \&c. - (m+kn+kpn)\,tS_1^{v}.$$

V.

$$\int x^{m-1}\,\mathrm{d}x X^p =$$

$$(Ax^m + Bx^{m+n} + Cx^{m+2n} + \&c. + Tx^{m+(k-1)})\,X^{p+1} +$$

$$\int(A'x^{m-1} + B'x^{m+n-1} + C'x^{m+2n-1} + \&c. + T'x^{m+(k-1)k-1})X^{p+1}\mathrm{d}x$$

From this equation and the differentials, the coefficients $A, B, C \dots\dots T, A', B', C', \dots\dots T'$ may be determined. Their general values admit of representation, but not with brevity.

TAB. I.

$$\int \frac{x^m dx}{a+bx}$$

$$a + bx = X$$

$$\int \frac{dx}{X} = \frac{1}{b} \log. X = \log. X^{\frac{1}{b}} \ *$$

$$\int \frac{xdx}{X} = \frac{x}{b} - \frac{a}{b^2} \log. X$$

$$\int \frac{x^2dx}{X} = \frac{x^2}{2b} - \frac{ax}{b^2} + \frac{a^2}{b^3} \log X$$

$$\int \frac{x^3dx}{X} = \frac{x^3}{3b} - \frac{ax^2}{2b^2} + \frac{a^2x}{b^3} - \frac{a^3}{b^4} \log. X$$

$$\int \frac{x^4dx}{X} = \frac{x^4}{4b} - \frac{ax^3}{3b^2} + \frac{a^2x^2}{2b^3} - \frac{a^3x}{b^4} + \frac{a^4}{b^5} \log. X$$

$$\int \frac{x^5dx}{X} = \frac{x^5}{5b} - \frac{ax^4}{4b^2} + \frac{a^2x^3}{3b^3} - \frac{a^3x^2}{2b^4} + \frac{a^4x}{b^5} - \frac{a^5}{b^6} \log. X$$

$$\int \frac{x^6dx}{X} = \frac{x^6}{6b} - \frac{ax^5}{5b^2} + \frac{a^2x^4}{4b^3} - \frac{a^3x^3}{3b^4} + \frac{a^4x^2}{2b^5} - \frac{a^5x}{b^6} + \frac{a^6}{b^7} \log X$$

$$\int \frac{x^7dx}{X} = \frac{x^7}{7b} - \frac{ax^6}{6b^2} + \frac{a^2x^5}{5b^3} - \frac{a^3x^4}{4b^4} + \frac{a^4x^3}{3b^5} - \frac{a^5x^2}{2b^6} + \frac{a^6x}{b^7} - \frac{a^7}{b^8} \log. X$$

$$\int \frac{x^8dx}{X} = \frac{x^8}{8b} - \frac{ax^7}{7b^2} + \frac{a^2x^6}{6b^3} - \frac{a^3x^5}{5b^4} + \frac{a^4x^4}{4b^5} - \frac{a^5x^3}{3b^6} + \frac{a^6x^2}{2b^7} - \frac{a^7x}{b^8} + \frac{a^8}{b^9} \log. X$$

$$\int \frac{x^9dx}{X} = \frac{x^9}{9b} - \frac{ax^8}{8b^2} + \frac{a^2x^7}{7b^3} - \frac{a^3x^6}{6b^4} + \frac{a^4x^5}{5b^5} - \frac{a^5x^4}{4b^6} + \frac{a^6x^3}{3b^7} - \frac{a^7x^2}{2b^8} + \frac{a^8x}{b^9} - \frac{a^9}{b^{10}} \log. X$$

* $\int \frac{dx}{X} = \frac{1}{b} \log. X + k = \frac{1}{b} \log. X + \frac{1}{b} \log. \mathrm{k} = \frac{1}{b} \log. \mathrm{k}X$
$= \log. \mathrm{k}^{\frac{1}{b}} X^{\frac{1}{b}} = \log. (\mathrm{k}X)^{\frac{1}{b}}$

TAB. II.

$$\int \frac{x^m dx}{(a+bx)^2}$$

$$a + bx = X$$

$$\int \frac{dx}{X^2} = -\frac{1}{bX}$$

$$\int \frac{x dx}{X^2} = \frac{a}{b^2 X} + \frac{1}{b^2} \log. X$$

$$\int \frac{x^2 dx}{X^2} = \left(\frac{x^2}{b} - \frac{2a^2}{b^3}\right)\frac{1}{X} - \frac{2a}{b^3} \log. X$$

$$\int \frac{x^3 dx}{X^2} = \left(\frac{x^3}{2b} - \frac{3ax^2}{2b^2} + \frac{3a^3}{b^4}\right)\frac{1}{X} + \frac{3a^2}{b^4} \log. X$$

$$\int \frac{x^4 dx}{X^2} = \left(\frac{x^4}{3b} - \frac{2ax^3}{3b^2} + \frac{2a^2x^2}{b^3} - \frac{4a^4}{b^5}\right)\frac{1}{X} - \frac{4a^3}{b^5} \log. X$$

$$\int \frac{x^5 dx}{X^2} = \left(\frac{x^5}{4b} - \frac{5ax^4}{12b^2} + \frac{5a^2x^3}{6b^3} - \frac{5a^3x^2}{2b^4} + \frac{5a^5}{b^6}\right)\frac{1}{X} + \frac{5a^4}{b^6} \log. X$$

$$\int \frac{x^6 dx}{X^2} = \left(\frac{x^6}{5b} - \frac{8ax^5}{10b^2} + \frac{a^2x^4}{2b^3} - \frac{a^3x^3}{b^4} + \frac{3a^4x^2}{b^5} + \frac{6a^6}{b^7}\right)\frac{1}{X} - \frac{6a^5}{b^7} \log. X$$

$$\int \frac{x^7 dx}{X^2} = \left(\frac{x^7}{6b} - \frac{7ax^6}{30b^2} + \frac{7a^2x^5}{20b^3} - \frac{7a^3x^4}{12b^4} + \frac{7a^4x^3}{6b^5} - \frac{7a^5x^2}{2b^6} + \frac{7a^7}{b^8}\right)\frac{1}{X} + \frac{7a^6}{b^8} \log. X$$

$$\int \frac{x^8 dx}{X^2} = \left(\frac{x^8}{7b} - \frac{4ax^7}{21b^2} + \frac{4a^2x^6}{15b^3} - \frac{2a^3x^5}{5b^4} + \frac{2a^4x^4}{3b^5} - \frac{4a^5x^3}{3b^6} + \frac{4a^6x^2}{b^7} - \frac{8a^8}{b^9}\right)\frac{1}{X} - \frac{8a^7}{b^9} \log. X$$

$$\int \frac{x^9 dx}{X^2} = \left(\frac{x^9}{8b} - \frac{9ax^8}{56b^2} + \frac{3a^2x^7}{14b^3} - \frac{3a^3x^6}{10b^4} + \frac{9a^4x^5}{20b^5} - \frac{3a^5x^4}{4b^6} + \frac{3a^6x^3}{2b^7} - \frac{9a^7x^2}{2b^8} + \frac{9a^9}{b^{10}}\right)\frac{1}{X} + \frac{9a^8}{b^{10}} \log. X$$

TAB. III.

$$\int \frac{x^m dx}{(a+bx)^3}.$$

$$a + bx = X$$

$$\int \frac{dx}{X^3} = -\frac{1}{2bX^2}$$

$$\int \frac{x dx}{X^3} = -\left(\frac{x}{b} + \frac{a}{2b^2}\right)\frac{1}{X^2}$$

$$\int \frac{x^2 dx}{X^3} = \left(\frac{2ax}{b^2} + \frac{3a^2}{2b^3}\right)\frac{1}{X^2} + \frac{1}{b^3}\log X$$

$$\int \frac{x^3 dx}{X^3} = \left(\frac{x^2}{b} - \frac{6a^2x}{b^3} - \frac{9a^3}{2b^4}\right)\frac{1}{X^2} - \frac{3a}{b^4}\log X$$

$$\int \frac{x^4 dx}{X^3} = \left(\frac{x^4}{2b} - \frac{2ax^3}{b^2} + \frac{12a^3x}{b^4} + \frac{9a^4}{b^5}\right)\frac{1}{X^2} + \frac{6a^2}{b^5}\log X$$

$$\int \frac{x^5 dx}{X^3} = \left(\frac{x^5}{3b} - \frac{5ax^4}{6b^2} + \frac{10a^2x^3}{3b^3} - \frac{20a^4x}{b^5} - \frac{15a^5}{b^6}\right)\frac{1}{X^2} - \frac{10a^3}{b^6}\log X$$

$$\int \frac{x^6 dx}{X^3} = \left(\frac{x^6}{4b} - \frac{ax^5}{2b^2} + \frac{5a^2x^4}{4b^3} - \frac{5a^3x^3}{b^4} + \frac{30a^5x}{b^6} + \frac{45a^6}{2b^7}\right)\frac{1}{X^2}$$
$$+ \frac{15a^4}{b^7}\log X$$

$$\int \frac{x^7 dx}{X^3} = \left(\frac{x^7}{5b} - \frac{7ax^6}{20b^2} + \frac{7a^2x^5}{10b^3} - \frac{7a^3x^4}{4b^4} + \frac{7a^4x^3}{b^5} - \frac{42a^6x}{b^7}\right.$$
$$\left. - \frac{63a^7}{2b^8}\right)\frac{1}{X^2} - \frac{21a^5}{b^8}\log X$$

$$\int \frac{x^8 dx}{X^3} = \left(\frac{x^8}{6b} - \frac{4ax^7}{15b^2} + \frac{7a^2x^6}{15b^3} - \frac{14a^3x^5}{15b^4} + \frac{7a^4x^4}{3b^5} - \frac{28a^5x^3}{3b^6}\right.$$
$$\left. + \frac{56a^7x}{b^8} + \frac{42a^8}{b^9}\right)\frac{1}{X^2} + \frac{28a^6}{b^9}\log X$$

$$\int \frac{x^9 dx}{X^3} = \left(\frac{x^9}{7b} - \frac{3ax^8}{14b^2} + \frac{12a^2x^7}{35} - \frac{3a^3x^6}{5b^4} + \frac{6a^4x^5}{5b^5} - \frac{3a^5x^4}{b^6}\right.$$
$$\left. + \frac{12a^6x^3}{b^7} - \frac{72a^8x}{b^9} - \frac{54a^9}{b^{10}}\right)\frac{1}{X^2} - \frac{36a^7}{b^{10}}\log X$$

TAB. IV.

$$\int \frac{x^m dx}{(a+bx)^4}$$

$$a + bx = X$$

$$\int \frac{dx}{X^4} = -\frac{1}{3bX^3}$$

$$\int \frac{x dx}{X^4} = -\left(\frac{x}{2b} + \frac{a}{6b^2}\right)\frac{1}{X^3}$$

$$\int \frac{x^2 dx}{X^4} = -\left(\frac{x^2}{b} + \frac{ax}{b^2} + \frac{a^2}{3b^3}\right)\frac{1}{X^3}$$

$$\int \frac{x^3 dx}{X^4} = \left(\frac{3ax^2}{b^2} + \frac{9a^2x}{2b^3} + \frac{11a^3}{6b^4}\right)\frac{1}{X^3} + \frac{1}{b^4}\log. X$$

$$\int \frac{x^4 dx}{X^4} = \left(\frac{x^4}{b} - \frac{12a^2x^2}{b^3} - \frac{18a^3x}{b^4} - \frac{22a^4}{3b^5}\right)\frac{1}{X^3} - \frac{4a}{b^5}\log. X$$

$$\int \frac{x^5 dx}{X^4} = \left(\frac{x^5}{2b} - \frac{5ax^4}{2b^2} + \frac{30a^3x^2}{b^4} + \frac{45a^4x}{b^5} + \frac{55a^5}{3b^6}\right)\frac{1}{X^3} + \frac{10a^2}{b^6}\log. X$$

$$\int \frac{x^6 dx}{X^4} = \left(\frac{x^6}{3b} - \frac{ax^5}{b^2} + \frac{5a^2x^4}{b^3} - \frac{63a^4x^2}{b^5} - \frac{90a^5x}{b^6} - \frac{110a^6}{3b^7}\right)\frac{1}{X^3} - \frac{20a^3}{b^7}\log. X$$

$$\int \frac{x^7 dx}{X^4} = \left(\frac{x^7}{4b} - \frac{7ax^6}{12b^2} + \frac{7a^2x^5}{4b^3} - \frac{35a^3x^4}{4b^4} + \frac{105a^5x^2}{b^6} + \frac{315a^6x}{2b^7} + \frac{385a^7}{6b^8}\right)\frac{1}{X^3} + \frac{35a^4}{b^8}\log. X$$

$$\int \frac{x^8 dx}{X^4} = \left(\frac{x^8}{5b} - \frac{2ax^7}{5b^2} + \frac{14a^2x^6}{15b^3} - \frac{14a^3x^5}{5b^4} + \frac{14a^4x^4}{b^5} - \frac{168a^6x^2}{b^7} - \frac{252a^7x}{b^8} - \frac{308a^8}{3b^9}\right)\frac{1}{X^3} - \frac{56a^5}{b^9}\log. X$$

TAB. V.

$$\int \frac{x^m dx}{(a+bx)^5}$$

$$a + bx = X$$

$$\int \frac{dx}{X^5} = -\frac{1}{4bX^4}$$

$$\int \frac{x dx}{X^5} = -\left(\frac{x}{3b} + \frac{a}{12b^2}\right)\frac{1}{X^4}$$

$$\int \frac{x^2 dx}{X^5} = -\left(\frac{x^2}{2b} + \frac{ax}{3b^2} + \frac{a^2}{12b^3}\right)\frac{1}{X^4}$$

$$\int \frac{x^3 dx}{X^5} = -\left(\frac{x^3}{b} + \frac{3ax^2}{2b^2} + \frac{a^2x}{b^3} + \frac{a^3}{4b^4}\right)\frac{1}{X^4}$$

$$\int \frac{x^4 dx}{X^5} = \left(\frac{4ax^3}{b^2} + \frac{9a^2x^2}{b^3} + \frac{22a^3x}{3b^4} + \frac{25a^4}{12b^5}\right)\frac{1}{X^4} + \frac{1}{b^5}\log. X$$

$$\int \frac{x^5 dx}{X^5} = \left(\frac{x^5}{b} - \frac{20a^2x^3}{b^3} - \frac{45a^3x^2}{b^4} - \frac{110a^4x}{3b^5} - \frac{125a^5}{12b^6}\right)\frac{1}{X^4} - \frac{5a}{b^6}\log. X$$

$$\int \frac{x^6 dx}{X^5} = \left(\frac{x^6}{2b} - \frac{3ax^5}{b^2} + \frac{60a^3x^3}{b^4} + \frac{135a^4x^2}{b^5} + \frac{110a^5x}{b^6} + \frac{125a^6}{4b^7}\right)\frac{1}{X^4} + \frac{5a^2}{b^7}\log. X$$

$$\int \frac{x^7 dx}{X^5} = \left(\frac{x^7}{3b} - \frac{7ax^6}{6b^2} + \frac{7a^2x^5}{b^3} - \frac{140a^4x^3}{b^5} - \frac{315a^5x^2}{b^6} - \frac{770a^6x}{3b^7} - \frac{875a^7}{12b^8}\right)\frac{1}{X^4} - \frac{35a^3}{b^8}\log. X$$

$$\int \frac{x^8 dx}{X^5} = \left(\frac{x^8}{4b} - \frac{2ax^7}{3b^2} + \frac{7a^2x^6}{3b^3} - \frac{14a^3x^5}{b^4} + \frac{280a^5x^3}{b^6} + \frac{630a^6x^2}{b^7} + \frac{1540a^7x}{3b^8} + \frac{875a^8}{6b^9}\right)\frac{1}{X^4} + \frac{70a^4}{b^9}\log. X$$

TAB. VI.

$$\int \frac{x^m dx}{(a+bx)^6}$$

$$a + bx = X$$

$$\int \frac{dx}{X^6} = -\frac{1}{5bX^5}$$

$$\int \frac{x dx}{X^6} = -\left(\frac{x}{4b} + \frac{a}{20b^2}\right)\frac{1}{X^5}$$

$$\int \frac{x^2 dx}{X^6} = -\left(\frac{x^2}{3b} + \frac{ax}{6b^2} + \frac{a^2}{30b^3}\right)\frac{1}{X^5}$$

$$\int \frac{x^3 dx}{X^6} = -\left(\frac{x^3}{2b} + \frac{ax^2}{2b^2} + \frac{a^2x}{4b^3} + \frac{a^3}{20b^4}\right)\frac{1}{X^5}$$

$$\int \frac{x^4 dx}{X^6} = -\left(\frac{x^4}{b} + \frac{2ax^3}{b^2} + \frac{2a^2x^2}{b^3} + \frac{a^3x}{b^4} + \frac{a^4}{5b^5}\right)\frac{1}{X^5}$$

$$\int \frac{x^5 dx}{X^6} = \left(\frac{5ax^4}{b^2} + \frac{15a^2x^3}{b^3} + \frac{55a^3x^2}{3b^4} + \frac{125a^4x}{12b^5} + \frac{137a^5}{60b^6}\right)\frac{1}{X^5} + \frac{1}{b^6}\log. X$$

$$\int \frac{x^6 dx}{X^6} = \left(\frac{x^6}{b} - \frac{30a^2x^4}{b^3} - \frac{90a^3x^3}{b^4} - \frac{110a^4x^2}{b^5} - \frac{125a^5x}{2b^6} - \frac{137a^6}{10b^7}\right)\frac{1}{X^5} - \frac{6a}{b^7}\log. X$$

$$\int \frac{x^7 dx}{X^6} = \left(\frac{x^7}{2b} - \frac{7ax^6}{2b^2} + \frac{105a^3x^4}{b^4} + \frac{315a^4x^3}{b^5} + \frac{385a^5x^2}{b^6} + \frac{875a^6x}{4b^7} + \frac{959a^7}{20b^8}\right)\frac{1}{X^5} + \frac{21a^2}{b^8}\log. X$$

$$\int \frac{x^8 dx}{X^6} = \left(\frac{x^8}{3b} - \frac{4ax^7}{3b^2} + \frac{28a^2x^6}{3b^3} - \frac{280a^4x^4}{b^5} - \frac{840a^5x^3}{b^6} - \frac{3080a^6x^2}{3b^7} - \frac{1750a^7x}{3b^8} - \frac{1918a^8}{15b^9}\right)\frac{1}{X^5} - \frac{56a^3}{b^9}\log. X$$

TAB. VII.

$$\int \frac{dx}{x^m(a+bx)}$$

$$a + bx = X$$

$$\int \frac{dx}{xX} = \frac{1}{a}\log.\frac{x}{X} = -\frac{1}{a}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^2X} = -\frac{1}{ax} + \frac{b}{a^2}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^3X} = -\frac{1}{2ax^2} + \frac{b}{a^2x} - \frac{b^2}{a^3}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^4X} = -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2}{a^3x} + \frac{b^3}{a^4}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^5X} = -\frac{1}{4ax^4} + \frac{b}{3a^2x^3} - \frac{b^2}{2a^3x^2} + \frac{b^3}{a^4x} - \frac{b^4}{a^5}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^6X} = -\frac{1}{5ax^5} + \frac{b}{4a^2x^4} - \frac{b^2}{3a^3x^3} + \frac{b^3}{2a^4x^2} - \frac{b^4}{a^5x} + \frac{b^5}{a^6}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^7X} = -\frac{1}{6ax^6} + \frac{b}{5a^2x^5} - \frac{b^2}{4a^3x^4} + \frac{b^3}{3a^4x^3} - \frac{b^4}{2a^5x^2} + \frac{b^5}{a^6x} - \frac{b^6}{a^7}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^8X} = -\frac{1}{7ax^7} + \frac{b}{6a^2x^6} - \frac{b^2}{5a^3x^5} + \frac{b^3}{4a^4x^4} - \frac{b^4}{3a^5x^3} + \frac{b^5}{2a^6x^2} - \frac{b^6}{a^7x} + \frac{b^7}{a^8}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^9X} = -\frac{1}{8ax^8} + \frac{b}{7a^2x^7} - \frac{b^2}{6a^3x^6} + \frac{b^3}{5a^4x^5} - \frac{b^4}{4a^5x^4} + \frac{b^5}{3a^6x^3} - \frac{b^6}{2a^7x^2} + \frac{b^7}{a^8x} - \frac{b^8}{a^9}\log.\frac{X}{x}$$

$$\log.\frac{x}{X} = \log.\frac{1}{\left(\frac{X}{x}\right)} = -\log.\frac{X}{x}$$

TAB. VIII.

$$\int \frac{dx}{x^m(a+bx)^2}$$

$$a + bx = X$$

$$\int \frac{dx}{xX^2} = \frac{1}{aX} - \frac{1}{a^2}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^2X^2} = \left(-\frac{1}{ax} - \frac{2b}{a^2}\right)\frac{1}{X} + \frac{2b}{a^3}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^3X^2} = \left(-\frac{1}{2ax^2} + \frac{3b}{2a^2x} + \frac{3b^2}{a^3}\right)\frac{1}{X} - \frac{3b^2}{a^4}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^4X^2} = \left(-\frac{1}{3ax^3} + \frac{2b}{3a^2x^2} - \frac{2b^2}{a^3x} - \frac{4b^3}{a^4}\right)\frac{1}{X} + \frac{4b^3}{a^5}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^5X^2} = \left(-\frac{1}{4ax^4} + \frac{5b}{12a^2x^3} - \frac{5b^2}{6a^3x^2} + \frac{5b^3}{2a^4x} + \frac{5b^4}{a^5}\right)\frac{1}{X}$$
$$- \frac{5b^4}{a^6}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^6X^2} = \left(-\frac{1}{5ax^5} + \frac{3b}{10a^2x^4} - \frac{b^2}{2a^3x^3} + \frac{b^3}{a^4x^2} - \frac{3b^4}{a^5x}\right.$$
$$\left. - \frac{6b^5}{a^6}\right)\frac{1}{X} + \frac{6b^5}{a^7}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^7X^2} = \left(-\frac{1}{6ax^6} + \frac{7b}{30a^2x^5} - \frac{7b^2}{20a^3x^4} + \frac{7b^3}{12a^4x^3} - \frac{7b^4}{6a^5x^2}\right.$$
$$\left. + \frac{7b^5}{2a^6x} + \frac{7b^6}{a^7}\right)\frac{1}{X} - \frac{7b^6}{a^8}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^8X^2} = \left(-\frac{1}{7ax^7} + \frac{4b}{21a^2x^6} - \frac{4b^2}{15a^3x^5} + \frac{2b^3}{5a^4x^4} - \frac{2b^4}{3a^5x^3}\right.$$
$$\left. + \frac{4b^5}{3a^6x^2} - \frac{4b^6}{a^7x} - \frac{8b^7}{a^8}\right)\frac{1}{X} + \frac{8b^7}{a^9}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^9X^2} = \left(-\frac{1}{8ax^8} + \frac{9b}{56a^2x^7} - \frac{3b^2}{14a^3x^5} + \frac{3b^3}{10a^4x^5} - \frac{9b^4}{20a^5x^4}\right.$$
$$\left. + \frac{3b^5}{4a^6x^3} - \frac{3b^6}{2a^7x^2} + \frac{9b^7}{2a^8x} + \frac{9b^8}{a^9}\right)\frac{1}{X} - \frac{9b^8}{a^{10}}\log.\frac{X}{x}$$

TAB. IX.

$$\int \frac{dx}{x^m (a+bx)^3}$$

$$a + bx = X$$

$$\int \frac{dx}{xX^3} = \left(\frac{3}{2a} + \frac{bx}{a^2}\right)\frac{1}{X^2} - \frac{1}{a^3}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^2X^3} = \left(-\frac{1}{ax} - \frac{9b}{2a^2} - \frac{8b^2x}{a^3}\right)\frac{1}{X^2} + \frac{3b}{a^4}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^3X^3} = \left(-\frac{1}{2ax^2} + \frac{2b}{a^2x} + \frac{9b^2}{a^3} + \frac{6b^3x}{a^4}\right)\frac{1}{X^2} - \frac{6b^2}{a^5}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^4X^3} = \left(-\frac{1}{3ax^3} + \frac{5b}{6a^2x^2} - \frac{10b^2}{3a^3x} - \frac{15b^3}{a^4} - \frac{10b^4x}{a^5}\right)\frac{1}{X^2} + \frac{10b^3}{a^7}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^5X^3} = \left(-\frac{1}{4ax^4} + \frac{b}{2a^2x^3} - \frac{5b^2}{4a^3x^2} + \frac{5b^3}{a^4x} + \frac{45b^4}{2a^5} + \frac{15b^5x}{a^6}\right)\frac{1}{X^2} - \frac{15b^4}{a^7}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^6X^3} = \left(-\frac{1}{5ax^5} + \frac{7b}{20a^2x^4} - \frac{7b^2}{10a^3x^3} + \frac{7b^3}{4a^4x^2} - \frac{7b^4}{a^5x} - \frac{63b^5}{2a^6} - \frac{21b^6x}{a^7}\right)\frac{1}{X^2} + \frac{21b^5}{a^8}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^7X^3} = \left(-\frac{1}{6ax^6} + \frac{4b}{15a^2x^5} - \frac{7b^2}{15a^3x^4} + \frac{14b^3}{15a^4x^3} - \frac{7b^4}{3a^5x^2} + \frac{28b^5}{3a^6x} + \frac{42b^6}{a^7} + \frac{28b^7x}{a^8}\right)\frac{1}{X^2} - \frac{28b^6}{a^9}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^8X^3} = \left(-\frac{1}{7ax^7} + \frac{3b}{14a^2x^6} - \frac{12b^2}{35a^3x^5} + \frac{3b^3}{5a^4x^4} - \frac{6b^4}{5a^5x^3} + \frac{3b^5}{a^6x^2} - \frac{12b^6}{a^7x} - \frac{54b^7}{a^8} - \frac{36b^8x}{a^9}\right)\frac{1}{X^2} + \frac{36b^7}{a^{10}}\log.\frac{X}{x}$$

TAB. XII.

$$\int \frac{dx}{x^m(a+bx)^6}$$

$$a + bx = X$$

$$\int \frac{dx}{xX^6} = \left(\frac{137}{60a} + \frac{77bx}{12a^2} + \frac{47b^2x^2}{6a^3} + \frac{9b^3x^3}{2a^4} + \frac{b^4x^4}{a^5}\right)\frac{1}{X^5} - \frac{1}{a^6}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^2X^6} = \left(-\frac{1}{ax} - \frac{137b}{10a^2} - \frac{77b^2x}{2a^3} - \frac{47b^3x^2}{a^4} - \frac{27b^4x^3}{a^5} - \frac{6b^5x^4}{a^6}\right)\frac{1}{X^5} + \frac{6b}{a^7}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^3X^6} = \left(-\frac{1}{2ax^2} + \frac{7b}{2a^2x} + \frac{959b^2}{20a^3} + \frac{539b^3x}{4a^4} + \frac{329b^4x^2}{2a^5} + \frac{189b^5x^3}{a^6} + \frac{21b^6x^4}{a^7}\right)\frac{1}{X^5} - \frac{21b^2}{a^8}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^4X^6} = \left(-\frac{1}{3ax^3} + \frac{4b}{3a^2x^2} - \frac{28b^2}{3a^3x} - \frac{1918b^3}{15a^4} - \frac{1078b^4x}{3a^5} - \frac{1316b^5x^2}{3a^6} - \frac{504b^6x^3}{a^7} - \frac{56b^7x^4}{a^8}\right)\frac{1}{X^5} + \frac{56b^3}{a^{10}}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^5X^6} = \left(-\frac{1}{4ax^4} + \frac{3b}{4a^2x^3} - \frac{3b^2}{a^3x^2} + \frac{21b^3}{a^4x} + \frac{2877b^4}{10a^5} + \frac{1617b^5x}{2a^6} + \frac{987b^6x^2}{a^7} + \frac{1134b^7x^3}{a^8} + \frac{126b^8x^4}{a^9}\right)\frac{1}{X^5} - \frac{126b^4}{a^{10}}\log.\frac{X}{x}$$

$$\int \frac{dx}{x^6X^6} = \left(-\frac{1}{5ax^5} + \frac{b}{2a^2x^4} - \frac{3b^2}{2a^3x^3} + \frac{6b^3}{a^4x^2} - \frac{42b^4}{a^5x} - \frac{2877b^5}{5a^6} - \frac{1617b^6x}{a^7} - \frac{1974b^7x^2}{a^8} - \frac{2268b^8x^3}{a^9} - \frac{252b^9x^4}{a^{10}}\right)\frac{1}{X^5} + \frac{252b^5}{a^{11}}\log.\frac{X}{x}$$

TAB. XIII.

$$\int\frac{dx}{(a+bx^2)^n}$$

$$a+bx^2=X$$

$$\int\frac{dx}{X}=\int\frac{dx}{X}\ \text{[see the following page.]}$$

$$\int\frac{dx}{X^2}=\frac{x}{2aX}+\frac{1}{2a}\int\frac{dx}{X}$$

$$\int\frac{dx}{X^3}=\left(\frac{1}{4aX^2}+\frac{3}{8a^2X}\right)x+\frac{3}{8a^2}\int\frac{dx}{X}$$

$$\int\frac{dx}{X^4}=\left(\frac{1}{6aX^3}+\frac{5}{24a^2X^2}+\frac{1}{16a^3X}\right)x+\frac{5}{16a^3}\int\frac{dx}{X}$$

$$\int\frac{dx}{X^5}=\left(\frac{1}{8aX^4}+\frac{7}{48a^2X^3}+\frac{35}{192a^3X^2}+\frac{35}{128a^4X}\right)x+\frac{35}{128a^4}\int\frac{dx}{X}$$

$$\int\frac{dx}{X^6}=\left(\frac{1}{10aX^5}+\frac{9}{80a^2X^4}+\frac{21}{160a^3X^3}+\frac{21}{128a^4X^2}+\frac{63}{256a^5X}\right)x+\frac{63}{256a^5}\int\frac{dx}{X}$$

$$\int\frac{dx}{X^7}=\left(\frac{1}{12aX^6}+\frac{11}{120a^2X^5}+\frac{33}{320a^3X^4}+\frac{77}{640a^4X^3}+\frac{77}{512a^5X^2}+\frac{231}{1024a^6X}\right)x+\frac{231}{1024a^6}\int\frac{dx}{X}$$

$$\int\frac{dx}{X^8}=\left(\frac{1}{14aX^7}+\frac{13}{168a^2X^6}+\frac{143}{1680a^3X^5}+\frac{429}{4480a^4X^4}+\frac{143}{1280a^5X^3}+\frac{143}{1024a^6X^2}+\frac{429}{2048a^7X}\right)x+\frac{429}{2048a^7}\int\frac{dx}{X}$$

$$\int\frac{dx}{X^9}=\left(\frac{1}{16aX^8}+\frac{15}{224a^2X^7}+\frac{65}{896a^3X^6}+\frac{143}{1792a^4X^5}+\frac{1287}{14336a^5X^4}+\frac{429}{4096a^6X^3}+\frac{2145}{16384a^7X^2}+\frac{6435}{32768a^8X}\right)x+\frac{6435}{32768a^8}\int\frac{dx}{X}$$

Note on the preceding Table.

In general, whether a and b be positive or negative, we have

$$\int \frac{dx}{a+bx^2} = \frac{1}{\sqrt{ab}} \text{ arc tang.} x\sqrt{\frac{b}{a}} = \frac{1}{2\sqrt{-ab}} \log. \frac{\sqrt{a}+x\sqrt{-b}}{\sqrt{a}-x\sqrt{-b}},$$

and, of these two expressions, that is always used which appears in a real form. Hence we obtain

$$\int \frac{dx}{a+bx^2} = \frac{1}{\sqrt{ab}} \text{ arc tang.} x\sqrt{\frac{b}{a}} = \frac{1}{\sqrt{ab}} \text{ arc sin. } \sqrt{\frac{bx^2}{a+bx^2}}$$

$$= \frac{1}{2\sqrt{ab}} \text{ arc sin. } \frac{2x\sqrt{ab}}{a+bx^2} = \frac{1}{\sqrt{ab}} \text{ arc cos. } \sqrt{\frac{a}{a+bx^2}}$$

$$= \frac{1}{2\sqrt{ab}} \text{ arc cos. } \frac{a-bx^2}{a+bx^2} = \frac{1}{\sqrt{ab}} \text{ arc cot. } \frac{\sqrt{a}}{x\sqrt{b}}$$

$$= \frac{1}{\sqrt{ab}} \text{ arc sec. } \sqrt{\frac{a+bx^2}{a}} = \frac{1}{2\sqrt{ab}} \text{ arc sec. } \frac{a+bx^2}{a-bx^2}$$

$$= \frac{1}{\sqrt{ab}} \text{ arc cosec. } \sqrt{\frac{a+bx^2}{bx^2}} = \frac{1}{2\sqrt{ab}} \text{ arc cosec. } \frac{a+bx^2}{2x\sqrt{ab}}$$

$$= \frac{1}{2\sqrt{ab}} \text{ arc sin. v. } \frac{2bx^2}{a+bx^2}$$

$$\int \frac{dx}{a-bx^2} = \frac{1}{2\sqrt{ab}} \log. \frac{\sqrt{a}+x\sqrt{b}}{\sqrt{a}-x\sqrt{b}} = \frac{1}{\sqrt{ab}} \log. \frac{\sqrt{a}+x\sqrt{b}}{\sqrt{(a-bx^2)}}$$

$$= -\frac{1}{2\sqrt{ab}} \log. \frac{\sqrt{a}-x\sqrt{b}}{\sqrt{a}+x\sqrt{b}} = -\frac{1}{\sqrt{ab}} \log. \frac{\sqrt{a}-x\sqrt{b}}{\sqrt{(a-bx^2)}}$$

$$\int \frac{dx}{-a+bx^2} = -\int \frac{dx}{a-bx^2}, \quad \int \frac{dx}{-a-bx^2} = -\int \frac{dx}{a+bx^2}.$$

In a particular case, we have

$$\int \frac{dx}{1+x^2} = \text{arc tang. } x = \text{arc sin. } \frac{x}{\sqrt{(1+x^2)}} = \tfrac{1}{2} \text{ arc sin. } \frac{2x}{1+x^2}$$

$$= \text{arc cos. } \frac{1}{\sqrt{(1+x^2)}} = \tfrac{1}{2} \text{ arc cos. } \frac{1-x^2}{1+x^2} = \text{arc cot. } \frac{1}{x}$$

$$= \text{arc sec. } \sqrt{(1+x^2)} = \tfrac{1}{2} \text{ arc sec. } \frac{1+x^2}{1-x^2} = \text{arc cosec. } \frac{\sqrt{(1+x^2)}}{x}$$

$$= \tfrac{1}{2} \text{ arc cosec. } \frac{1+x^2}{2x} = \tfrac{1}{2} \text{ arc sin. v. } \frac{2x^2}{1+x^2}$$

$$\int \frac{dx}{1-x^2} = \tfrac{1}{2} \log. \frac{1+x}{1-x} = -\tfrac{1}{2} \log. \frac{1-x}{1+x}.$$

In all these formulæ, the integral vanishes when $x = 0$. If it vanish when $x = h$, we have

$$\int \frac{dx}{a+bx^2} = \frac{1}{\sqrt{ab}} \text{arctang} \frac{(x-h)\sqrt{ab}}{a+bhx} = \frac{1}{\sqrt{ab}} \text{arc cos.} \frac{a+bhx}{\sqrt{(a+bh^2)(a+bx^2)}}$$

&c.

TAB. XIV.

$$\int \frac{x^m dx}{a + bx^2}$$

$$a + bx = X$$

$$\int \frac{dx}{X} = \int \frac{dx}{X}$$ [see p. 24.]

$$\int \frac{xdx}{X} = \frac{1}{2b} \log. X$$

$$\int \frac{x^2dx}{X} = \frac{x}{b} - \frac{a}{b}\int \frac{dx}{X}$$

$$\int \frac{x^3dx}{X} = \frac{x^2}{2b} - \frac{a}{b}\int \frac{xdx}{X}$$

$$\int \frac{x^4dx}{X} = \frac{x^3}{3b} - \frac{ax}{b^2} + \frac{a^2}{b^2}\int \frac{dx}{X}$$

$$\int \frac{x^5dx}{X} = \frac{x^4}{4b} - \frac{ax^2}{2b^2} + \frac{a^2}{b^2}\int \frac{xdx}{X}$$

$$\int \frac{x^6dx}{X} = \frac{x^5}{5b} - \frac{ax^3}{3b^2} + \frac{a^2x}{b^3} - \frac{a^3}{b^3}\int \frac{dx}{X}$$

$$\int \frac{x^7dx}{X} = \frac{x^6}{6b} - \frac{ax^4}{4b^2} + \frac{a^2x^2}{2b^3} - \frac{a^3}{b^3}\int \frac{xdx}{X}$$

$$\int \frac{x^8dx}{X} = \frac{x^7}{7b} - \frac{ax^5}{5b^2} + \frac{a^2x^3}{3b^3} - \frac{a^3x}{b^4} + \frac{a^4}{b^4}\int \frac{dx}{X}$$

$$\int \frac{x^9dx}{X} = \frac{x^8}{8b} - \frac{ax^6}{6b^2} + \frac{a^2x^4}{4b^3} - \frac{a^3x^2}{2b^4} + \frac{a^4}{b^4}\int \frac{dx}{X}$$

$$\int \frac{x^{10}dx}{X} = \frac{x^9}{9b} - \frac{ax^7}{7b^2} + \frac{a^2x^5}{5b^3} - \frac{a^3x^3}{3b^4} + \frac{a^4x}{b^5} - \frac{a^5}{b^5}\int \frac{dx}{X}$$

$$\int \frac{x^{11}dx}{X} = \frac{x^{10}}{10b} - \frac{ax^8}{8b^2} + \frac{a^2x^6}{6b^3} - \frac{a^3x^4}{4b^4} + \frac{a^4x^2}{2b^5} - \frac{a^5}{b^5}\int \frac{xdx}{X}$$

$$\int \frac{x^{12}dx}{X} = \frac{x^{11}}{11b} - \frac{ax^9}{9b^2} + \frac{a^2x^7}{7b^3} - \frac{a^3x^5}{5b^4} + \frac{a^4x^3}{3b^5} - \frac{a^5x}{b^6} + \frac{a^6}{b^6}\int \frac{dx}{X}$$

TAB. XV.

$$\int \frac{x^m dx}{(a+bx^2)^2}$$

$$a + bx^2 = X$$

$$\int \frac{dx}{X^2} = \frac{x}{2aX} + \frac{1}{2a} \int \frac{dx}{X}$$

$$\int \frac{x dx}{X^2} = -\frac{1}{2bX}$$

$$\int \frac{x^2 dx}{X^2} = -\frac{x}{2bX} + \frac{1}{2b} \int \frac{dx}{X}$$

$$\int \frac{x^3 dx}{X^2} = \frac{a}{2b^2 X} + \frac{1}{2b^2} \log. X$$

$$\int \frac{x^4 dx}{X^2} = \left(\frac{x^3}{b} + \frac{3ax}{2b^2}\right) \frac{1}{X} - \frac{3a}{2b^2} \int \frac{dx}{X}$$

$$\int \frac{x^5 dx}{X^2} = \left(\frac{x^4}{2b} - \frac{a^2}{b^3}\right) \frac{1}{X} - \frac{a}{b^3} \log. X$$

$$\int \frac{x^6 dx}{X^2} = \left(\frac{x^5}{3b} - \frac{5ax^3}{3b^2} - \frac{5a^2x}{2b^3}\right) \frac{1}{X} + \frac{5a^2}{2b^3} \int \frac{dx}{X}$$

$$\int \frac{x^7 dx}{X^2} = \left(\frac{x^6}{4b} - \frac{3ax^4}{4b^2} + \frac{3a^3}{2b^4}\right) \frac{1}{X} + \frac{3a^2}{2b^4} \log. X$$

$$\int \frac{x^8 dx}{X^2} = \left(\frac{x^7}{5b} - \frac{7ax^5}{15b^2} + \frac{7a^2x^3}{3b^3} + \frac{7a^3x}{2b^4}\right) \frac{1}{X} - \frac{7a^3}{2b^4} \int \frac{dx}{X}$$

$$\int \frac{x^9 dx}{X^2} = \left(\frac{x^8}{6b} - \frac{ax^6}{3b^2} + \frac{a^2x^4}{b^3} - \frac{2a^4}{b^5}\right) \frac{1}{X} - \frac{2a^3}{b^5} \log. X$$

$$\int \frac{x^{10} dx}{X^2} = \left(\frac{x^9}{7b} - \frac{9ax^7}{35b^2} + \frac{3a^2x^5}{5b^3} - \frac{3a^3x^3}{b^4} - \frac{9a^4x}{2b^5}\right) \frac{1}{X} + \frac{9a^4}{2b^5} \int \frac{dx}{X}$$

$$\int \frac{x^{11} dx}{X^2} = \left(\frac{x^{10}}{8b} - \frac{5ax^8}{24b^2} + \frac{5a^2x^6}{12b^3} - \frac{5a^3x^4}{4b^4} + \frac{5a5}{2b^5}\right) \frac{1}{X} + \frac{5a^4}{2b^6} \log. X$$

TAB. XVI.

$$\int \frac{x^m dx}{(a+bx^2)^3}$$

$$a + bx^2 = X$$

$$\int \frac{dx}{X^3} = \left(\frac{3bx^3}{8a^2} + \frac{5x}{8a}\right)\frac{1}{X^2} + \frac{3}{8a^2}\int\frac{dx}{X}$$

$$\int \frac{x dx}{X^3} = -\frac{1}{4bX^2}$$

$$\int \frac{x^2 dx}{X^3} = \left(\frac{x^3}{8a} - \frac{x}{8b}\right)\frac{1}{X^2} + \frac{1}{8ab}\int\frac{dx}{X}$$

$$\int \frac{x^3 dx}{X^3} = \left(-\frac{x^2}{2b} - \frac{a}{4b^2}\right)\frac{1}{X^2}$$

$$\int \frac{x^4 dx}{X^3} = \left(-\frac{5x^3}{8b} - \frac{3ax}{8b^2}\right)\frac{1}{X^2} + \frac{3}{8b^2}\int\frac{dx}{X}$$

$$\int \frac{x^5 dx}{X^3} = \left(\frac{ax^2}{b^2} + \frac{3a^2}{4b^3}\right)\frac{1}{X^2} + \frac{1}{2b^3}\log. X$$

$$\int \frac{x^6 dx}{X^3} = \left(\frac{x^5}{b} + \frac{25ax^3}{8b^2} + \frac{15a^2x}{8b^3}\right)\frac{1}{X^2} - \frac{15a}{8b^3}\int\frac{dx}{X}$$

$$\int \frac{x^7 dx}{X^3} = \left(\frac{x^6}{2b} - \frac{3a^2x^2}{b^3} - \frac{9a^3}{4b^4}\right)\frac{1}{X^2} - \frac{3a}{2b^4}\log. X$$

$$\int \frac{x^8 dx}{X^3} = \left(\frac{x^7}{3b} - \frac{7ax^5}{3b^2} - \frac{175a^2x^3}{24b^3} - \frac{35a^3x}{8b^4}\right)\frac{1}{X^2} + \frac{35a^2}{8b^4}\int\frac{dx}{X}$$

$$\int \frac{x^9 dx}{X^3} = \left(\frac{x^8}{4b} - \frac{ax^6}{b^2} + \frac{6a^3x^2}{b^4} + \frac{9a^4}{2b^5}\right)\frac{1}{X^2} + \frac{3a^3}{b^5}\log. X$$

$$\int \frac{x^{10} dx}{X^3} = \left(\frac{x^9}{5b} - \frac{3ax^7}{5b^2} + \frac{21a^2x^5}{5b^3} + \frac{105a^3x^3}{8b^4} + \frac{63a^4x}{8b^5}\right)\frac{1}{X^2} - \frac{63a^5}{8b^5}\int\frac{dx}{X}$$

TAB. XVII.

$$\int \frac{x^m dx}{(a+bx^2)^4}$$

$$a + bx^2 = X$$

$$\int \frac{dx}{X^4} = \left(\frac{5b^2x^5}{16a^3} + \frac{5bx^3}{6a^2} + \frac{11x}{16a}\right)\frac{1}{X^3} + \frac{5}{16a^3}\int\frac{dx}{X}$$

$$\int \frac{xdx}{X^4} = -\frac{1}{6bX^3}$$

$$\int \frac{x^2dx}{X^4} = \left(\frac{bx^5}{16a^2} + \frac{x^3}{6a} - \frac{x}{16b}\right)\frac{1}{X^3} + \frac{1}{16a^2b}\int\frac{dx}{X}$$

$$\int \frac{x^3dx}{X^4} = \left(-\frac{x^2}{4b} - \frac{a}{12b^2}\right)\frac{1}{X^3}$$

$$\int \frac{x^4dx}{X^4} = \left(\frac{x^5}{16a} - \frac{x^3}{6b} - \frac{ax}{16b^2}\right)\frac{1}{X^3} + \frac{1}{16ab^2}\int\frac{dx}{X}$$

$$\int \frac{x^5dx}{X^4} = \left(-\frac{x^4}{2b} - \frac{ax^2}{2b^2} - \frac{a^2}{6b^3}\right)\frac{1}{X^3}$$

$$\int \frac{x^6dx}{X^4} = \left(-\frac{11x^5}{16b} - \frac{5ax^3}{6b^2} - \frac{5a^2x}{16b^3}\right)\frac{1}{X^3} + \frac{5}{16b^3}\int\frac{dx}{X}$$

$$\int \frac{x^7dx}{X^4} = \left(\frac{3ax^4}{2b^2} + \frac{9a^2x^2}{4b^3} + \frac{11a^3}{12b^4}\right)\frac{1}{X^3} + \frac{1}{2b^4}\log. X$$

$$\int \frac{x^8dx}{X^4} = \left(\frac{x^7}{5b} + \frac{77ax^5}{16b^2} + \frac{35a^2x^3}{6b^3} + \frac{35a^3x}{16b^4}\right)\frac{1}{X^3} - \frac{35a}{16b^4}\int\frac{dx}{X}$$

$$\int \frac{x^9dx}{X^4} = \left(\frac{x^8}{2b} - \frac{6a^2x^4}{b^3} - \frac{9a^3x^2}{b^4} - \frac{11a^4}{3b^5}\right)\frac{1}{X^3} - \frac{2a}{b^5}\log. X$$

$$\int \frac{x^{10}dx}{X^4} = \left(\frac{x^9}{3b} - \frac{3ax^7}{b^2} - \frac{231a^2x^5}{16b^3} - \frac{35a^3x^3}{2b^4} - \frac{105a^4x}{16b^5}\right)\frac{1}{X^3} + \frac{105a^2}{16b^5}\int\frac{dx}{X}$$

$$\int \frac{x^{11}dx}{X^4} = \left(\frac{x^{10}}{4b} - \frac{5ax^8}{4b^2} + \frac{15a^3x^4}{b^4} + \frac{45a^4x^2}{2b^5} + \frac{55a^5}{6b^6}\right)\frac{1}{X^3} + \frac{5a^2}{b^6}\log. X$$

TAB. XVIII.

$$\int \frac{x^m dx}{(a+bx^2)^5}$$

$$a + bx^2 = X$$

$$\int \frac{dx}{X^5} = \left(\frac{35b^3x^7}{128a^4} + \frac{385b^2x^5}{384a^3} + \frac{511bx^3}{384a^2} + \frac{93x}{128a}\right)\frac{1}{X^4} + \frac{35}{128a^4}\int\frac{dx}{X}$$

$$\int \frac{xdx}{X^5} = -\frac{1}{8bX^4}$$

$$\int \frac{x^2dx}{X^5} = \left(\frac{5b^2x^7}{128a^3} + \frac{55bx^5}{384a^2} + \frac{73x^3}{384a} - \frac{5x}{128b}\right)\frac{1}{X^4} + \frac{5}{128a^3b}\int\frac{dx}{X}$$

$$\int \frac{x^3dx}{X^5} = \left(-\frac{x^2}{6b} - \frac{a}{24b^2}\right)\frac{1}{X^4}$$

$$\int \frac{x^4dx}{X^5} = \left(\frac{3bx^7}{128a^2} + \frac{11x^5}{128a} - \frac{11x^3}{128b} - \frac{3ax}{128b^2}\right)\frac{1}{X^4} + \frac{3}{128a^2b^2}\int\frac{dx}{X}$$

$$\int \frac{x^5dx}{X^5} = \left(-\frac{x^4}{4b} - \frac{ax^2}{6b^2} - \frac{a^2}{24b^3}\right)\frac{1}{X^4}$$

$$\int \frac{x^6dx}{X^5} = \left(\frac{5x^7}{128a} - \frac{73x^5}{384b} - \frac{55ax^3}{384b^2} - \frac{5a^2x}{128b^3}\right)\frac{1}{X^4} + \frac{5}{128ab^3}\int\frac{dx}{X}$$

$$\int \frac{x^7dx}{X^5} = \left(-\frac{x^6}{2b} - \frac{3ax^4}{4b^2} - \frac{a^2x^2}{2b^3} - \frac{a^3}{8b^4}\right)\frac{1}{X^4}$$

$$\int \frac{x^8dx}{X^5} = \left(-\frac{93x^7}{128b} - \frac{511ax^5}{384b^2} - \frac{385a^2x^3}{384b^3} - \frac{35a^3x}{128b^4}\right)\frac{1}{X^4} + \frac{35}{128b^4}\int\frac{dx}{X}$$

$$\int \frac{x^9dx}{X^5} = \left(\frac{2ax^6}{b^2} + \frac{9a^2x^4}{2b^3} + \frac{11a^3x^2}{3b^4} + \frac{25a^4}{24b^5}\right)\frac{1}{X^4} + \frac{1}{2b^5}\log. X$$

$$\int \frac{x^{10}dx}{X^5} = \left(\frac{x^9}{b} + \frac{837ax^7}{128b^2} + \frac{1533a^2x^5}{128b^3} + \frac{1155a^3x^3}{128b^4} + \frac{315a^4x}{128b^5}\right)\frac{1}{X^4}$$
$$- \frac{315a}{128b^5}\int\frac{dx}{X}$$

$$\int \frac{x^{11}dx}{X^5} = \left(\frac{x^{10}}{2b} - \frac{10a^2x^6}{b^3} - \frac{45a^3x^4}{2b^4} - \frac{55a^4x^2}{3b^5} - \frac{125a^5}{24b^6}\right)\frac{1}{X^4}$$
$$- \frac{5a}{2b^6}\log. X$$

TAB. XIX.

$$\int \frac{x^m dx}{(a+bx^2)^6}$$

$$a + bx^2 = X$$

$$\int \frac{dx}{X^6} = \left(\frac{63b^4x^9}{256a^5} + \frac{147b^3x^7}{128a^4} + \frac{21b^2x^5}{10a^3} + \frac{237bx^3}{128a^2} + \frac{193x}{256a}\right)\frac{1}{X^5} + \frac{63}{256a^5}\int\frac{dx}{X}$$

$$\int \frac{x\,dx}{X^6} = -\frac{1}{10bX^5}$$

$$\int \frac{x^2dx}{X^6} = \left(\frac{7b^3x^9}{256a^4} + \frac{49b^2x^7}{384a^3} + \frac{7bx^5}{30a^2} + \frac{79x^3}{384a} - \frac{7x}{256b}\right)\frac{1}{X^5} + \frac{7}{256a^4b}\int\frac{dx}{X}$$

$$\int \frac{x^3dx}{X^6} = \left(-\frac{x^2}{8b} - \frac{a}{40b^2}\right)\frac{1}{X^5}$$

$$\int \frac{x^4dx}{X^6} = \left(\frac{3b^2x^9}{256a^3} + \frac{7bx^7}{128a^2} + \frac{x^5}{10a} - \frac{7x^3}{128b} - \frac{3ax}{256b^2}\right)\frac{1}{X^5} + \frac{3}{256a^3b^2}\int\frac{dx}{X}$$

$$\int \frac{x^5dx}{X^6} = \left(-\frac{x^4}{6b} - \frac{ax^2}{12b^2} - \frac{a^2}{60b^3}\right)\frac{1}{X^5}$$

$$\int \frac{x^6dx}{X^6} = \left(\frac{3bx^9}{256a^2} + \frac{7x^7}{128a} - \frac{x^5}{10b} - \frac{7ax^3}{128b^2} - \frac{3a^2x}{256b^3}\right)\frac{1}{X^5} + \frac{3}{256a^2b^3}\int\frac{dx}{X}$$

$$\int \frac{x^7dx}{X^6} = \left(-\frac{x^6}{4b} - \frac{ax^4}{4b^2} - \frac{a^2x^2}{8b^3} - \frac{a^3}{40b^4}\right)\frac{1}{X^5}$$

$$\int \frac{x^8dx}{X^6} = \left(\frac{7x^9}{256a} - \frac{79x^7}{384b} - \frac{7ax^5}{30b^2} - \frac{49a^2x^3}{384b^3} - \frac{7a^3x}{256b^4}\right)\frac{1}{X^5} + \frac{7}{256ab^4}\int\frac{dx}{X}$$

$$\int \frac{x^9dx}{X^6} = \left(-\frac{x^8}{2b} - \frac{ax^6}{b^2} - \frac{a^2x^4}{b^3} - \frac{a^3x^2}{2b^4} - \frac{a^4}{10b^5}\right)\frac{1}{X^5}$$

TAB. XX

$$\int \frac{dx}{x^m(a+bx^2)}$$

$$a + bx^2 = X$$

$$\int \frac{dx}{xX} = \frac{1}{2a}\log.\frac{x^2}{X} = \frac{1}{a}\log.\frac{x}{\sqrt{X}} = -\frac{1}{2a}\log.\frac{X}{x^2} = -\frac{1}{a}\log.\frac{\sqrt{X}}{x}$$

$$\int \frac{dx}{x^2X} = -\frac{1}{ax} - \frac{b}{a}\int\frac{dx}{X}$$

$$\int \frac{dx}{x^3X} = -\frac{1}{2ax^2} - \frac{b}{a}\int\frac{dx}{xX}$$

$$\int \frac{dx}{x^4X} = -\frac{1}{3ax^3} + \frac{b}{a^2x} + \frac{b^2}{a^2}\int\frac{dx}{X}$$

$$\int \frac{dx}{x^5X} = -\frac{1}{4ax^4} + \frac{b}{2a^2x^2} + \frac{b^2}{a^2}\int\frac{dx}{xX}$$

$$\int \frac{dx}{x^6X} = -\frac{1}{5ax^5} + \frac{b}{3a^2x^3} - \frac{b^2}{a^3x} - \frac{b^3}{a^3}\int\frac{dx}{X}$$

$$\int \frac{dx}{x^7X} = -\frac{1}{6ax^6} + \frac{b}{4a^2x^4} - \frac{b^2}{2a^3x^2} - \frac{b^3}{a^3}\int\frac{dx}{xX}$$

$$\int \frac{dx}{x^8X} = -\frac{1}{7ax^7} + \frac{b}{5a^2x^5} - \frac{b^2}{3a^3x^3} + \frac{b^3}{a^4x} + \frac{b^4}{a^4}\int\frac{dx}{X}$$

$$\int \frac{dx}{x^9X} = -\frac{1}{8ax^8} + \frac{b}{6a^2x^6} - \frac{b^2}{4a^3x^4} + \frac{b^3}{2a^4x^2} + \frac{b^4}{a^4}\int\frac{dx}{xX}$$

$$\int \frac{dx}{x^{10}X} = -\frac{1}{9ax^9} + \frac{b}{7a^2x^7} - \frac{b^2}{5a^3x^5} + \frac{b^3}{3a^4x^3} - \frac{b^5}{a^5x} - \frac{b^5}{a^5}\int\frac{dx}{X}$$

$$\int \frac{dx}{x^{11}X} = -\frac{1}{10ax^{10}} + \frac{b}{8a^2x^8} - \frac{b^2}{6a^3x^6} + \frac{b^3}{4a^4x^4} - \frac{b^4}{2a^5x^2} - \frac{b^5}{a^5}\int\frac{dx}{xX}$$

$$\int \frac{dx}{x^{12}X} = -\frac{1}{11ax^{11}} + \frac{b}{9ax^9} - \frac{b^2}{7a^3x^7} + \frac{b^3}{5a^4x^5} - \frac{b^4}{3a^5x^3} + \frac{b^6}{a^6x} + \frac{b^6}{a^6}\int\frac{dx}{X}$$

TAB. XXI.

$$\int \frac{dx}{x^m(a+bx^2)^2}$$

$$a + bx^2 = X$$

$$\int \frac{dx}{xX^2} = \frac{1}{2aX} + \frac{1}{a}\int \frac{dx}{xX}$$

$$\int \frac{dx}{x^2X^2} = \left(-\frac{1}{ax} - \frac{3bx}{2a^2}\right)\frac{1}{X} - \frac{3b}{2a^2}\int \frac{dx}{X}$$

$$\int \frac{dx}{x^3X^2} = \left(-\frac{1}{2ax^2} - \frac{b}{a^2}\right)\frac{1}{X} - \frac{2b}{a^2}\int \frac{dx}{xX}$$

$$\int \frac{dx}{x^4X^2} = \left(-\frac{1}{3ax^3} + \frac{5b}{3a^2x} + \frac{5b^2x}{2a^3}\right)\frac{1}{X} + \frac{5b^2}{2a^3}\int \frac{dx}{X}$$

$$\int \frac{dx}{x^5X^2} = \left(-\frac{1}{4ax^4} + \frac{3b}{4a^2x^2} + \frac{3b^2}{2a^3}\right)\frac{1}{X} + \frac{3b^2}{a^3}\int \frac{dx}{xX}$$

$$\int \frac{dx}{x^6X^2} = \left(-\frac{1}{5ax^5} + \frac{7b}{15a^2x^3} - \frac{7b^2}{3a^3x} - \frac{7b^3x}{2a^4}\right)\frac{1}{X} - \frac{7b^3}{2a^4}\int \frac{dx}{X}$$

$$\int \frac{dx}{x^7X^2} = \left(-\frac{1}{6ax^6} + \frac{b}{3a^2x^4} - \frac{b^2}{a^3x^2} - \frac{2b^3}{a^4}\right)\frac{1}{X} - \frac{4b^3}{a^4}\int \frac{dx}{xX}$$

$$\int \frac{dx}{x^8X^2} = \left(-\frac{1}{7ax^7} + \frac{9b}{35a^2x^5} - \frac{3b^2}{5a^3x^3} + \frac{3b^3}{a^4x} + \frac{9b^4x}{2a^5}\right)\frac{1}{X} + \frac{9b^4}{2a^5}\int \frac{dx}{X}$$

$$\int \frac{dx}{x^9X^2} = \left(-\frac{1}{8ax^8} + \frac{5b}{24a^2x^6} - \frac{5b^2}{12a^3x^4} + \frac{5b^3}{4a^4x^2} + \frac{5b^3}{2a^5}\right)\frac{1}{X} + \frac{5b^4}{a^5}\int \frac{dx}{xX}$$

$$\int \frac{dx}{x^{10}X^2} = \left(-\frac{1}{9ax^9} + \frac{11b}{63a^2x^7} - \frac{11b^2}{35a^3x^5} + \frac{11b^3}{15a^4x^3} - \frac{11b^4}{3a^5x} - \frac{11b^5x}{2a^6}\right)\frac{1}{X} - \frac{11b^5}{2a^6}\int \frac{dx}{X}$$

$$\int \frac{dx}{x^{11}X^2} = \left(-\frac{1}{10ax^{10}} + \frac{3b}{20a^2x^8} - \frac{b^2}{4a^3x^6} + \frac{b^3}{2a^4x^4} - \frac{3b^4}{2a^5x^2} - \frac{3b^5}{a^6}\right)\frac{1}{X} - \frac{6b^5}{a^6}\int \frac{dx}{xX}$$

TAB. XXII.

$$\int \frac{dx}{x^m(a+bx^2)^3}$$

$$a + bx^2 = X$$

$$\int \frac{\mathrm{d}x}{xX^3} = \left(\frac{3}{4a} + \frac{bx^2}{2a^2}\right)\frac{1}{X^2} + \frac{1}{a^3}\int\frac{\mathrm{d}x}{xX}$$

$$\int \frac{\mathrm{d}x}{x^2X^3} = \left(-\frac{1}{ax} - \frac{25bx}{8a^2} - \frac{15b^2x^3}{8a^3}\right)\frac{1}{X^2} - \frac{15b}{8a^3}\int\frac{\mathrm{d}x}{X}$$

$$\int \frac{\mathrm{d}x}{x^3X^3} = \left(-\frac{1}{2ax^2} - \frac{9b}{4a^2} - \frac{3b^2x^2}{2a^3}\right)\frac{1}{X^2} - \frac{3b}{a^3}\int\frac{\mathrm{d}x}{xX}$$

$$\int \frac{\mathrm{d}x}{x^4X^3} = \left(-\frac{1}{3ax^3} + \frac{7b}{3a^2x} + \frac{175b^2x}{24a^3} + \frac{35b^3x^3}{8a^4}\right)\frac{1}{X^2} + \frac{35b^2}{8a^4}\int\frac{\mathrm{d}}{X}$$

$$\int \frac{\mathrm{d}x}{x^5X^3} = \left(-\frac{1}{4ax^4} + \frac{b}{a^2x^2} + \frac{9b^2}{2a^3} + \frac{3b^3x^2}{a^4}\right)\frac{1}{X^2} + \frac{6b^2}{a^4}\int\frac{dx}{xX}$$

$$\int \frac{\mathrm{d}x}{x^6X^3} = \left(-\frac{1}{5ax^4} + \frac{3b}{5a^2x^3} - \frac{21b^2}{5a^3x} - \frac{105b^3x}{8a^4} - \frac{63b^4x^3}{8a^5}\right)\frac{1}{X^2} - \frac{63b^3}{8a^5}\int\frac{\mathrm{d}x}{X}$$

$$\int \frac{\mathrm{d}x}{x^7X^3} = \left(-\frac{1}{6ax^6} + \frac{5b}{12a^2x^4} - \frac{5b^2}{3a^3x^2} - \frac{15b^3}{2a^4} - \frac{5b^4x^2}{a^5}\right)\frac{1}{X^2} - \frac{10b^3}{a^5}\int\frac{\mathrm{d}x}{xX}$$

$$\int \frac{\mathrm{d}x}{x^8X^3} = \left(-\frac{1}{7ax^7} + \frac{11b}{35x^2x^5} - \frac{33b^2}{35a^3x^3} + \frac{33b^3}{5a^4x} + \frac{165b^4x}{8a^5} + \frac{99b^5x^3}{8a^6}\right)\frac{1}{X^2} + \frac{99b^4}{8a^6}\int\frac{\mathrm{d}x}{X}$$

$$\int \frac{\mathrm{d}x}{x^9X^3} = \left(-\frac{1}{8ax^8} + \frac{b}{4a^2x^6} - \frac{5b^2}{8a^3x^4} + \frac{5b^3}{2a^4x^2} + \frac{45b^4}{4a^5} + \frac{15b^5x^2}{2a^6}\right)\frac{1}{X^2} + \frac{15b^4}{a^6}\int\frac{\mathrm{d}x}{xX}$$

$$\int \frac{\mathrm{d}x}{x^{10}X^3} = \left(-\frac{1}{9ax^9} + \frac{13b}{63a^2x^7} - \frac{143b^2}{315a^3x^5} + \frac{143b^3}{105a^4x^3} - \frac{143b^4}{15a^5x} - \frac{715b^5x}{24a^6} - \frac{143b^6x^3}{8a^7}\right)\frac{1}{X^2} - \frac{143b^5}{8a^7}\int\frac{\mathrm{d}x}{X}$$

TAB. XXIII.

$$\int \frac{dx}{x^m(a+bx^2)^4}$$

$$a + bx^2 = X$$

$$\int \frac{dx}{xX^4} = \left(\frac{11}{12a} + \frac{5bx^2}{4a^2} + \frac{b^2x^4}{2a^3}\right)\frac{1}{X^3} + \frac{1}{a^3}\int\frac{dx}{xX}$$

$$\int \frac{dx}{x^2X^4} = \left(-\frac{1}{ax} - \frac{77bx}{16a^2} - \frac{35b^2x^3}{6a^3} - \frac{35b^3x^5}{16a^4}\right)\frac{1}{X^3} - \frac{35b}{16a^4}\int\frac{dx}{X}$$

$$\int \frac{dx}{x^3X^4} = \left(-\frac{1}{2ax^2} - \frac{11b}{3a^2} - \frac{5b^2x^2}{a^3} - \frac{2b^3x^4}{a^4}\right)\frac{1}{X^3} - \frac{4b}{a^4}\int\frac{dx}{xX}$$

$$\int \frac{dx}{x^4X^4} = \left(-\frac{1}{3ax^3} + \frac{3b}{a^2x} + \frac{231b^2x}{16a^3} + \frac{35b^3x^3}{2a^4} + \frac{105b^4x^5}{16a^5}\right)\frac{1}{X^3} + \frac{105b^2}{16a^5}\int\frac{dx}{X}$$

$$\int \frac{dx}{x^5X^4} = \left(-\frac{1}{4ax^4} + \frac{5b}{4a^2x^2} + \frac{55b^2}{6a^3} + \frac{25b^3x^2}{2a^4} + \frac{5b^4x^4}{a^5}\right)\frac{1}{X^3} + \frac{10b^3}{a^5}\int\frac{dx}{xX}$$

$$\int \frac{dx}{x^6X^4} = \left(-\frac{1}{5ax^5} + \frac{11b}{15a^2x^3} - \frac{33b^2}{5a^3x} - \frac{2541b^3x}{80a^4} - \frac{77b^4x^3}{2a^5} - \frac{231b^5x^5}{16a^6}\right)\frac{1}{X^3} - \frac{231b^3}{16a^6}\int\frac{dx}{X}$$

$$\int \frac{dx}{x^7X^4} = \left(-\frac{1}{6ax^6} + \frac{b}{2a^2x^4} - \frac{5b^2}{2a^3x^2} - \frac{55b^3}{3a^4} - \frac{25b^4x^2}{a^5} - \frac{10b^5x^4}{a^6}\right)\frac{1}{X^3} - \frac{20b^3}{a^6}\int\frac{dx}{xX}$$

$$\int \frac{dx}{x^8X^4} = \left(-\frac{1}{7ax^7} + \frac{13b}{35a^2x^5} - \frac{143b^2}{105a^3x^3} + \frac{429b^3}{35a^4x} + \frac{4719b^4x}{80a^5} + \frac{143b^5x^3}{2a^6} + \frac{429b^6x^5}{16a^7}\right)\frac{1}{X^3} + \frac{429b^4}{16a^7}\int\frac{dx}{X}$$

$$\int \frac{dx}{x^9X^4} = \left(-\frac{1}{8ax^8} + \frac{7b}{24a^2x^6} - \frac{7b^2}{8a^3x^4} + \frac{35b^3}{8a^4x^2} + \frac{385b^4}{12a^5} + \frac{175b^5x^2}{4a^6} + \frac{35b^6x^4}{2a^7}\right)\frac{1}{X^3} + \frac{35b^4}{a^7}\int\frac{dx}{xX}$$

TAB. XXIV.

$$\int \frac{dx}{x^m(a+bx^2)^5}$$

$$a + bx^2 = X$$

$$\int \frac{dx}{xX^5} = \left(\frac{25}{24a} + \frac{13bx^2}{6a^2} + \frac{7b^2x^4}{4a^3} + \frac{b^3x^6}{2a^4}\right)\frac{1}{X^4} + \frac{1}{a^4}\int \frac{dx}{xX}$$

$$\int \frac{dx}{x^2X^5} = \left(-\frac{1}{ax} - \frac{837bx}{128a^2} - \frac{1533b^2x^3}{128a^3} - \frac{1155b^3x^5}{128a^4} - \frac{315b^4x^7}{128a^5}\right)\frac{1}{X^4} - \frac{315b}{128a^5}\int \frac{dx}{X}$$

$$\int \frac{dx}{x^3X^5} = \left(-\frac{1}{2ax^2} - \frac{125b}{24a^2} - \frac{65b^2x^2}{6a^3} - \frac{35b^3x^4}{4a^4} - \frac{5b^4x^6}{2a^5}\right)\frac{1}{X^4} - \frac{5b}{a}\int \frac{dx}{xX}$$

$$\int \frac{dx}{x^4X^5} = \left(-\frac{1}{3ax^3} + \frac{11b}{3a^2x} + \frac{3069b^2x}{128a^3} + \frac{5621b^3x^3}{128a^4} + \frac{4235b^4x^5}{128a^5} + \frac{1155b^5x^7}{128a^6}\right)\frac{1}{X^4} + \frac{1155b^2}{128a^6}\int \frac{dx}{X}$$

$$\int \frac{dx}{x^5X^5} = \left(-\frac{1}{4ax^4} + \frac{3b}{2a^2x^2} + \frac{125b^2}{8a^3} + \frac{65b^3x^2}{2a^4} + \frac{105b^4x^4}{4a^5} + \frac{15b^5x^6}{2a^6}\right)\frac{1}{X^4} + \frac{15b^3}{a^6}\int \frac{dx}{xX}$$

$$\int \frac{dx}{x^6X^5} = \left(-\frac{1}{5ax^5} + \frac{13b}{15a^2x^3} - \frac{143b^2}{15a^3x} - \frac{39897b^3x}{640a^4} - \frac{73073b^4x^3}{640a^5} - \frac{110111b^5x^5}{128a^6} - \frac{3003b^6x^7}{128a^7}\right)\frac{1}{X^4} - \frac{3003b^3}{128a^7}\int \frac{dx}{X}$$

$$\int \frac{dx}{x^7X^5} = \left(-\frac{1}{6ax^6} + \frac{7b}{12a^2x^4} - \frac{7b^2}{2a^3x^2} - \frac{875b^3}{24a^4} - \frac{455b^4x^2}{6a^5} - \frac{245b^5x^4}{4a^6} - \frac{35b^6x^6}{2a^7}\right)\frac{1}{X^4} - \frac{35b^3}{a^7}\int \frac{dx}{xX}$$

$$\int \frac{dx}{x^8X^5} = \left(-\frac{1}{7ax^7} + \frac{3b}{7a^2x^5} - \frac{13b^2}{7a^3x^3} + \frac{143b^3}{7a^4x} + \frac{119691b^4x}{896a^5} + \frac{31317b^5x^3}{128a^6} + \frac{23595b^6x^5}{128a^7} + \frac{6435b^7x^7}{128a^8}\right)\frac{1}{X^4} + \frac{6435b^4}{128a^8}\int \frac{dx}{X}$$

TAB. XXV.

$$\int \frac{dx}{x^m(a+bx^2)^6}$$

$$a + bx^2 = X$$

$$\int \frac{dx}{xX^6} = \left(\frac{137}{120a} + \frac{77bx^2}{24a^2} + \frac{47b^2x^4}{12a^3} + \frac{9b^3x^6}{4a^4} + \frac{b^4x^8}{2a^5}\right)\frac{1}{X^5} + \frac{1}{a^5}\int \frac{dx}{xX}$$

$$\int \frac{dx}{x^2X^6} = \left(-\frac{1}{ax} - \frac{2123bx}{256a^2} - \frac{2607b^2x^3}{128a^3} - \frac{231b^3x^5}{10a^4} - \frac{1617b^4x^7}{128a^5}\right.$$
$$\left. - \frac{693b^5x^9}{256a^6}\right)\frac{1}{X^5} - \frac{693b}{256a^6}\int \frac{dx}{X}$$

$$\int \frac{dx}{x^3X^6} = \left(-\frac{1}{2ax^2} - \frac{137b}{20a^2} - \frac{77b^2x^2}{4a^3} - \frac{47b^3x^4}{2a^4} - \frac{27b^4x^6}{2a^5}\right.$$
$$\left. - \frac{3b^5x^8}{a^6}\right)\frac{1}{X^5} - \frac{6b}{a^6}\int \frac{dx}{xX}$$

$$\int \frac{dx}{x^4X^6} = \left(-\frac{1}{3ax^3} + \frac{13b}{3a^2x} + \frac{27599b^2x}{768a^3} + \frac{11297b^3x^3}{128a^4} + \frac{1001b^4x^5}{10a^5}\right.$$
$$\left. + \frac{7007b^5x^7}{128a^6} + \frac{3003b^6x^9}{256a^7}\right)\frac{1}{X^5} + \frac{3003b^2}{256a^7}\int \frac{dx}{X}$$

$$\int \frac{dx}{x^5X^6} = \left(-\frac{1}{4ax^4} + \frac{7b}{4a^2x^2} + \frac{959b^2}{40a^3} + \frac{539b^3x^2}{8a^4} + \frac{329b^4x^4}{4a^5}\right.$$
$$\left. + \frac{189b^5x^6}{4a^6} + \frac{21b^6x^8}{2a^7}\right)\frac{1}{X^5} + \frac{21b^2}{a^7}\int \frac{dx}{xX}$$

$$\int \frac{dx}{x^6X^6} = \left(-\frac{1}{5ax^5} + \frac{b}{a^2x^3} - \frac{13}{a^3x} - \frac{27599b^3x}{256a^4} - \frac{33891b^4x^3}{128a^5}\right.$$
$$\left. - \frac{3003b^5x^5}{10a^6} - \frac{21021b^6x^7}{128a^7} - \frac{9009b^7x^9}{256a^8}\right)\frac{1}{X^5} - \frac{9009b^3}{256a^8}\int \frac{dx}{X}$$

$$\int \frac{dx}{x^7X^6} = \left(-\frac{1}{6ax^6} + \frac{2b}{3a^2x^4} - \frac{14b^2}{3a^3x^2} - \frac{959b^3}{15a^4} - \frac{539b^4x^2}{3a^5}\right.$$
$$\left. - \frac{658b^5x^4}{3a^6} - \frac{126b^6x^6}{a^7} - \frac{28b^7x^8}{a^8}\right)\frac{1}{X^5} - \frac{56b^3}{a^8}\int \frac{dx}{xX}$$

$$\int \frac{dx}{x^8X^6} = \left(-\frac{1}{7ax^7} + \frac{17b}{35a^2x^5} + \frac{17b^2}{7a^3x^3} + \frac{221b^3}{7a^4x} + \frac{469183b^4x}{1792a^5}\right.$$
$$\left. + \frac{576147b^5x^3}{896a^6} + \frac{7293b^6x^5}{10a^7} + \frac{51051b^7x^7}{128a^8} + \frac{21879b^8x^9}{256a^9}\right)\frac{1}{X^5}$$
$$+ \frac{21879b^4}{256a^9}\int \frac{dx}{X}$$

TAB. XXVI

$$\int \frac{dx}{(a+bx+cx^2)^n}$$

$$a + bx + cx^2 = X,\quad 4ac - b^2 = k$$

$$\int \frac{dx}{X} = \int \frac{dx}{X}$$ [see the following page.]

$$\int \frac{dx}{X^2} = \frac{2cx+b}{kX} + \frac{2c}{k}\int \frac{dx}{X}$$

$$\int \frac{dx}{X^3} = \left(\frac{1}{2kX^2} + \frac{3c}{k^2X}\right)(2cx+b) + \frac{6c^2}{k^2}\int \frac{dx}{X}$$

$$\int \frac{dx}{X^4} = \left(\frac{1}{3kX^3} + \frac{5c}{3k^2X^2} + \frac{10c^2}{k^3X}\right)(2cx+b) + \frac{20c^3}{k^3}\int \frac{dx}{X}$$

$$\int \frac{dx}{X^5} = \left(\frac{1}{4kX^4} + \frac{7c}{6k^2X^3} + \frac{35c^2}{6k^3X^2} + \frac{35c^3}{k^4X}\right)(2cx+b) + \frac{70c^4}{k^4X}\int \frac{dx}{X}$$

$$\int \frac{dx}{X^6} = \left(\frac{1}{5kX^5} + \frac{9c}{10k^2X^4} + \frac{21c^2}{5k^3X^3} + \frac{21c^3}{k^4X^2} + \frac{126c^4}{k^5X}\right)(2cx+b) + \frac{252c^5}{k^5}\int \frac{dx}{X}$$

$$\int \frac{dx}{X^7} = \left(\frac{1}{6kX^6} + \frac{11c}{15k^2X^5} + \frac{33c^2}{10k^3X^4} + \frac{77c^3}{5k^4X^3} + \frac{77c^4}{k^5X^2} + \frac{462c^5}{k^6X}\right) \times (2cx+b) + \frac{924c^6}{k^6}\int \frac{dx}{X}$$

$$\int \frac{dx}{X^8} = \left(\frac{1}{7kX^7} + \frac{13c}{21k^2X^6} + \frac{286c^2}{105k^3X^5} + \frac{858c^3}{70k^4X^4} + \frac{286c^4}{5k^5X^3} + \frac{286c^5}{k^6X^2} + \frac{1716c^6}{k^7X}\right)(2cx+b) + \frac{3432c^7}{k^7}\int \frac{dx}{X}$$

$$\int \frac{dx}{X^9} = \left(\frac{1}{8kX^8} + \frac{15c}{28k^2X^7} + \frac{65c^2}{28k^3X^6} + \frac{143c^3}{14k^4X^5} + \frac{1287c^4}{28k^5X^4} + \frac{429c^5}{2k^6X^3} + \frac{2145c^6}{2k^7X^2} + \frac{6435c^7}{k^8X}\right)(2cx+b) + \frac{13870c^8}{k^8}\int \frac{dx}{X}$$

Note on the preceding Table.

When X retains its signification in the preceding page, we have in general

$$\int \frac{dx}{X} = \frac{2}{\sqrt{(4ac - b^2)}} \text{ arc tang.} \frac{2cx+b}{\sqrt{(4ac-b^2)}}$$

$$= \frac{1}{\sqrt{(b^2 - 4ac)}} \log. \frac{2cx+b-\sqrt{(b^2-4ac)}}{2cx+b+\sqrt{(b^2-4ac)}}$$

The first form is real when $4ac-b^2$ is positive; the second is so when $4ac-b^2$ is negative. Hence there arises

I. $4ac - b^2$ *positive* $(4ac - b^2 = k.)$

$$\int \frac{dx}{X} = \frac{2}{\sqrt{k}} \text{ arc tang.} \frac{2cx+b}{\sqrt{k}} = \frac{2}{\sqrt{k}} \text{ arc cot.} \frac{\sqrt{k}}{2cx+b} = \frac{2}{\sqrt{k}} \text{ arc sec.} \frac{2\sqrt{cX}}{\sqrt{k}}$$

$$= \frac{2}{\sqrt{k}} \text{ arccosec.} \frac{2\sqrt{cX}}{2cx+b} = \frac{2}{\sqrt{k}} \text{ arc cos.} \frac{\sqrt{k}}{2\sqrt{cX}} = \frac{2}{\sqrt{k}} \text{ arc sin.} \frac{2cx+b}{2\sqrt{cX}}$$

$$= \frac{1}{\sqrt{k}} \text{ arc sin.} \frac{(2cx+b)\sqrt{k}}{2cX} = \frac{1}{\sqrt{k}} \text{ arc cos.} \left(\frac{k}{2cX} - 1\right)$$

$$= \frac{1}{\sqrt{k}} \text{ arc sin. vers.} \frac{(2cx+b)^2}{2cX}.$$

and when $\int \frac{dx}{X}$ vanishes by putting $x=0$,

$$\int \frac{dx}{X} = \frac{2}{\sqrt{k}} \text{arc tang.} \frac{x\sqrt{k}}{2a+bx} = \frac{2}{\sqrt{k}} \text{ arc cot.} \frac{2a+bx}{x\sqrt{k}} = \frac{2}{\sqrt{k}} \text{arc sec.} \frac{2\sqrt{aX}}{2a+bx}$$

$$= \frac{2}{\sqrt{k}} \text{ arc cosc.} \frac{2\sqrt{aX}}{x\sqrt{k}} = \frac{2}{\sqrt{k}} \text{ arc sin.} \frac{x\sqrt{k}}{2\sqrt{aX}} = \frac{2}{\sqrt{k}} \text{ arc cos.} \frac{2a+bx}{2\sqrt{aX}}$$

$$= \frac{1}{\sqrt{k}} \text{ arc sin.} \frac{(2ax+bx^2)\sqrt{k}}{2aX} = \frac{1}{\sqrt{k}} \text{ arc sin. vers } \frac{kx^2}{2aX}.$$

II. $4ac-b^2$ *negative* $(b^2-4ac = k')$.

$$\int \frac{dx}{X} = \frac{1}{\sqrt{k'}} \log. \frac{2cx+b-\sqrt{k'}}{2cx+b+\sqrt{k'}} = \frac{2}{\sqrt{k'}} \log. \frac{2cx+b-\sqrt{k'}}{2\sqrt{cX}}$$

and when the Integral vanishes by putting $x=0$

$$\int \frac{dx}{X} = \frac{1}{\sqrt{k'}} \log. \frac{(b+\sqrt{k'})(2cx+b-\sqrt{k'})}{(b-\sqrt{k'})(2cx+b+\sqrt{k'})}.$$

In both kinds of Integrals, $\sqrt{k}$ and $\sqrt{k'}$ may be taken either positive or negative.

TAB. XXVII.

$$\int \frac{x^m dx}{a + bx + cx^2}$$

$$a + bx + cx^2 = X$$

$$\int \frac{dx}{X} = \int \frac{dx}{X} \text{ [see the preceding page.]}$$

$$\int \frac{x dx}{X} = \frac{1}{2c} \log. X - \frac{b}{2c} \int \frac{dx}{X}$$

$$\int \frac{x^2 dx}{X} = \frac{x}{c} - \frac{b}{2c^2} \log. X + \left(\frac{b^2}{2c^2} - \frac{a}{c}\right) \int \frac{dx}{X}$$

$$\int \frac{x^3 dx}{X} = \frac{x^2}{2c} - \frac{bx}{c^2} + \left(\frac{b^2}{2c^3} - \frac{a}{2c^2}\right) \log. X - \left(\frac{b^3}{2c^3} - \frac{3ab}{2c^2}\right) \int \frac{dx}{X}$$

$$\int \frac{x^4 dx}{X} = \frac{x^3}{3c} - \frac{bx^2}{2c^2} + \left(\frac{b^2}{c^3} - \frac{a}{c^2}\right) x - \left(\frac{b^3}{2c^4} - \frac{ab}{c^3}\right) \log. X$$
$$+ \left(\frac{b^4}{2c^4} - \frac{2ab^2}{c^3} + \frac{a^2}{c^2}\right) \int \frac{dx}{X}$$

$$\int \frac{x^5 dx}{X} = \frac{x^4}{4c} - \frac{b}{c} \int \frac{x^4 dx}{X} - \frac{a}{c} \int \frac{x^3 dx}{X}$$

$$\int \frac{x^6 dx}{X} = \frac{x^5}{5c} - \frac{bx^4}{4c^2} + \left(\frac{b^2}{c^2} - \frac{a}{c}\right) \int \frac{x^4 dx}{X} + \frac{ab}{c^2} \int \frac{x^3 dx}{X}$$

$$\int \frac{x^7 dx}{X} = \frac{x^6}{6c} - \frac{bx^5}{5c^2} + \left(\frac{b^2}{4c^3} - \frac{a}{4c^2}\right) x^4 - \left(\frac{b^3}{c^3} - \frac{2ab}{c^2}\right) \int \frac{x^4 dx}{X}$$
$$- \left(\frac{ab^2}{c^3} - \frac{a^2}{c^2}\right) \int \frac{x^3 dx}{X}$$

$$\int \frac{x^8 dx}{X} = \frac{x^7}{7c} - \frac{b}{c} \int \frac{x^7 dx}{X} - \frac{a}{c} \int \frac{x^6 dx}{X}$$

$$\int \frac{x^9 dx}{X} = \frac{x^8}{8c} - \frac{bx^7}{7c^2} + \left(\frac{b^2}{c^2} - \frac{a}{c}\right) \int \frac{x^7 dx}{X} + \frac{ab}{c^2} \int \frac{x^6 dx}{X}$$

TAB. XXX.

$$\int \frac{x^m dx}{(a + bx + cx^2)^4}$$

$$a + bx + cx^2 = X, \quad 4ac - b^2 = k.$$

$$\int \frac{dx}{X^4} = \left(\frac{1}{3kX^3} + \frac{5c}{3k^2X^2} + \frac{10c^2}{k^3X}\right)(2cx + b) + \frac{20c^3}{k^3}\int \frac{dx}{X}$$

$$\int \frac{x\,dx}{X^4} = -\frac{1}{6cX^3} - \frac{b}{2c}\int \frac{dx}{X^4}$$

$$\int \frac{x^2dx}{X^4} = \left(-\frac{x}{5c} + \frac{b}{15c^2}\right)\frac{1}{X^3} + \left(\frac{b^2}{5c^2} + \frac{a}{5c}\right)\int \frac{dx}{X^4}$$

$$\int \frac{x^3dx}{X^4} = \left[-\frac{x^2}{4c} + \frac{bx}{20c^2} - \left(\frac{b^2}{60c^3} + \frac{a}{12c^2}\right)\right]\frac{1}{X^3}$$

$$- \left(\frac{b^3}{20c^3} + \frac{3ab}{10c^2}\right)\int \frac{dx}{X^4}$$

$$\int \frac{x^4dx}{X^4} = \left(-\frac{x^3}{3c} - \frac{ax}{5c^2} + \frac{ab}{15c^3}\right)\frac{1}{X^3} + \left(\frac{ab^2}{5c^3} + \frac{a^2}{5c^2}\right)\int \frac{dx}{X^4}$$

$$\int \frac{x^5dx}{X^4} = \left(-\frac{x^4}{c} - \frac{bx^3}{6c^2} - \frac{ax^2}{2c^2} - \frac{a^2}{6c^3}\right)\frac{1}{X^3} + \frac{a^2b}{2c^3}\int \frac{dx}{X^4}$$

$$\int \frac{x^6dx}{X^4} = \left[-\frac{x^5}{c} - \frac{bx^4}{c^2} - \left(\frac{b^2}{3c^3} + \frac{5a}{3c^2}\right)x^3 - \frac{abx^2}{c^3} - \frac{a^2x}{c^3}\right]\frac{1}{X^3}$$

$$+ \frac{a^3}{c^3}\int \frac{dx}{X^4}$$

$$\int \frac{x^7dx}{X^4} = \frac{1}{c}\int \frac{x^5dx}{X^3} - \frac{a}{c}\int \frac{x^5dx}{X^4} - \frac{b}{c}\int \frac{x^6dx}{X^4}$$

$$\int \frac{x^8dx}{X^4} = \frac{x^7}{cX^3} - \frac{4b}{c}\int \frac{x^7dx}{X^4} - \frac{7a}{c}\int \frac{x^6dx}{X^4}$$

$$\int \frac{x^9dx}{X^4} = \left(\frac{x^8}{2c} - \frac{5bx^7}{2c^2}\right)\frac{1}{X^3} + \left(\frac{10b^2}{c^2} - \frac{4a}{c}\right)\int \frac{x^7dx}{X^4} + \frac{35ab}{2c^2}\int \frac{x^6dx}{X^4}$$

TAB. XXXI.

$$\int \frac{x^m dx}{(a + bx + cx^2)^5}$$

$$a + bx + cx^2 = X, \quad 4ac - b^2 = k$$

$$\int \frac{dx}{X^5} = \left(\frac{1}{4kX^4} + \frac{7c}{6k^2X^3} + \frac{35c^2}{4k^3X^2} + \frac{35c^3}{k^4X}\right)(2cx + b) + \frac{70c^4}{k^4}\int \frac{dx}{X}$$

$$\int \frac{xdx}{X^5} = -\frac{1}{8cX^4} - \frac{b}{2c}\int \frac{dx}{X^5}$$

$$\int \frac{x^2dx}{X^5} = \left(-\frac{x}{7c} + \frac{3b}{56c^2}\right)\frac{1}{X^4} + \left(\frac{3b^2}{14c^2} + \frac{a}{7c}\right)\int \frac{dx}{X^5}$$

$$\int \frac{x^3dx}{X^5} = \left(-\frac{x^2}{6c} + \frac{bx}{21c^2} - \frac{b^2}{56c^3} - \frac{a}{24c^2}\right)\frac{1}{X^4} - \left(\frac{b^3}{14c^3} + \frac{3ab}{14c^2}\right)\int \frac{dx}{X^5}$$

$$\int \frac{x^4dx}{X^5} = \left[-\frac{x^3}{5c} + \frac{bx^2}{30c^2} - \left(\frac{b^2}{105c^3} + \frac{3a}{35c^2}\right)x + \frac{b^3}{280c^4} + \frac{17ab}{420c^3}\right]\frac{1}{X^4}$$
$$+ \left(\frac{b^4}{70c^4} + \frac{6ab^2}{35c^3} + \frac{3a^2}{35c^2}\right)\int \frac{dx}{X^5}$$

$$\int \frac{x^5dx}{X^5} = \left(-\frac{x^4}{4c} - \frac{ax^2}{6c^2} + \frac{abx}{21c^3} - \frac{ab^2}{56c^4} - \frac{a^2}{24c^3}\right)\frac{1}{X^4}$$
$$- \left(\frac{ab^3}{14c^4} + \frac{3a^2b}{14c^3}\right)\int \frac{dx}{X^5}$$

$$\int \frac{x^6dx}{X^5} = -\frac{x^5}{3cX^4} + \frac{b}{3c}\int \frac{x^5dx}{X^5} + \frac{5a}{3c}\int \frac{x^4dx}{X^5}$$

$$\int \frac{x^7dx}{X^5} = \left(-\frac{x^6}{2c} - \frac{bx^5}{3c^2}\right)\frac{1}{X^4} + \left(\frac{b^2}{3c^2} + \frac{3a}{c}\right)\int \frac{x^5dx}{X^5} + \frac{5ab}{3c^2}\int \frac{x^4dx}{X^5}$$

$$\int \frac{x^8dx}{X^5} = \left[-\frac{x^7}{c} - \frac{3bx^6}{2c^2} - \left(\frac{b^2}{c^3} + \frac{7a}{3c^2}\right)x^5\right]\frac{1}{X^4} + \left(\frac{b^3}{c^3} + \frac{17ab}{3c^2}\right)\times$$
$$\int \frac{x^5dx}{X^5} + \left(\frac{5ab^2}{c^3} + \frac{35a^2}{3c^2}\right)\int \frac{x^4dx}{X^5}$$

TAB. XXXII.

$$\int \frac{x^m dx}{(a + bx + cx^2)^6}$$

$$a + bx + cx^2 = X, \quad 4ac - b^2 = k$$

$$\int \frac{dx}{X^6} = \left(\frac{1}{5kX^5} + \frac{9c}{10k^2X^4} + \frac{21c^2}{5k^3X^3} + \frac{21c^3}{k^4X^2} + \frac{126c^4}{k^5X}\right)(2cx + b) + \frac{252c^5}{k^5}\int \frac{dx}{X}$$

$$\int \frac{x\,dx}{X^6} = -\frac{1}{10cX^5} - \frac{b}{2c}\int \frac{dx}{X^6}$$

$$\int \frac{x^2dx}{X^6} = \left(-\frac{x}{9c} + \frac{2b}{45c^2}\right)\frac{1}{X^5} + \left(\frac{2b^2}{9c^2} + \frac{a}{9c}\right)\int \frac{dx}{X^6}$$

$$\int \frac{x^3dx}{X^6} = \left(-\frac{x^2}{8c} + \frac{bx}{24c^2} - \frac{b^2}{60c^3} - \frac{a}{40c^2}\right)\frac{1}{X^5} - \left(\frac{b^3}{12c^3} + \frac{ab}{6c^2}\right)\int \frac{dx}{X}$$

$$\int \frac{x^4dx}{X^6} = \left[-\frac{x^3}{7c} + \frac{bx^2}{28c^2} - \left(\frac{b^2}{84c^3} + \frac{a}{21c^2}\right)x + \frac{b^3}{210c^4} + \frac{11ab}{420c^3}\right)\int \frac{dx}{X} + \left(\frac{b^4}{42c^4} + \frac{ab^2}{7c^3} + \frac{a^2}{21c^2}\right)\int \frac{dx}{X}$$

$$\int \frac{x^5dx}{X^6} = -\frac{x^4}{6cX^5} - \frac{b}{6c}\int \frac{x^4dx}{X^6} + \frac{2a}{3c}\int \frac{x^3dx}{X^6}$$

$$\int \frac{x^6dx}{X^6} = -\frac{x^5}{5cX^5} + \frac{a}{c}\int \frac{x^4dx}{X^6}$$

$$\int \frac{x^7dx}{X^6} = \left(-\frac{x^6}{4c} - \frac{bx^5}{20c^2} - \frac{ax^4}{4c^2}\right)\frac{1}{X^5} + \frac{a^2}{c^2}\int \frac{x^3dx}{X^6}$$

$$\int \frac{x^8dx}{X^6} = \left[-\frac{x^7}{3c} - \frac{bx^6}{6c^2} - \left(\frac{b^2}{30c^3} + \frac{7a}{15c^2}\right)x^5 - \frac{abx^4}{6c}\right]\frac{1}{X} + \frac{7a^2}{3c^2}\int \frac{x^4dx}{X^6} + \frac{2a^2b}{3c^3}\int \frac{x^3dx}{X^6}$$

TAB. XXXIII.

$$\int \frac{dx}{x^m (a + bx + cx^2)}$$

$$a + bx + cx^2 = X$$

$$\int \frac{dx}{xX} = \frac{1}{2a} \log. \frac{x^2}{X} - \frac{b}{2a} \int \frac{dx}{X} \text{*}$$

$$\int \frac{dx}{x^2X} = -\frac{1}{ax} - \frac{b}{2a^2} \log. \frac{x^2}{X} + \left(\frac{b^2}{2a^2} - \frac{c}{a}\right) \int \frac{dx}{X}$$

$$\int \frac{dx}{x^3X} = -\frac{1}{2ax^2} + \frac{b}{a^2x} + \left(\frac{b^2}{2a^3} - \frac{c}{2a^2}\right) \log. \frac{x^2}{X} - \left(\frac{b^3}{2a^3} - \frac{3bc}{2a^2}\right) \int \frac{dx}{X}$$

$$\int \frac{dx}{x^4X} = -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \left(\frac{b^2}{a^3} - \frac{c}{a^2}\right) \frac{1}{x} - \left(\frac{b^3}{2a^4} - \frac{bc}{a^3}\right) \log. \frac{x^2}{X}$$
$$+ \left(\frac{b^4}{2a^4} - \frac{2b^2c}{a^3} + \frac{c^2}{a^2}\right) \int \frac{dx}{X}$$

$$\int \frac{dx}{x^5X} = -\frac{1}{4ax^4} - \frac{b}{a} \int \frac{dx}{x^4X} - \frac{c}{a} \int \frac{dx}{x^3X}$$

$$\int \frac{dx}{x^6X} = -\frac{1}{5ax^5} + \frac{b}{4a^2x^4} + \left(\frac{b^2}{a^2} - \frac{c}{a}\right) \int \frac{dx}{x^4X} + \frac{bc}{a^2} \int \frac{dx}{x^3X}$$

$$\int \frac{dx}{x^7X} = -\frac{1}{6ax^6} + \frac{b}{5a^2x^5} - \left(\frac{b^2}{4a^3} - \frac{c}{4a^2}\right) \frac{1}{x^4} - \left(\frac{b^3}{a^3} - \frac{2bc}{a^2}\right) \int \frac{dx}{x^4X}$$
$$- \left(\frac{b^2c}{a^3} - \frac{c^2}{a^2}\right) \int \frac{dx}{x^3X}$$

$$\int \frac{dx}{x^8X} = -\frac{1}{7ax^7} + \frac{b}{6a^2x^6} - \left(\frac{b^2}{5a^3} - \frac{c}{5a^2}\right) \frac{1}{x^5} + \left(\frac{b^3}{4a^4} - \frac{bc}{2a^3}\right) \frac{1}{x^4}$$
$$+ \left(\frac{b^4}{a^4} - \frac{3b^2c}{a^3} + \frac{c^2}{a^2}\right) \int \frac{dx}{x^4X} + \left(\frac{b^3c}{a^4} - \frac{2bc^2}{a^3}\right) \int \frac{dx}{x^3X}$$

* The Integral $\int \frac{dx}{xX}$ does not vanish when $x = 0$, because then $\log. \frac{x^2}{X} = \log. 0 = -\infty$. Moreover, we have $\log. \frac{x^2}{X} = -\log. \frac{X}{x^2}$.

TAB. XXXIV.

$$\int \frac{dx}{x^m (a+bx+cx^2)^2}$$

$$a + bx + cx^2 = X$$

$$\int \frac{dx}{xX^2} = \frac{1}{2aX} + \frac{1}{2a^2} \log. \frac{x^2}{X} - \frac{b}{2a}\int\frac{dx}{X^2} - \frac{b}{2a^2}\int\frac{dx}{X}$$

$$\int \frac{dx}{x^2X^2} = \left(-\frac{1}{ax} - \frac{b}{a^2}\right)\frac{1}{X} - \frac{b}{a^3}\log. \frac{x^2}{X} + \left(\frac{b^2}{a^2} - \frac{3c}{a}\right)\int\frac{dx}{X^2} + \frac{b^2}{a^3}\int\frac{dx}{X}$$

$$\int \frac{dx}{x^3X^2} = \left(-\frac{1}{2ax^2} + \frac{3b}{2a^2x} + \frac{3b^2}{2a^3} - \frac{c}{a^2}\right)\frac{1}{X} + \left(\frac{3b^3}{2a^4} - \frac{c}{a^3}\right)\log. \frac{x^2}{X} - \left(\frac{3b^3}{2a^3} - \frac{11bc}{2a^2}\right)\int\frac{dx}{X^2} - \left(\frac{3b^3}{2a^4} - \frac{bc}{a}\right)\int\frac{dx}{X}$$

$$\int \frac{dx}{x^4X^2} = \left[-\frac{1}{3ax^3} + \frac{2b}{3a^2x^2} - \left(\frac{2b^2}{a^3} - \frac{5c}{3a^2}\right)\frac{1}{x} - \frac{2b^3}{a^4} + \frac{3bc}{a^3}\right]\frac{1}{X} - \left(\frac{2b^3}{a^5} - \frac{3bc}{a^4}\right)\log. \frac{x^2}{X} + \left(\frac{2b^4}{a^4} - \frac{9b^2c}{a^3} + \frac{5c^2}{a^2}\right)\int\frac{dx}{X^2} + \left(\frac{2b^4}{a^5} - \frac{3b^2c}{a^4}\right)\int\frac{dx}{X}$$

$$\int \frac{dx}{x^5X^2} = -\frac{1}{4ax^4X} - \frac{5}{4a}\int\frac{dx}{x^4X^2} - \frac{3c}{2a}\int\frac{dx}{x^3X^2}$$

$$\int \frac{dx}{x^6X^2} = \left(-\frac{1}{5ax^5} + \frac{3b}{10a^2x^4}\right)\frac{1}{X} + \left(\frac{3b^2}{2a^2} - \frac{7c}{5a}\right)\int\frac{dx}{x^4X^2} + \frac{9bc}{5a^2}\int\frac{dx}{x^3X^2}$$

$$\int \frac{dx}{x^7X^2} = \left[-\frac{1}{6ax^6} + \frac{7b}{30a^2x^5} - \left(\frac{7b^2}{20a^3} - \frac{c}{3a^2}\right)\frac{1}{x^4}\right]\frac{1}{X} - \left(\frac{7b^3}{4a^3} - \frac{33bc}{10a^2}\right)\int\frac{dx}{x^4X^2} - \left(\frac{21b^2c}{10a^3} - \frac{2c^2}{a^2}\right)\int\frac{dx}{x^3X^2}$$

$$\int \frac{dx}{x^8X^2} = -\frac{1}{7ax^7X} - \frac{8b}{7a}\int\frac{dx}{x^7X^2} - \frac{9c}{7a}\int\frac{dx}{x^6X^2}$$

TAB. XXXV.

$$\int \frac{dx}{x^m (a + bx + cx^2)^3}$$

$$a + bx + cx^2 = X$$

$$\int \frac{dx}{xX^3} = \frac{1}{4aX^2} + \frac{1}{2a^2X} + \frac{1}{2a^3} \log. \frac{x^2}{X} - \frac{b}{2a} \int \frac{dx}{X^3} - \frac{b}{2a^2} \int \frac{dx}{X^2} - \frac{b}{2a^3} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^2X^3} = - \frac{1}{axX^2} - \frac{3b}{a} \int \frac{dx}{xX^3} - \frac{5c}{a} \int \frac{dx}{X^3}$$

$$\int \frac{dx}{x^3X^3} = \left(- \frac{1}{2ax^2} + \frac{2b}{a^2x}\right) \frac{1}{X^2} + \left(\frac{6b^2}{a^2} - \frac{3c}{a}\right) \int \frac{dx}{xX^3} + \frac{10bc}{a^2} \int \frac{dx}{X^3}$$

$$\int \frac{dx}{x^4X^3} = \left[- \frac{1}{3ax^3} + \frac{5b}{6a^2x^2} - \left(\frac{10b^2}{3a^3} - \frac{7c}{3a^2}\right) \frac{1}{x}\right] \frac{1}{X^2} - \left(\frac{10b^3}{a^3} - \frac{12bc}{a^2}\right) \int \frac{dx}{xX^3} - \left(\frac{50b^2c}{3a^3} - \frac{35c^2}{3a^2}\right) \int \frac{dx}{X^3}$$

$$\int \frac{dx}{x^5X^3} = - \frac{1}{4ax^4X^2} - \frac{3b}{2a} \int \frac{dx}{x^4X^3} - \frac{2c}{a} \int \frac{dx}{x^3X^3}$$

$$\int \frac{dx}{x^6X^3} = \left(- \frac{1}{5ax^5} + \frac{7b}{20a^2x^4}\right) \frac{1}{X^2} + \left(\frac{21b^2}{10a^2} - \frac{9c}{5a}\right) \int \frac{dx}{x^4X^3} + \frac{14bc}{5a^2} \int \frac{dx}{x^3X^3}$$

$$\int \frac{dx}{x^7X^3} = \left[- \frac{1}{6ax^6} + \frac{4b}{15a^2x^5} - \left(\frac{7b^2}{15a^3} - \frac{5c}{12a^2}\right)\right] \frac{1}{X^2} - \left(\frac{14b^3}{5a^3} - \frac{49bc}{10a^2}\right) \int \frac{dx}{x^4X^3} - \left(\frac{56b^2c}{15a^3} - \frac{10c^2}{3a^2}\right) \int \frac{dx}{x^3X^3}$$

$$\int \frac{dx}{x^8X^3} = - \frac{1}{7ax^7X^2} - \frac{9b}{7a} \int \frac{dx}{x^7X^3} - \frac{11c}{7a} \int \frac{dx}{x^6X^3}$$

TAB. XXXVI.

$$\int \frac{dx}{x^m (a + bx + cx^2)^4}$$

$$a + bx + cx^2 = X$$

$$\int \frac{dx}{xX^4} = \frac{1}{6aX^3} + \frac{1}{4a^2X^2} + \frac{1}{2a^3X} + \frac{1}{2a^4} \log. \frac{x^2}{X} - \frac{b}{2a} \int \frac{dx}{X^4}$$
$$- \frac{b}{2a^2} \int \frac{dx}{X^3} - \frac{b}{2a^3} \int \frac{dx}{X^2} - \frac{b}{2a^4} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^2X^4} = - \frac{1}{axX^3} - \frac{4b}{a} \int \frac{dx}{xX^4} - \frac{7c}{a} \int \frac{dx}{X^4}$$

$$\int \frac{dx}{x^3X^4} = \left(- \frac{1}{2ax^2} + \frac{5b}{2ax^2} \right) \frac{1}{X^3} + \left(\frac{10b^2}{a^2} - \frac{4c}{a} \right) \int \frac{dx}{xX^4}$$
$$+ \frac{35bc}{2a^2} \int \frac{dx}{X^4}$$

$$\int \frac{dx}{x^4X^4} = \left[- \frac{1}{3ax^3} + \frac{b}{a^2x^2} - \left(\frac{5b^2}{a^3} - \frac{3c}{a^2} \right) \frac{1}{x} \right] \frac{1}{X^3}$$
$$- \left(\frac{20b^3}{a^3} - \frac{20bc}{a^2} \right) \int \frac{dx}{xX^4} - \left(\frac{35b^2c}{a^3} - \frac{21c^2}{a^2} \right) \int \frac{dx}{X^4}$$

$$\int \frac{dx}{x^5X^4} = - \frac{1}{4ax^4X^3} - \frac{7b}{4a} \int \frac{dx}{x^4X^4} - \frac{5c}{2a} \int \frac{dx}{x^3X^4}$$

$$\int \frac{dx}{x^6X^4} = \left(- \frac{1}{5ax^5} + \frac{2b}{5a^2x^4} \right) \frac{1}{X^3} + \left(\frac{14b^2}{5a^2} - \frac{11c}{5a} \right) \int \frac{dx}{x^4X^4}$$
$$+ \frac{4bc}{a^2} \int \frac{dx}{x^3X^4}$$

$$\int \frac{dx}{x^7X^4} = \left[- \frac{1}{6ax^6} + \frac{3b}{10a^2x^5} - \left(\frac{3b^2}{5a^3} - \frac{c}{2a^2} \right) \frac{1}{x^4} \right] \frac{1}{X^3}$$
$$- \left(\frac{21b^3}{5a^3} - \frac{34bc}{5a^2} \right) \int \frac{dx}{x^4X^4} - \left(\frac{6b^2c}{a^3} - \frac{5c^2}{a^2} \right) \int \frac{dx}{x^3X^4}$$

$$\int \frac{dx}{x^8X^4} = - \frac{1}{7ax^7X^3} - \frac{10b}{7a} \int \frac{dx}{x^7X^4} - \frac{13c}{7a} \int \frac{dx}{x^6X^4}$$

TAB. XXXVII.

$$\int \frac{dx}{x^m (a + bx + cx^2)^5}$$

$$a + bx + cx^2 = X$$

$$\int \frac{dx}{xX^5} = \frac{1}{8aX^4} + \frac{1}{6a^2X^3} + \frac{1}{4a^3X^2} + \frac{1}{2a^4X} + \frac{1}{2a^5} \log. \frac{x^2}{X} - \frac{b}{2a} \int \frac{dx}{X^5}$$
$$- \frac{b}{2a^2} \int \frac{dx}{X^4} - \frac{b}{2a^3} \int \frac{dx}{X^3} - \frac{b}{2a^4} \int \frac{dx}{X^2} - \frac{b}{2a^5} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^2X^5} = - \frac{1}{axX^4} - \frac{5b}{a} \int \frac{dx}{xX^5} - \frac{9c}{a} \int \frac{dx}{X^5}$$

$$\int \frac{dx}{x^3X^5} = \left(- \frac{1}{2ax^2} + \frac{3b}{a^2x} \right) \frac{1}{X^4} + \left(\frac{15b^2}{a^2} - \frac{5c}{a} \right) \int \frac{dx}{xX^5} + \frac{27bc}{a^2} \int \frac{dx}{X^5}$$

$$\int \frac{dx}{x^4X^5} = \left[- \frac{1}{3ax^3} + \frac{7b}{6a^2x^2} - \left(\frac{7b^2}{a^3} - \frac{11c}{3a^2} \right) \frac{1}{x} \right] \frac{1}{X^4}$$
$$- \left(\frac{35b^3}{a^3} - \frac{30bc}{a^2} \right) \int \frac{dx}{xX^5} - \left(\frac{63b^2c}{a^3} - \frac{33c^2}{a^2} \right) \int \frac{dx}{X^5}$$

$$\int \frac{dx}{x^5X^5} = - \frac{1}{4ax^4X^4} - \frac{2b}{a} \int \frac{dx}{x^4X^5} - \frac{3c}{a} \int \frac{dx}{x^3X^5}$$

$$\int \frac{dx}{x^6X^5} = \left(- \frac{1}{5ax^5} + \frac{9b}{20a^2x^4} \right) \frac{1}{X^4} + \left(\frac{18b^2}{5a^2} - \frac{13c}{5a} \right) \int \frac{dx}{x^4X^5}$$
$$+ \frac{39c^2}{5a^2} \int \frac{dx}{x^3X^5}$$

$$\int \frac{dx}{x^7X^5} = \left[- \frac{1}{6ax^6} + \frac{b}{3a^2x^5} - \left(\frac{3b^2}{4a^3} - \frac{7c}{12a^2} \right) \frac{1}{x^4} \right] \frac{1}{X^4}$$
$$- \left(\frac{6b^3}{a^3} - \frac{9bc}{a^2} \right) \int \frac{dx}{x^4X^5} - \left(\frac{13b^2c}{a^3} - \frac{7c^2}{a^2} \right) \int \frac{dx}{x^3X^5}$$

$$\int \frac{dx}{x^8X^5} = - \frac{1}{7ax^7} - \frac{11b}{7a} \int \frac{dx}{x^7X^5} - \frac{15c}{7a} \int \frac{dx}{x^6X^5}$$

TAB. XXXVIII.

$$\int \frac{dx}{x^n (a + bx + cx^2)^6}.$$

$$a + bx + cx^2 = X$$

$$\int \frac{dx}{xX^6} = \frac{1}{10aX^5} + \frac{1}{8a^2X^4} + \frac{1}{6a^3X^3} + \frac{1}{4a^4X^2} + \frac{1}{2a^5X} + \frac{1}{2a^6} \log \frac{x^2}{X}$$
$$- \frac{b}{2a}\int\frac{dx}{X^6} - \frac{b}{2a^2}\int\frac{dx}{X^5} - \frac{b}{2a^3}\int\frac{dx}{X^4} - \frac{b}{2a^4}\int\frac{dx}{X^3}$$
$$- \frac{b}{2a^5}\int\frac{dx}{X^2} - \frac{b}{2a^6}\int\frac{dx}{X}$$

$$\int \frac{dx}{x^2X^6} = - \frac{1}{axX^5} - \frac{6b}{a}\int\frac{dx}{xX^6} - \frac{11c}{a}\int\frac{dx}{X^6}$$

$$\int \frac{dx}{x^3X^6} = \left(-\frac{1}{2ax^2} + \frac{7b}{2a^2x}\right)\frac{1}{X^5} + \left(\frac{21b^2}{a^2} - \frac{6c}{a}\right)\int\frac{dx}{xX^6} - \frac{77bc}{a^2}\int\frac{dx}{X^6}$$

$$\int \frac{dx}{x^4X^6} = \left[-\frac{1}{3ax^3} + \frac{4b}{3a^2x^2} - \left(\frac{28b^2}{3a^3} - \frac{13c}{3a^2}\right)\frac{1}{x}\right]\frac{1}{X^5}$$
$$- \left(\frac{56b^3}{a^3} - \frac{42bc}{a^2}\right)\int\frac{dx}{xX^6} - \left(\frac{616b^2c}{3a^3} - \frac{143c^2}{3a^2}\right)\int\frac{dx}{X^6}$$

$$\int \frac{dx}{x^5X^6} = - \frac{1}{4ax^4X^5} - \frac{9b}{4a}\int\frac{dx}{x^4X^6} - \frac{7c}{2a}\int\frac{dx}{x^3X^6}$$

$$\int \frac{dx}{x^6X^6} = \left(-\frac{1}{5ax^5} + \frac{b}{2a^2x^4}\right)\frac{1}{X^5} + \left(\frac{9b^2}{2a^2} - \frac{3c}{a}\right)\int\frac{dx}{x^4X^6} + \frac{7bc}{a^2}\int\frac{dx}{x^3X^6}$$

$$\int \frac{dx}{x^7X^6} = \left[-\frac{1}{6ax^6} + \frac{11b}{30a^2x^5} - \left(\frac{11b^2}{12a^3} - \frac{2c}{3a^2}\right)\frac{1}{x^4}\right]\frac{1}{X^5}$$
$$- \left(\frac{33b^3}{4a^3} - \frac{23bc}{2a^2}\right)\int\frac{dx}{x^4X^6} - \left(\frac{77b^2c}{6a^3} - \frac{28c^2}{3a^2}\right)\int\frac{dx}{x^3X^6}$$

$$\int \frac{dx}{x^8X^6} = - \frac{1}{7ax^7X^5} - \frac{12b}{7a}\int\frac{dx}{x^7X^6} - \frac{17c}{7a}\int\frac{dx}{x^6X^6}$$

TAB. XXXIX.

$$\int \frac{x^m dx}{a+bx^3}$$

(a and b positive or negative.)

$$a+bx^3 = X, \quad \sqrt[3]{\frac{a}{b}} = k$$

$$\int\frac{dx}{X} = \frac{1}{3bk^2}\left(\tfrac{1}{2}\log.\frac{(x+k)^2}{x^2-kx+k^2} + \sqrt{3}.\ \text{arc tang.}\ \frac{x\sqrt{3}}{2k-x}\right)$$

$$\int\frac{xdx}{X} = \frac{-1}{3bk}\left(\tfrac{1}{2}\log.\frac{(x+k)^2}{x^2-kx+k^2} - \sqrt{3}.\ \text{arc. tang.}\ \frac{x\sqrt{3}}{2k-x}\right)$$

$$\int\frac{x^2dx}{X} = \frac{1}{3b}\log. X$$

$$\int\frac{x^3dx}{X} = \frac{x}{b} - \frac{a}{b}\int\frac{dx}{X}$$

$$\int\frac{x^4dx}{X} = \frac{x^2}{2b} - \frac{a}{b}\int\frac{xdx}{X}$$

$$\int\frac{x^5dx}{X} = \frac{x^3}{3b} - \frac{a}{3b^2}\log. X$$

$$\int\frac{x^6dx}{X} = \frac{x^4}{4b} - \frac{ax}{b^2} + \frac{a^2}{b^2}\int\frac{dx}{X}$$

$$\int\frac{x^7dx}{X} = \frac{x^5}{5b} - \frac{ax^2}{2b^2} + \frac{a^2}{b^2}\int\frac{xdx}{X}$$

$$\int\frac{x^8dx}{X} = \frac{x^6}{6b} - \frac{ax^3}{3b^2} + \frac{a^2}{3b^3}\log. X$$

$$\int\frac{x^9dx}{X} = \frac{x^7}{7b} - \frac{ax^4}{4b^2} + \frac{a^2x}{b^3} - \frac{a^3}{b^3}\int\frac{dx}{X}$$

The Integrals $\int\frac{dx}{X}, \int\frac{xdx}{X}$ are here considered evanescent when $x=0$. Moreover $\log.\frac{(x+k)^2}{x^2-kx+k^2} = \log.\frac{b.(x+k)^3}{X}$, or introducing the constant, $=\log.\frac{(x+k)^3}{X} = 3\log.\frac{x+k}{\sqrt[3]{X}}$.

TAB. XL.

$$\int \frac{x^m dx}{(a+bx^3)^2}, \quad \int \frac{x^m dx}{(a+bx^3)^3}$$

$$a + bx^3 = X$$

$$\int \frac{dx}{X^2} = \frac{x}{3aX} + \frac{2}{3a}\int \frac{dx}{X}$$

$$\int \frac{xdx}{X^2} = \frac{x^2}{3aX} + \frac{1}{3a}\int \frac{xdx}{X}$$

$$\int \frac{x^2dx}{X^2} = -\frac{1}{3bX}$$

$$\int \frac{x^3dx}{X^2} = -\frac{x}{3bX} + \frac{1}{3b}\int \frac{dx}{X}$$

$$\int \frac{x^4dx}{X^2} = -\frac{x^2}{3bX} + \frac{2}{3b}\int \frac{xdx}{X}$$

$$\int \frac{x^5dx}{X^2} = \frac{a}{3b^2X} + \frac{1}{3b^2}\log. X$$

$$\int \frac{x^6dx}{X^2} = \left(\frac{x^4}{b} + \frac{4ax}{3b^2}\right)\frac{1}{X} - \frac{4a}{3b^2}\int \frac{dx}{X}$$

$$\int \frac{dx}{X^3} = \left(\frac{5bx^4}{18a^2} + \frac{4x}{9a}\right)\frac{1}{X^2} + \frac{5}{9a^2}\int \frac{dx}{X}$$

$$\int \frac{xdx}{X^3} = \left(\frac{2bx^5}{9a^2} + \frac{7x^2}{18a}\right)\frac{1}{X^2} + \frac{2}{9a^2}\int \frac{xdx}{X}$$

$$\int \frac{x^2dx}{X^3} = -\frac{1}{6bX^2}$$

$$\int \frac{x^3dx}{X^3} = \left(\frac{x^4}{18a} - \frac{x}{9b}\right)\frac{1}{X^2} + \frac{1}{9ab}\int \frac{dx}{X}$$

$$\int \frac{x^4dx}{X^3} = \left(\frac{x^5}{9a} - \frac{x^2}{18b}\right)\frac{1}{X^2} + \frac{1}{9ab}\int \frac{xdx}{X}$$

$$\int \frac{x^5dx}{X^3} = \frac{x^6}{6aX^2}$$

$$\int \frac{x^6dx}{X^3} = \left(-\frac{7x^4}{18b} - \frac{2ax}{9b^2}\right)\frac{1}{X^2} + \frac{2}{9b^2}\int \frac{dx}{X}$$

TAB. XLI.

$$\int \frac{dx}{x^m (a + bx^3)}, \quad \int \frac{dx}{x^m (a+bx^3)^2}$$

$$a + bx^3 = X$$

$$\int \frac{dx}{xX} = \frac{\log. x}{a} - \frac{\log. X}{3a} = \frac{1}{3a} \log. \frac{x^3}{X} = -\frac{1}{3a} \log. \frac{X}{x^3}$$

$$\int \frac{dx}{x^2X} = -\frac{1}{ax} - \frac{b}{a} \int \frac{xdx}{X}$$

$$\int \frac{dx}{x^3X} = -\frac{1}{2ax^2} - \frac{b}{a} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^4X} = -\frac{1}{3ax^3} + \frac{b}{3a^2} \log. \frac{X}{x^3}$$

$$\int \frac{dx}{x^5X} = -\frac{1}{4ax^4} + \frac{b}{a^2x} + \frac{b^2}{a^2} \int \frac{xdx}{X}$$

$$\int \frac{dx}{x^6X} = -\frac{1}{5ax^5} + \frac{b}{2a^2x^2} + \frac{b^2}{a^2} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^7X} = -\frac{1}{6ax^6} + \frac{b}{3a^2x^3} - \frac{b^2}{3a^3} \log. \frac{X}{x^3}$$

$$\int \frac{dx}{xX^2} = \frac{1}{3aX} - \frac{1}{3a^2} \log. \frac{X}{x^3}$$

$$\int \frac{dx}{x^2X^2} = \left(-\frac{1}{ax} - \frac{4bx^2}{3a^2}\right) \frac{1}{X} - \frac{4b}{3a^2} \int \frac{xdx}{X}$$

$$\int \frac{dx}{x^3X^2} = \left(-\frac{1}{2ax^2} - \frac{5bx}{6a^2}\right) \frac{1}{X} - \frac{5b}{3a^2} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^4X^2} = \left(-\frac{1}{3ax^3} - \frac{2b}{3a^2}\right) \frac{1}{X} + \frac{2b}{3a^3} \log. \frac{X}{x^3}$$

$$\int \frac{dx}{x^5X^2} = \left(-\frac{1}{4ax^4} + \frac{7b}{4a^2x} + \frac{7b^2x^2}{3a^3}\right) \frac{1}{X} + \frac{7b^2}{3a^3} \int \frac{xdx}{X}$$

$$\int \frac{dx}{x^6X^2} = \left(-\frac{1}{5ax^5} + \frac{4b}{5a^2x^2} + \frac{4b^2x}{3a^3}\right) \frac{1}{X} + \frac{8b^2}{3a^3} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^7X^2} = \left(-\frac{1}{6ax^6} + \frac{b}{2a^2x^3} + \frac{b^2}{a^3}\right) \frac{1}{X} \log. \frac{X}{x^3}$$

TAB. XLII. *a*.

$$\int \frac{x^m dx}{a + bx^4}$$

(*a* and *b* have the same signs.)

$$a + bx^4 = X,\ \sqrt[4]{\frac{a}{b}} = k$$

$$\int \frac{dx}{X} = \frac{1}{4bk^3\sqrt{2}}\left(\log.\frac{x^2+kx\sqrt{2}+k^2}{x^2-kx\sqrt{2}+k^2} + 2\text{arc tang.}\frac{kx\sqrt{2}}{k^2-x^2}\right)$$

$$\int \frac{xdx}{X} = -\frac{1}{2bk^2}\text{ arc tang. } x^2\sqrt{\frac{b}{a}}$$

$$\int \frac{x^2dx}{X} = \frac{1}{4bk\sqrt{2}}\left(-\log.\frac{x^2+kx\sqrt{2}+k^2}{x^2-kx\sqrt{2}+k^2} + 2\text{ arc tang.}\frac{kx\sqrt{2}}{k^2-x^2}\right)$$

$$\int \frac{x^3dx}{X} = \frac{1}{4b}\log. X$$

$$\int \frac{x^4dx}{X} = \frac{x}{b} - \frac{a}{b}.\int \frac{dx}{X}$$

$$\int \frac{x^5dx}{X} = \frac{x^2}{2b} - \frac{a}{b}\int \frac{xdx}{X}$$

$$\int \frac{x^6dx}{X} = \frac{x^3}{3b} - \frac{a}{b}\int \frac{x^2dx}{X}$$

$$\int \frac{x^7dx}{X} = \frac{x^4}{4b} - \frac{a}{b}\int \frac{x^3dx}{X}$$

$$\int \frac{x^8dx}{X} = \frac{x^5}{5b} - \frac{ax}{b^2} + \frac{a^2}{b^2}.\int \frac{dx}{X}$$

$$\int \frac{x^9dx}{X} = \frac{x^6}{6b} - \frac{ax^2}{2b} + \frac{a^2}{b^2}\int \frac{xdx}{X}$$

We have $\log.\frac{x^2+kx\sqrt{2}+k^2}{x^2-kx\sqrt{2}+k^2}+\text{const.}=2\log.\frac{x^2+kx\sqrt{2}+k^2}{\sqrt{X}}$ $+\text{ const.}$, and $\text{arc tang.}\frac{kx\sqrt{2}}{k^2-x^2} = \text{arc sec.}\frac{\sqrt{X}}{\sqrt{a}-x^2\sqrt{b}} = \text{arc cos.}$ $\frac{\sqrt{a}-x^2\sqrt{b}}{\sqrt{X}}$

TAB. XLII. b

$$\int \frac{x^m dx}{a + bx^4}$$

(a and b having different signs.)

$$a + bx^4 = X, \quad \sqrt[4]{-\frac{a}{b}} = k$$

$$\int \frac{dx}{X} = -\frac{1}{4bk^3}\left(\log. \frac{x+k}{x-k} + 2 \text{ arc tang. } \frac{x}{k}\right)$$

$$\int \frac{xdx}{X} = -\frac{1}{4bk^2} \log. \frac{x^2+k^2}{x^2-k^2} \, *$$

$$\int \frac{x^2dx}{X} = -\frac{1}{4bk}\left(\log. \frac{x+k}{x-k} - 2 \text{ arc tang. } \frac{x}{k}\right)$$

The remaining Integrals as in Tab. XLII. a.

* $\log. \frac{x^2+k^2}{x^2-k^2} + \text{const.} = \log. \frac{k^2+x^2}{k^2-x^2} + \text{const}$

In the same manner

$$\log. \frac{x+k}{x-k} + \text{const.} = \log. \frac{k+x}{k-x} + \text{const.}$$

TAB. XLII. a.

$$\int \frac{x^m dx}{a + bx^4}$$

(*a* and *b* have the same signs.)

$$a + bx^4 = X, \quad \sqrt[4]{\frac{a}{b}} = k$$

$$\int \frac{dx}{X} = \frac{1}{4bk^3\sqrt{2}} \left(\log. \frac{x^2 + kx\sqrt{2} + k^2}{x^2 - kx\sqrt{2} + k^2} + 2 \text{arc tang.} \frac{kx\sqrt{2}}{k^2 - x^2}\right)$$

$$\int \frac{xdx}{X} = -\frac{1}{2bk^2} \text{arc tang.} x^2\sqrt{\frac{b}{a}}$$

$$\int \frac{x^2dx}{X} = \frac{1}{4bk\sqrt{2}} \left(-\log. \frac{x^2 + kx\sqrt{2} + k^2}{x^2 - kx\sqrt{2} + k^2} + 2 \text{arc tang.} \frac{kx\sqrt{2}}{k^2 - x^2}\right)$$

$$\int \frac{x^3dx}{X} = \frac{1}{4b} \log. X$$

$$\int \frac{x^4dx}{X} = \frac{x}{b} - \frac{a}{b} \int \frac{dx}{X}$$

$$\int \frac{x^5dx}{X} = \frac{x^2}{2b} - \frac{a}{b} \int \frac{xdx}{X}$$

$$\int \frac{x^6dx}{X} = \frac{x^3}{3b} - \frac{a}{b} \int \frac{x^2dx}{X}$$

$$\int \frac{x^7dx}{X} = \frac{x^4}{4b} - \frac{a}{b} \int \frac{x^3dx}{X}$$

$$\int \frac{x^8dx}{X} = \frac{x^5}{5b} - \frac{ax}{b^2} + \frac{a^2}{b^2} \int \frac{dx}{X}$$

$$\int \frac{x^9dx}{X} = \frac{x^6}{6b} - \frac{ax^2}{2b} + \frac{a^2}{b^2} \int \frac{xdx}{X}$$

We have $\log. \frac{x^2 + kx\sqrt{2} + k^2}{x^2 - kx\sqrt{2} + k^2} + \text{const.} = 2 \log. \frac{x^2 + kx\sqrt{2} + k^2}{\sqrt{X}}$ + const., and $\text{arc tang.} \frac{kx\sqrt{2}}{k^2 - x^2} = \text{arc sec.} \frac{\sqrt{X}}{\sqrt{a} - x^2\sqrt{b}} = \text{arc cos.} \frac{\sqrt{a} - x^2\sqrt{b}}{\sqrt{X}}$

TAB. XLII. b

$$\int \frac{x^m dx}{a + bx^4}$$

(a and b having different signs.)

$$a + bx^4 = X, \ \sqrt[4]{-\frac{a}{b}} = k$$

$$\int \frac{dx}{X} = -\frac{1}{4bk^3}\left(\log. \frac{x+k}{x-k} + 2 \text{ arc tang. } \frac{x}{k}\right)$$

$$\int \frac{x dx}{X} = -\frac{1}{4bk^2} \log. \frac{x^2+k^2}{x^2-k^2} *$$

$$\int \frac{x^2 dx}{X} = -\frac{1}{4bk}\left(\log. \frac{x+k}{x-k} - 2 \text{ arc tang. } \frac{x}{k}\right)$$

The remaining Integrals as in Tab. XLII. a.

* $\log. \frac{x^2+k^2}{x^2-k^2} + \text{const.} = \log. \frac{k^2+x^2}{k^2-x^2} + \text{const}$

In the same manner

$$\log. \frac{x+k}{x-k} + \text{const.} = \log. \frac{k+x}{k-x} + \text{const.}$$

TAB. XLIII.

$$\int\frac{x^m dx}{(a+bx^4)^2},\ \int\frac{x^m dx}{(a+bx^4)^3}$$

$$a+bx^4=X$$

$$\int\frac{dx}{X^2}=\frac{x}{4aX}+\frac{3}{4a}\int\frac{dx}{X}$$

$$\int\frac{xdx}{X^2}=\frac{x^2}{4aX}+\frac{1}{2a}\int\frac{xdx}{X}$$

$$\int\frac{x^2dx}{X^2}=\frac{x^3}{4aX}+\frac{1}{4a}\int\frac{x^2dx}{X}$$

$$\int\frac{x^3dx}{X^2}=-\frac{1}{4bX}$$

$$\int\frac{x^4dx}{X^2}=-\frac{x}{4bX}+\frac{1}{4b}\int\frac{dx}{X}$$

$$\int\frac{x^5dx}{X^2}=-\frac{x^2}{4bX}+\frac{1}{2b}\int\frac{xdx}{X}$$

$$\int\frac{x^6dx}{X^2}=-\frac{x^3}{4bX}+\frac{3}{4b}\int\frac{x^2dx}{X}$$

$$\int\frac{dx}{X^3}=\left(\frac{7bx^5}{32a^2}+\frac{11x}{32a}\right)\frac{1}{X^2}+\frac{21}{32a^2}\int\frac{dx}{X}$$

$$\int\frac{xdx}{X^3}=\left(\frac{3bx^6}{16a^2}+\frac{5x^2}{16a}\right)\frac{1}{X^2}+\frac{3}{8a^2}\int\frac{xdx}{X}$$

$$\int\frac{x^2dx}{X^3}=\left(\frac{5bx^7}{32a^2}+\frac{9x^3}{32a}\right)\frac{1}{X^2}+\frac{5}{32a^2}\int\frac{x^2dx}{X}$$

$$\int\frac{x^3dx}{X^3}=-\frac{1}{8bX^2}$$

$$\int\frac{x^4dx}{X^3}=\left(\frac{x^5}{32a}-\frac{3x}{32b}\right)\frac{1}{X^2}+\frac{3}{32ab}\int\frac{dx}{X}$$

$$\int\frac{x^5dx}{X^3}=\left(\frac{x^6}{16a}-\frac{x^2}{16b}\right)\frac{1}{X^2}+\frac{1}{8ab}\int\frac{xdx}{X}$$

$$\int\frac{x^6dx}{X^3}=\left(\frac{3x^7}{32a}-\frac{x^3}{32ab}\right)\frac{1}{X^2}+\frac{3}{32ab}\int\frac{x^2dx}{X}$$

TAB. XLIV.

$$\int \frac{dx}{x^m(a+bx^4)}, \quad \int \frac{dx}{x^m(a+bx^4)^2}$$

$$a + bx^4 = X$$

$$\int \frac{dx}{xX} = \frac{\log. x}{a} - \frac{\log. X}{4a} = \frac{1}{4a} \log. \frac{x^4}{X} = -\frac{1}{4a} \log. \frac{X}{x^4}$$

$$\int \frac{dx}{x^2X} = -\frac{1}{ax} - \frac{b}{a} \int \frac{x^2dx}{X}$$

$$\int \frac{dx}{x^3X} = -\frac{1}{2ax^2} - \frac{b}{a} \int \frac{xdx}{X}$$

$$\int \frac{dx}{x^4X} = -\frac{1}{3ax^3} - \frac{b}{a} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^5X} = -\frac{1}{4ax^4} - \frac{b}{a} \int \frac{dx}{xX}$$

$$\int \frac{dx}{x^6X} = -\frac{1}{5ax^5} + \frac{b}{a^2x} + \frac{b^2}{a^2} \int \frac{x^2dx}{X}$$

$$\int \frac{dx}{x^7X} = -\frac{1}{6ax^6} + \frac{b}{2a^2x^2} + \frac{b^2}{a^2} \int \frac{xdx}{X}$$

$$\int \frac{dx}{xX^2} = \frac{1}{4aX} + \frac{1}{a} \int \frac{dx}{xX}$$

$$\int \frac{dx}{x^2X^2} = \left(-\frac{1}{ax} - \frac{5bx^3}{4a^2}\right) \frac{1}{X} - \frac{5b}{4a^2} \int \frac{x^2dx}{X}$$

$$\int \frac{dx}{x^3X^2} = \left(-\frac{1}{2ax^2} - \frac{3bx^2}{4a^2}\right) \frac{1}{X} - \frac{3b}{2a^2} \int \frac{xdx}{X}$$

$$\int \frac{dx}{x^4X^2} = \left(-\frac{1}{3ax^3} - \frac{7bx}{12a^2}\right) \frac{1}{X} - \frac{7b}{4a^2} \int \frac{dx}{X}$$

$$\int \frac{dx}{x^5X^2} = \left(-\frac{1}{4ax^4} - \frac{b}{2a^2}\right) \frac{1}{X} - \frac{2b}{a^2} \int \frac{dx}{xX}$$

$$\int \frac{dx}{x^6X^2} = \left(-\frac{1}{5ax^5} + \frac{9b}{5a^2x} + \frac{9b^2x^3}{4a^3}\right) \frac{1}{X} + \frac{9b^2}{4a^3} \int \frac{x^2dx}{X}$$

$$\int \frac{dx}{x^7X^2} = \left(-\frac{1}{6ax^6} + \frac{5b}{6a^2x^2} + \frac{5b^2x^2}{4a^3}\right) \frac{1}{X} + \frac{5b^2}{2a^3} \int \frac{xdx}{X}$$

TAB. XLV.

$$\int \frac{x^m dx}{a + bx^5}$$

(*a* and *b* positive or negative.)

$$a + bx^5 = X, \sqrt[5]{\frac{a}{b}} = k$$

$$x^2 - 2kx \cos. 36^\circ + k^2 = Y$$

$$x^2 + 2kx \cos. 72^\circ + k^2 = Y'$$

$$\frac{x \sin. 36^\circ}{k - x \cos. 36^\circ} = Z, \frac{x \sin. 72^\circ}{k + x \cos. 72^\circ} = Z'$$

$$\int \frac{dx}{X} = \frac{1}{5bk^4} \left\{ \begin{array}{c} -\cos. 36^\circ \log. Y + 2 \sin. 36^\circ \text{ arc tang. } Z \\ + \cos. 72^\circ \log. Y' + 2 \sin. 72^\circ \text{ arc tang. } Z' \\ + \log. (x + k) \end{array} \right\}$$

$$\int \frac{xdx}{X} = \frac{1}{5bk^3} \left\{ \begin{array}{c} -\cos. 72^\circ \log. Y + 2 \sin. 72^\circ \text{ arc tang. } Z \\ + \cos. 36^\circ \log. Y' - 2 \sin. 36^\circ \text{ arc tang. } Z' \\ - \log. (x + k) \end{array} \right\}$$

$$\int \frac{x^2dx}{X} = \frac{1}{5bk^2} \left\{ \begin{array}{c} \cos. 72^\circ \log. Y + 2 \sin. 72^\circ \text{ arc tang. } Z \\ -\cos. 36^\circ \log. Y' - 2 \sin. 36^\circ \text{ arc tang. } Z' \\ + \log. (x + k) \end{array} \right\}$$

$$\int \frac{x^3dx}{X} = \frac{1}{5bk} \left\{ \begin{array}{c} \cos. 36^\circ \log. Y + 2 \sin. 36^\circ \text{ arc tang. } Z \\ -\cos. 72^\circ \log. Y' + 2 \sin. 72^\circ \text{ arc tang. } Z' \\ - \log. (x + k) \end{array} \right\}$$

$$\int \frac{x^4dx}{X} = \frac{1}{5b} \log. X$$

$$\int \frac{x^5dx}{X} = \frac{x}{b} - \frac{a}{b} \int \frac{dx}{X}$$

$$\int \frac{x^6dx}{X} = \frac{x^2}{2b} - \frac{a}{b} \int \frac{xdx}{X}$$

$$\int \frac{x^7dx}{X} = \frac{x^3}{3b} - \frac{a}{b} \int \frac{x^2dx}{X}$$

$$\int \frac{x^8dx}{X} = \frac{x^4}{4b} + \frac{a}{b} \int \frac{x^3dx}{X}$$

$$\int \frac{x^9dx}{X} = \frac{x^5}{5b} - \frac{a}{b} \int \frac{x^4dx}{X}$$

TAB. XLVI.

$$\int \frac{x^m \mathrm{d}x}{(a+bx^5)^2}, \quad \int \frac{\mathrm{d}x}{x^m(a+bx^5)}$$

$$a + bx^5 = X$$

$$\int \frac{\mathrm{d}x}{X^2} = \frac{x}{5aX} + \frac{4}{5a}\int \frac{\mathrm{d}x}{X}$$

$$\int \frac{x\mathrm{d}x}{X^2} = \frac{x^2}{5aX} + \frac{3}{5a}\int \frac{x\mathrm{d}x}{X}$$

$$\int \frac{x^2\mathrm{d}x}{X^2} = \frac{x^3}{5aX} + \frac{2}{5a}\int \frac{x^2\mathrm{d}x}{X}$$

$$\int \frac{x^3\mathrm{d}x}{X^2} = \frac{x^4}{5aX} + \frac{1}{5a}\int \frac{x^3\mathrm{d}x}{X}$$

$$\int \frac{x^4\mathrm{d}x}{X^2} = -\frac{1}{5bX}$$

$$\int \frac{x^5\mathrm{d}x}{X^2} = -\frac{x}{5bX} + \frac{1}{5b}\int \frac{\mathrm{d}x}{X}$$

$$\int \frac{x^6\mathrm{d}x}{X^2} = -\frac{x^2}{5bX} + \frac{2}{5b}\int \frac{x\mathrm{d}x}{X}$$

$$\int \frac{x^7\mathrm{d}x}{X^2} = -\frac{x^3}{5bX} + \frac{3}{5b}\int \frac{x^2\mathrm{d}x}{X}$$

$$\int \frac{\mathrm{d}x}{xX} = \frac{\log.\, x}{a} - \frac{\log.\, X}{5a} = \frac{1}{5a}\log.\frac{x^5}{X} = -\frac{1}{5a}\log.\frac{X}{x^5}$$

$$\int \frac{\mathrm{d}x}{x^2X} = -\frac{1}{ax} - \frac{b}{a}\int \frac{x^3\mathrm{d}x}{X}$$

$$\int \frac{\mathrm{d}x}{x^3X} = -\frac{1}{2ax^2} - \frac{b}{a}\int \frac{x^2\mathrm{d}x}{X}$$

$$\int \frac{\mathrm{d}x}{x^4X} = -\frac{1}{3ax^3} - \frac{b}{a}\int \frac{x\mathrm{d}x}{X}$$

$$\int \frac{\mathrm{d}x}{x^5X} = -\frac{1}{4ax^4} - \frac{b}{a}\int \frac{\mathrm{d}x}{X}$$

$$\int \frac{\mathrm{d}x}{x^6X} = -\frac{1}{5ax^5} - \frac{b}{a}\int \frac{\mathrm{d}x}{xX}$$

$$\int \frac{\mathrm{d}x}{x^7X} = -\frac{1}{6ax^6} + \frac{b}{a^2x} + \frac{b^2}{a^2}\int \frac{x^3\mathrm{d}x}{X}$$

TAB. XLVII. *a*.

$$\int \frac{x^m dx}{a+bx^6}$$

(*a* and *b* have the same signs.)

$$a + bx^6 = X, \ \sqrt[6]{\frac{a}{b}} = k$$

$$\int \frac{dx}{X} = \frac{1}{6bk^5}\left(\frac{\sqrt{3}}{2}\log.\frac{x^2+kx\sqrt{3}+k^2}{x^2-kx\sqrt{3}+k^2} + \text{arc. tang.}\frac{3kx\,(k^2-x^2)}{x^4-4k^2x^2+k^4}\right)$$

$$\int \frac{xdx}{X} = \frac{1}{6bk^4}\left(\tfrac{1}{2}\log.\frac{(x^2+k^2)^2}{x^4-k^2x^2+k^4} + \sqrt{3}\,.\,\text{arc tang.}\frac{x^2\sqrt{3}}{2k^2-x^2}\right)$$

$$\int \frac{x^2dx}{X} = \frac{1}{3bk^3}\ \text{arc tang.}\ x^3\sqrt{\frac{b}{a}}$$

$$\int \frac{x^3dx}{X} = \frac{1}{6bk^2}\left(\tfrac{1}{2}\log.\frac{x^4-k^2x^2+k^4}{(x^2+k^2)^2} + \sqrt{3}.\ \text{arc tang.}\frac{x^2\sqrt{3}}{2k^2-x^2}\right)$$

$$\int \frac{x^4dx}{X} = \frac{1}{6bk}\left(\frac{\sqrt{3}}{2}\log.\frac{x^2-kx\sqrt{3}+k^2}{x^2+kx\sqrt{3}+k^2} + \text{arc tang.}\frac{3kx(k^2-x^2)}{x^4-4k^2x^2+k^4}\right)$$

$$\int \frac{x^5dx}{X} = \frac{1}{6b}\log.X$$

$$\int \frac{x^6dx}{X} = \frac{x}{b} - \frac{a}{b}\int\frac{dx}{X}$$

$$\int \frac{x^7dx}{X} = \frac{x^2}{2b} - \frac{a}{b}\int\frac{xdx}{X}$$

$$\int \frac{x^8dx}{X} = \frac{x^3}{3b} - \frac{a}{b}\int\frac{x^2dx}{X}$$

$$\int \frac{x^9dx}{X} = \frac{x^4}{4b} - \frac{a}{b}\int\frac{x^3dx}{X}$$

These Integrals vanish altogether when $x=0$. We also have

$$\log.\frac{x^4-k^2x^2+k^4}{(x^2+k^2)^2} + \text{const.} = \log.\frac{X}{(x^2+k^2)^3} = 3\log.\frac{\sqrt[3]{X}}{x^2+k^2}.$$

TAB. XLVII. *b.*

$$\int \frac{x^m dx}{a + bx^6}$$

(*a* and *b* having different signs.)

$$a + bx^6 = X, \quad \sqrt[6]{-\frac{a}{b}} = k$$

$$\int \frac{dx}{X} = \frac{-1}{6bk^5}\left(\tfrac{1}{2}\log.\frac{(x+k)^2(x^2+kx+k^2)}{(x-k)^2(x^2-kx+k^2)} + \sqrt{3}.\ \text{arc tang.}\frac{kx\sqrt{3}}{k^2-x^2}\right)$$

$$\int \frac{x dx}{X} = \frac{-1}{6bk^4}\left(\tfrac{1}{2}\log.\frac{x^4+k^2x^2+k^4}{(x^2-k^2)^2} + \sqrt{3}.\ \text{arc tang.}\frac{x^2\sqrt{3}}{2k^2+x^2}\right)$$

$$\int \frac{x^2 dx}{X} = \frac{-1}{6bk^3}\left(-\log.\frac{x^3+k^3}{x^3-k^3}\right.$$

$$\int \frac{x^3 dx}{X} = \frac{-1}{6bk^2}\left(\tfrac{1}{2}\log.\frac{x^4+k^2x^2+k^4}{(x^2-k^2)^2} - \sqrt{3}\ \text{arc tang.}\frac{x^2\sqrt{3}}{2k^2+x^2}\right)$$

$$\int \frac{x^4 dx}{X} = \frac{-1}{6bk}\left(\tfrac{1}{2}\log.\frac{(x+k)^2(x^2+kx+k^2)}{(x-k)^2(x^2-kx+k^2)} - \sqrt{3}\,\text{arc tang.}\frac{kx\sqrt{3}}{k^2-x^2}\right)$$

The remaining Integrals as in Tab. XLVII. *a.*

The Integrals vanish altogether when $x = 0$. Also

$$\log.\frac{x^4+k^2x^2+k^4}{(x^2-k^2)^2} + \text{const.} = \log.\frac{X}{(x^2-k^2)^3} = 3\log.\frac{\sqrt[3]{X}}{x^2-k^2}.$$

TAB. XLVIII.

$$\int \frac{x^m \mathrm{d}x}{(a+bx^6)^2},\quad \int \frac{\mathrm{d}x}{x^m(a+bx^6)}$$

$$a+bx^6=X$$

$$\int \frac{\mathrm{d}x}{X^2}=\frac{x}{6aX}+\frac{5}{6a}\int\frac{\mathrm{d}x}{X}$$

$$\int \frac{x\,\mathrm{d}x}{X^2}=\frac{x^2}{6aX}+\frac{2}{3a}\int\frac{x\,\mathrm{d}x}{X}$$

$$\int \frac{x^2\mathrm{d}x}{X^2}=\frac{x^3}{6aX}+\frac{1}{2a}\int\frac{x^2\mathrm{d}x}{X}$$

$$\int \frac{x^3\mathrm{d}x}{X^2}=\frac{x^4}{6aX}+\frac{1}{3a}\int\frac{x^3\mathrm{d}x}{X}$$

$$\int \frac{x^4\mathrm{d}x}{X^2}=\frac{x^5}{6aX}+\frac{1}{6a}\int\frac{x^4\mathrm{d}x}{X}$$

$$\int \frac{x^5\mathrm{d}x}{X^2}=-\frac{1}{6bX}$$

$$\int \frac{x^6\mathrm{d}x}{X^2}=-\frac{x}{6bX}+\frac{1}{6b}\int\frac{\mathrm{d}x}{X}$$

$$\int \frac{x^7\mathrm{d}x}{X^2}=-\frac{x^2}{6bX}+\frac{1}{3b}\int\frac{x\,\mathrm{d}x}{X}$$

$$\int \frac{\mathrm{d}x}{xX}=\frac{\log. x}{a}-\frac{\log. X}{6a}=\frac{1}{6a}\log.\frac{x^6}{X}=-\frac{1}{6a}\log.\frac{X}{x^6}$$

$$\int \frac{\mathrm{d}x}{x^2X}=-\frac{1}{ax}-\frac{b}{a}\int\frac{x^4\mathrm{d}x}{X}$$

$$\int \frac{\mathrm{d}x}{x^3X}=-\frac{1}{2ax^2}-\frac{b}{a}\int\frac{x^3\mathrm{d}x}{X}$$

$$\int \frac{\mathrm{d}x}{x^4X}=-\frac{1}{3ax^3}-\frac{b}{a}\int\frac{x^2\mathrm{d}x}{X}$$

$$\int \frac{\mathrm{d}x}{x^5X}=-\frac{1}{4ax^4}-\frac{b}{a}\int\frac{x\,\mathrm{d}x}{X}$$

$$\int \frac{\mathrm{d}x}{x^6X}=-\frac{1}{5ax^5}-\frac{b}{a}\int\frac{\mathrm{d}x}{X}$$

$$\int \frac{\mathrm{d}x}{x^7X}=-\frac{1}{6ax^6}-\frac{b}{a}\int\frac{\mathrm{d}x}{xX}$$

TAB. XLIX. a.

$$\int \frac{x^m dx}{a + bx^2 + cx^4}$$

($b^2 - 4ac$ a positive quantity.)

$$a + bx^2 + cx^4 = X$$
$$\tfrac{1}{2}b - \tfrac{1}{2}\sqrt{(b^2 - 4ac)} = f,\ \tfrac{1}{2}b + \tfrac{1}{2}\sqrt{(b^2 - 4ac)} = g$$
$$\sqrt{(b^2 - 4ac)} = g - f = h$$

$$\int \frac{dx}{X} = \frac{c}{h}\left[\int \frac{dx}{cx^2+f} - \int \frac{dx}{cx^2+g}\right]$$

$$\int \frac{xdx}{X} = \frac{1}{2h}\log.\frac{cx^2+f}{cx^2+g}$$

$$\int \frac{x^2dx}{X} = \frac{g}{h}\int \frac{dx}{cx^2+g} - \frac{f}{h}\int \frac{dx}{cx^2+f}$$

$$\int \frac{x^3dx}{X} = \frac{1}{2h}\left[g\log.(cx^2+g) - f\log.(cx^2+f)\right]$$

$$\int \frac{x^4dx}{X} = \frac{x}{c} - \frac{a}{c}\int \frac{dx}{X} - \frac{b}{c}\int \frac{x^2dx}{X}$$

$$\int \frac{x^5dx}{X} = \frac{x^2}{2c} - \frac{a}{c}\int \frac{xdx}{X} - \frac{b}{c}\int \frac{x^3dx}{X}$$

$$\int \frac{x^6dx}{X} = \frac{x^3}{3c} - \frac{bx}{c^2} + \frac{ab}{c^2}\int \frac{dx}{X} + \left(\frac{b^2}{c^2} - \frac{a}{c}\right)\int \frac{x^2dx}{X}$$

$$\int \frac{x^7dx}{X} = \frac{x^4}{4c} - \frac{bx^2}{2c^2} + \frac{ab}{c^2}\int \frac{xdx}{X} + \left(\frac{b^2}{c^2} - \frac{a}{c}\right)\int \frac{x^3dx}{X}$$

$$\int \frac{x^8dx}{X} = \frac{x^5}{5c} - \frac{bx^3}{3c^2} + \left(\frac{b^2}{c^3} - \frac{a}{c^2}\right)x - \left(\frac{ab^2}{c^3} - \frac{a^2}{c^2}\right)\int \frac{dx}{X}$$
$$- \left(\frac{b^3}{c^3} - \frac{2ab}{c^2}\right)\int \frac{x^2dx}{X}$$

$$\int \frac{x^9dx}{X} = \frac{x^6}{6c} - \frac{bx^4}{4c^2} + \left(\frac{b^2}{2c^3} - \frac{a}{2c^2}\right)x^2 - \left(\frac{ab^2}{c^3} - \frac{a^2}{c^2}\right)\int \frac{xdx}{X}$$
$$- \left(\frac{b^3}{c^3} - \frac{2ab}{c^2}\right)\int \frac{x^3dx}{X}$$

$$\int \frac{x^{10}dx}{X} = \frac{x^7}{7c} - \frac{a}{c}\int \frac{x^6dx}{X} - \frac{b}{c}\int \frac{x^8dx}{X}$$

TAB. XLIX. *b.*

$$\int \frac{x^m dx}{a + bx^2 + cx^4}$$

($b^2 - 4ac$ a negative quantity.)

$$a + bx^2 + cx^4 = X, \quad \sqrt[4]{\frac{a}{c}} = f$$

α an angle, whose cosine $= -\dfrac{b}{2\sqrt{ac}}$

$$\int \frac{dx}{X} = \frac{1}{4cf^3 \sin.\alpha} \left\{ \begin{array}{l} \sin.\frac{\alpha}{2} \log.\dfrac{x^2 + 2fx\cos.\frac{\alpha}{2} + f^2}{x^2 - 2fx\cos.\frac{\alpha}{2} + f^2} \\ + 2\cos.\frac{\alpha}{2} \text{ arc tang.} \dfrac{2fx\sin.\frac{\alpha}{2}}{f^2 - x^2} \end{array} \right\}$$

$$\int \frac{xdx}{X} = \frac{1}{2cf^2 \sin.\alpha} \text{ arc tang.} \frac{f^2 \sin.\alpha}{f^2\cos.\alpha - x^2}$$

$$\int \frac{x^2dx}{X} = \frac{1}{4cf \sin.\alpha} \left\{ \begin{array}{l} \sin.\frac{\alpha}{2} \log.\dfrac{x^2 - 2fx\cos.\frac{\alpha}{2} + f^2}{x^2 + 2fx\cos.\frac{\alpha}{2} + f^2} \\ + 2\cos.\frac{\alpha}{2} \text{ arc tang.} \dfrac{2fx\sin.\frac{\alpha}{2}}{f^2 - x^2} \end{array} \right\}$$

$$\int \frac{x^3dx}{X} = \frac{1}{4c \sin.\alpha} \left\{ \begin{array}{l} \sin.\alpha \log.(x^4 - 2f^2x^2\sin.\alpha + f^4) \\ + 2\cos.\alpha \text{ arc tang.} \dfrac{2fx\sin.\frac{\alpha}{2}}{f^2 - x^2} \end{array} \right\}$$

The remaining Integrals as in Tab. XLIX *a.*

An angle whose cosine $= -\dfrac{b}{2\sqrt{ac}}$ may always be found; and the angle is acute or obtuse according as this quantity is positive or negative.

TAB. L.

$$\int \frac{x^m dx}{(a+bx^2+cx^4)^2}$$

$$a+bx^2+cx^4=X,\ 2a\,(b^2-4ac)=k$$

$$\int\frac{dx}{X^2}=[bcx^3+(b^2-2ac)\,x]\frac{1}{kX}+\frac{b^2-6ac}{k}\int\frac{dx}{X}+\frac{bc}{k}\int\frac{x^2dx}{X}$$

$$\int\frac{xdx}{X^2}=[bcx^4+(b^2-2ac)\,x^2]\frac{1}{kX}-\frac{4ac}{k}\int\frac{xdx}{X}$$

$$\int\frac{x^2dx}{X^2}=[bcx^5+(b^2-2ac)\,x^3]\frac{1}{kX}-\frac{bx}{k}+\frac{ab}{k}\int\frac{dx}{X}-\frac{2ac}{k}\int\frac{x^2dx}{X}$$

$$\int\frac{x^3dx}{X^2}=[bcx^6+(b^2-2ac)\,x^4]\frac{1}{kX}-\frac{bx^2}{k}+\frac{2ab}{k}\int\frac{xdx}{X}$$

$$\int\frac{x^4dx}{X^2}=-\frac{x}{3cX}+\frac{a}{3c}\int\frac{dx}{X^2}-\frac{b}{3c}\int\frac{x^2dx}{X}$$

$$\int\frac{x^5dx}{X^2}=-\frac{x^2}{2cX}+\frac{a}{c}\int\frac{xdx}{X^2}$$

$$\int\frac{x^6dx}{X^2}=\left(-\frac{x^3}{c}-\frac{bx}{3c^2}\right)\frac{1}{X}+\frac{ab}{3c^2}\int\frac{dx}{X^2}-\left(\frac{b^2}{3c^2}-\frac{3a}{c}\right)\int\frac{x^2dx}{X^2}$$

$$\int\frac{x^7dx}{X^2}=[bcx^{10}+(b^2-2ac)\,x^8]\ \frac{1}{kX}+\frac{8ac-6b^2}{k}\int\frac{x^7dx}{X}-\frac{6bc}{k}\int\frac{x^9dx}{X}$$

$$\int\frac{x^8dx}{X^2}=\frac{x^5}{cX}-\frac{5a}{c}\int\frac{x^4dx}{X^2}-\frac{3b}{c}\int\frac{x^6dx}{X^2}$$

$$\int\frac{x^9dx}{X^2}=\frac{x^6}{2cX}-\frac{3a}{c}\int\frac{x^5dx}{X^2}-\frac{2b}{c}\int\frac{x^7dx}{X^2}$$

TAB. LI.

$$\int \frac{dx}{x^m(a+bx^2+cx^4)}$$

$$a + bx^2 + cx^4 = X$$

$$\int \frac{dx}{xX} = \frac{\log. x}{a} - \frac{b}{a}\int\frac{xdx}{X} - \frac{c}{a}\int\frac{x^3dx}{X}$$

$$\int \frac{dx}{x^2X} = -\frac{1}{ax} - \frac{b}{a}\int\frac{dx}{X} - \frac{c}{a}\int\frac{x^2dx}{X}$$

$$\int \frac{dx}{x^3X} = -\frac{1}{2ax^2} - \frac{b}{a}\int\frac{dx}{xX} - \frac{c}{a}\int\frac{xdx}{X}$$

$$\int \frac{dx}{x^4X} = -\frac{1}{3ax^3} + \frac{b}{a^2x} + \left(\frac{b^2}{a^2} - \frac{c}{a}\right)\int\frac{dx}{X} + \frac{bc}{a^2}\int\frac{x^2dx}{X}$$

$$\int \frac{dx}{x^5X} = -\frac{1}{4ax^4} + \frac{b}{2a^2x^2} + \left(\frac{b^2}{a^2} - \frac{c}{a}\right)\int\frac{dx}{xX} + \frac{bc}{a^2}\int\frac{xdx}{X}$$

$$\int \frac{dx}{x^6X} = -\frac{1}{5ax^4} + \frac{b}{3a^2x^3} - \left(\frac{b^2}{a^3} - \frac{c}{a^2}\right)\frac{1}{x} - \left(\frac{b^3}{a^3} - \frac{2bc}{a^2}\right)\int\frac{dx}{X}$$
$$- \left(\frac{b^2c}{a^3} - \frac{c^2}{a^2}\right)\int\frac{x^2dx}{X}$$

$$\int \frac{dx}{x^7X} = -\frac{1}{6ax^6} - \frac{b}{a}\int\frac{dx}{x^5X} - \frac{c}{a}\int\frac{dx}{x^3X}$$

$$\int \frac{dx}{x^8X} = -\frac{1}{7ax^7} - \frac{b}{a}\int\frac{dx}{x^6X} - \frac{c}{a}\int\frac{dx}{x^4X}$$

$$\int \frac{dx}{x^9X} = -\frac{1}{8ax^8} + \frac{b}{6a^2x^6} + \left(\frac{b^2}{a^2} - \frac{c}{a}\right)\int\frac{dx}{x^5X} + \frac{bc}{a^2}\int\frac{dx}{x^3X}$$

$$\int \frac{dx}{x^{10}X} = -\frac{1}{9ax^9} + \frac{b}{7a^2x^7} + \left(\frac{b^2}{a^2} - \frac{c}{a}\right)\int\frac{dx}{x^6X} + \frac{bc}{a^2}\int\frac{dx}{x^4X}$$

$$\int \frac{dx}{x^{11}X} = -\frac{1}{10ax^{10}} + \frac{b}{8a^2x^8} - \left(\frac{b^2}{6a^3} - \frac{c}{6a^2}\right)\frac{1}{x^6} - \left(\frac{b^3}{a^3} - \frac{2bc}{a^2}\right)\int\frac{dx}{x^5X}$$
$$- \left(\frac{b^2c}{a^3} - \frac{c^2}{a^2}\right)\int\frac{dx}{x^3X}$$

TAB. LII.

$$\int \frac{dx}{x^m(a+bx^2+cx^4)^2}$$

$$a+bx^2+cx^4=X,\quad 2a\,(b^2-4ac)=k$$

$$\int\frac{dx}{xX^2}=\frac{bcx^2+b^2-2ac}{kX}+\frac{\log.\,x}{a^2}+\left(\frac{2bc}{k}-\frac{b}{a^2}\right)\int\frac{x\,dx}{X}-\frac{bc}{a^2}\int\frac{x^3dx}{X}$$

$$\int\frac{dx}{x^2X^2}=-\frac{1}{axX}-\frac{3b}{a}\int\frac{dx}{X^2}-\frac{5c}{a}\int\frac{x^2dx}{X^2}$$

$$\int\frac{dx}{x^3X^2}=-\frac{1}{2ax^2X}-\frac{2b}{a}\int\frac{dx}{xX^2}-\frac{3c}{a}\int\frac{x\,dx}{X^2}$$

$$\int\frac{dx}{x^4X^2}=\left(-\frac{1}{3ax^3}+\frac{5b}{3a^2x}\right)\frac{1}{X}+\left(\frac{5b^2}{a^2}-\frac{7c}{3a}\right)\int\frac{dx}{X^2}+\frac{25c^2}{3a^2}\int\frac{x^2dx}{X^2}$$

$$\int\frac{dx}{x^5X^2}=\left(-\frac{1}{4ax^4}+\frac{3b}{4a^2x^2}\right)\frac{1}{X}+\left(\frac{3b^2}{a^2}-\frac{2c}{a}\right)\int\frac{dx}{xX^2}+\frac{9bc}{2a^2}\int\frac{x\,dx}{X^2}$$

$$\int\frac{dx}{x^6X^2}=-\frac{1}{5ax^5X}-\frac{7b}{5a}\int\frac{dx}{x^4X^2}-\frac{9c}{5a}\int\frac{dx}{x^2X^2}$$

$$\int\frac{dx}{x^7X^2}=-\frac{1}{6ax^6X}-\frac{4b}{3a}\int\frac{dx}{x^5X^2}-\frac{5c}{3a}\int\frac{dx}{x^3X^2}$$

$$\int\frac{dx}{x^8X^2}=\left(-\frac{1}{7ax^7}+\frac{9b}{35a^2x^5}\right)\frac{1}{X}+\left(\frac{9b^2}{5a^2}-\frac{11c}{7a}\right)\int\frac{dx}{x^4X^2}+\frac{81bc}{35a^2}\int\frac{dx}{x^2X^2}$$

$$\int\frac{dx}{x^9X^2}=\left(-\frac{1}{8ax^8}+\frac{15b}{24a^2x^6}\right)\frac{1}{X}+\left(\frac{5b^2}{3a^2}-\frac{3c}{2a}\right)\int\frac{dx}{x^5X^2}+\frac{25bc}{12a^2}\int\frac{dx}{x^3X}$$

TAB. LIII. *a*.

$$\int \frac{x^m dx}{a + bx^3 + cx^6}$$

(b^2—$4ac$ a separate quantity.)

$$a + bx^3 + cx^6 = X$$
$$\tfrac{1}{2}b - \tfrac{1}{2}\sqrt{(b^2 - 4ac)} = f,\ \tfrac{1}{2}b + \tfrac{1}{2}\sqrt{(b^2 - 4ac)} = g$$
$$\sqrt{(b^2 - 4ac)} = g - f = h$$

$$\int \frac{dx}{X} = \frac{c}{h}\left[\int \frac{dx}{cx^3 + f} - \int \frac{dx}{cx^3 + g}\right]$$

$$\int \frac{xdx}{X} = \frac{c}{h}\left[\int \frac{xdx}{cx^3 + f} - \int \frac{xdx}{cx^3 + g}\right]$$

$$\int \frac{x^2dx}{X} = \frac{1}{3h} \log. \frac{cx^3 + f}{cx^3 + g}$$

$$\int \frac{x^3dx}{X} = \frac{g}{h}\int \frac{dx}{cx^3 + g} - \frac{f}{h}\int \frac{dx}{cx^3 + f}$$

$$\int \frac{x^4dx}{X} = \frac{g}{h}\int \frac{xdx}{cx^3 + g} - \frac{f}{h}\int \frac{xdx}{cx^3 + f}$$

$$\int \frac{x^5dx}{X} = \frac{g}{3h} \log. (cx^3 + g) - \frac{f}{3h} \log. (cx^3 + f)$$

$$\int \frac{x^6dx}{X} = \frac{x}{c} - \frac{a}{c}\int \frac{dx}{X} - \frac{b}{c}\int \frac{x^3dx}{X}$$

$$\int \frac{x^7dx}{X} = \frac{x^2}{2c} - \frac{a}{c}\int \frac{xdx}{X} - \frac{b}{c}\int \frac{x^4dx}{X}$$

$$\int \frac{x^8dx}{X} = \frac{x^3}{3c} - \frac{a}{c}\int \frac{x^2dx}{X} - \frac{b}{c}\int \frac{x^5dx}{X}$$

$$\int \frac{x^9dx}{X} = \frac{x^4}{4c} - \frac{bx}{c^2} + \frac{ab}{c^2}\int \frac{dx}{X} + \left(\frac{b^2}{c^2} - \frac{a}{c}\right)\int \frac{x^3dx}{X}$$

$$\int \frac{x^{10}dx}{X} = \frac{x^5}{5c} - \frac{bx^2}{2c^2} + \frac{ab}{c^2}\int \frac{xdx}{X} + \left(\frac{b^2}{c^2} - \frac{a}{c}\right)\int \frac{x^4dx}{X}$$

TAB. LIII. *b*.

$$\int \frac{x^m dx}{(a + bx^3 + cx^6}$$

($b^2 - 4ac$ a negative quantity.)

$$a + bx^3 + cx^6 = X, \sqrt[6]{\frac{a}{c}} = f$$

α an angle, whose cosine $= -\dfrac{b}{2\sqrt{ac}}$

$$\frac{\alpha}{3} = \varphi', \; 120° + \frac{\alpha}{3} = \varphi'', \; 240° + \frac{\alpha}{3} = \varphi'''$$

$$x^2 - 2fx \cos. \varphi' + f^2 = Y'$$

$$x^2 - 2fx \cos. \varphi'' + f^2 = Y''$$

$$x^2 - 2fx \cos. \varphi''' + f^2 = Y'''$$

$$\frac{x \sin. \varphi'}{f - x \cos. \varphi'} = Z', \; \frac{x \sin. \varphi''}{f - x \cos. \varphi''} = Z'',$$

$$\frac{x \sin. \varphi'''}{f - x \cos. \varphi'''} = Z'''$$

$$\int \frac{dx}{X} = \frac{1}{6cf^5 \sin. \alpha} \left\{ \begin{array}{l} -\sin 2\varphi' \log Y' + 2 \cos 2\varphi' \text{ arc tang } Z' \\ -\sin 2\varphi'' \log Y'' + 2 \cos 2\varphi'' \text{ arc tang } Z'' \\ -\sin 2\varphi''' \log Y''' + 2 \cos 2\varphi''' \text{ arc tang } Z''' \end{array} \right\}$$

$$\int \frac{x dx}{X} = \frac{1}{6cf^4 \sin. \alpha} \left\{ \begin{array}{l} -\sin \varphi' \log X' + 2 \cos \varphi' \text{ arc tang } Z' \\ -\sin \varphi'' \log Y'' + 2 \cos \varphi'' \text{ arc tang } Z'' \\ -\sin \varphi''' \log Y''' + 2 \cos \varphi''' \text{ arc tang } Z''' \end{array} \right\}$$

$$\int \frac{x^2 dx}{X} = \frac{1}{3cf^3 \sin. \alpha} \text{ arc. tang. } \frac{x^3 \sin. \alpha}{f^3 - x^3 \cos. \alpha}$$

$$\int \frac{x^3 dx}{X} = \frac{1}{6cf^2 \sin. \alpha} \left\{ \begin{array}{l} \sin \varphi' \log Y' + 2 \cos \varphi' \text{ arc tang } Z' \\ +\sin \varphi'' \log Y'' + 2 \cos \varphi'' \text{ arc tang } Z'' \\ +\sin \varphi''' \log Y''' + 2 \cos \varphi''' \text{ arc tang } Z''' \end{array} \right\}$$

$$\int \frac{x^4 dx}{X} = \frac{1}{6cf \sin, \alpha} \left\{ \begin{array}{l} \sin 2\varphi' \log Y' + 2 \cos 2\varphi' \text{ arc tang } Z' \\ +\sin 2\varphi'' \log Y'' + 2 \cos 2\varphi'' \text{ arc tang } Z'' \\ +\sin 2\varphi''' \log Y''' + 2 \cos 2\varphi''' \text{ arc tang } Z''' \end{array} \right\}$$

$$\int \frac{x^5 dx}{X} = \frac{1}{6c \sin. \alpha} \left\{ \begin{array}{l} \sin 3\varphi' \log Y' + 2 \cos 3\varphi' \text{ arc tang } Z' \\ +\sin 3\varphi'' \log Y'' + 2 \cos 3\varphi'' \text{ arc tang } Z'' \\ +\sin 3\varphi''' \log Y''' + 2 \cos 3\varphi''' \text{ arc tang } Z''' \end{array} \right\}$$

TAB. LIV.

$$\int \frac{x^m \mathrm{d}x}{(a+bx^3+cx^6)^2},\ \frac{\mathrm{d}x}{x^m(a+bx^3+cx^6)}$$

$$a + bx^3 + cx^6 = X,\ 3a\,(b^2 - 4ac) = k$$

$$\int \frac{\mathrm{d}x}{X^2} = [bcx^4+(b^2-2ac)\,x]\,\frac{1}{kX} + \frac{2b^2-10ac}{k}\int\frac{\mathrm{d}x}{X} + \frac{2bc}{k}\int\frac{x^3\mathrm{d}x}{X}$$

$$\int \frac{x\mathrm{d}x}{X^2} = [bcx^5+(b^2-2ac)x^2]\,\frac{1}{kX} + \frac{b^2-8ac}{k}\int\frac{x\mathrm{d}x}{X} + \frac{bc}{k}\int\frac{x^4\mathrm{d}x}{X}$$

$$\int \frac{x^2\mathrm{d}x}{X^2} = [bcx^6+(b^2-2ac)x^3]\,\frac{1}{kX} - \frac{6ac}{k}\int\frac{x^2\mathrm{d}x}{X}$$

$$\int \frac{x^3\mathrm{d}x}{X^2} = [bcx^7+(b^2-2ac)\,x^4]\,\frac{1}{kX} - \frac{b^2+4ac}{k}\int\frac{x^3\mathrm{d}x}{X} - \frac{bc}{k}\int\frac{x^6\mathrm{d}x}{X}$$

$$\int \frac{\mathrm{d}x}{xX} = \frac{\log x}{a} - \frac{b}{a}\int\frac{x^2\mathrm{d}x}{X} - \frac{c}{a}\int\frac{x^5\mathrm{d}x}{X}$$

$$\int \frac{\mathrm{d}x}{x^2X} = -\frac{1}{ax} - \frac{b}{a}\int\frac{x\mathrm{d}x}{X} - \frac{c}{a}\int\frac{x^4\mathrm{d}x}{X}$$

$$\int \frac{\mathrm{d}x}{x^3X} = -\frac{1}{2ax^2} - \frac{b}{a}\int\frac{\mathrm{d}x}{X} - \frac{c}{a}\int\frac{x^3\mathrm{d}x}{X}$$

$$\int \frac{\mathrm{d}x}{x^4X} = -\frac{1}{3ax^3} - \frac{b}{a}\int\frac{\mathrm{d}x}{xX} - \frac{c}{a}\int\frac{x^2\mathrm{d}x}{X}$$

$$\int \frac{\mathrm{d}x}{x^5X} = -\frac{1}{4ax^4} + \frac{b}{a^2x} + \left(\frac{b^2}{a^2} - \frac{c}{a}\right)\int\frac{x\mathrm{d}x}{X} + \frac{bc}{a^2}\int\frac{x^4\mathrm{d}x}{X}$$

$$\int \frac{\mathrm{d}x}{x^6X} = -\frac{1}{5ax^5} + \frac{b}{2a^2x^2} + \left(\frac{b^2}{a^2} - \frac{c}{a}\right)\int\frac{\mathrm{d}x}{X} + \frac{bc}{a^2}\int\frac{x^3\mathrm{d}x}{X}$$

$$\int \frac{\mathrm{d}x}{x^7X} = -\frac{1}{6ax^6} + \frac{b}{3a^2x^3} + \left(\frac{b^2}{a^2} - \frac{c}{a}\right)\int\frac{\mathrm{d}x}{xX} + \frac{bc}{a^2}\int\frac{x^2\mathrm{d}x}{X}$$

$$\int \frac{\mathrm{d}x}{x^8X} = -\frac{1}{7ax^7} + \frac{b}{4a^2x^4} - \left(\frac{b^2}{a^3} - \frac{c}{a^2}\right)\frac{1}{x} - \left(\frac{b^3}{a^3} - \frac{2bc}{a^2}\right)\int\frac{x\mathrm{d}x}{X}$$
$$- \left(\frac{b^2c}{a^3} - \frac{c^2}{a^2}\right)\int\frac{x^4\mathrm{d}x}{X}$$

TAB. LV

$$\int \frac{x^m dx}{X}$$

(X a product of binomial and trinomial factors.)

$$\int \frac{dx}{(x+f)(x+g)} = \frac{1}{g-f} \log. \frac{x+f}{x+g}$$

$$\int \frac{x dx}{(x+f)(x+g)} = \frac{1}{g-f} [g \log. (x+g) - f \log. (x+f)]$$

$$\int \frac{dx}{(x+f)(x+g)^2} = \frac{1}{(g-f)(x+g)} + \frac{1}{(g-f)^2} \log. \frac{x+f}{x+g}$$

$$\int \frac{x dx}{(x+f)(x+g)^2} = \frac{-g}{(g-f)(x+g)} - \frac{f}{(g-f)^2} \log. \frac{x+f}{x+g}$$

$$\int \frac{x^2 dx}{(x+f)(x+g)^2} = \frac{g^2}{(g-f)(x+g)} + \frac{f^2}{(g-f)^2} \log. (x+f) + \frac{g^2-2fg}{(g-f)^2} \log. (x+g)$$

$$\int \frac{dx}{(x+f)^2(x+g)^2} = \frac{-1}{(g-f)^2}\left(\frac{1}{x+f}+\frac{1}{x+g}\right) - \frac{2}{(g-f)^3} \log. \frac{x+f}{x+g}$$

$$\int \frac{x dx}{(x+f)^2(x+g)^2} = \frac{1}{(g-f)^2}\left(\frac{f}{x+f}+\frac{g}{x+g}\right) - \frac{f+g}{(g-f)^3} \log. \frac{x+f}{x+g}$$

$$\int \frac{x^2 dx}{(x+f)^2(x+g)^2} = \frac{-1}{(g-f)^2}\left(\frac{f^2}{x+f}+\frac{g^2}{x+g}\right) - \frac{2fg}{(g-f)^3} \log. \frac{x+f}{x+g}$$

$$\int \frac{x^3 dx}{(x+f)^2(x+g)^2} = \frac{1}{(g-f)^2}\left(\frac{f^3}{x+f}+\frac{g^3}{x+g}\right) + \frac{f^2(3g-f)}{(g-f)^3} \log. (x+f) + \frac{g^2(g-3f)}{(g-f)^3} \log. (x+g)$$

$$\int \frac{dx}{(x+f)(x+g)(x+h)} = \frac{1}{(g-f)(h-f)} \log. (x+f) + \frac{1}{(f-g)(h-g)} \log. (x+g) + \frac{1}{(f-h)(g-h)} \log. (x+h)$$

$$\int \frac{x dx}{(x+f)(x+g)(x+h)} = -\frac{f}{(g-f)(h-g)} \log. (x+f) - \frac{g}{(f-g)(h-g)} \log. (x+g) - \frac{h}{(f-h)(g-h)} \log. (x+h)$$

TAB. LV.

$$\int \frac{x^m dx}{X}$$

(X a product of binomial and trinomial factors.)

$$\int \frac{x^2 dx}{(x+f)(x+g)(x+h)} = \frac{f^2}{(g-f)(h-f)} \log.(x+f) + \frac{g^2}{(f-g)(h-g)} \log.(x+g) + \frac{h^2}{(f-h)(g-h)} \log.(x+h)$$

$$\int \frac{dx}{(x+f)(x^2+a)} = \frac{1}{f^2+a}\left[\log.\frac{x+f}{\sqrt{(x^2+a)}} + f\int\frac{dx}{x^2+a}\right]$$

$$\int \frac{x dx}{(x+f)(x^2+a)} = \frac{1}{f^2+a}\left[f \log.\frac{\sqrt{(x^2+a)}}{x+f} + a\int\frac{dx}{x^2+a}\right]$$

$$\int \frac{x^2 dx}{(x+f)(x^2+a)} = \frac{1}{f^2+a}\left[f^2 \log.(x+f) + \tfrac{1}{2} a \log.(x^2+a)\right] - \frac{af}{f^2+a}\int\frac{dx}{x^2+a}$$

$$\int \frac{dx}{(x^2+a)(x^2+b)} = \frac{1}{b-a}\left[\int\frac{dx}{x^2+a} - \int\frac{dx}{x^2+b}\right]$$

$$\int \frac{x dx}{(x^2+a)(x^2+b)} = \frac{1}{2(b-a)} \log.\frac{x^2+a}{x^2+b}$$

$$\int \frac{x^2 dx}{(x^2+a)(x^2+b)} = \frac{1}{a-b}\left[a\int\frac{dx}{x^2+a} - b\int\frac{dx}{x^2+b}\right]$$

$$\int \frac{dx}{(x+f)^2(x^2+a)} = \frac{1}{(f^2+a)^2}\left[f \log.\frac{(x+f)^2}{x^2+a} + (f^2-a)\int\frac{dx}{x^2+a}\right] - \frac{1}{(f+a)(x+f)}$$

$$\int \frac{x dx}{(x+f)^2(x^2+a)} = \frac{1}{(f^2+a)^2}\left[\frac{a-f^2}{2} \log.\frac{(x+f)^2}{x^2+a} + 2af\int\frac{dx}{x^2+a}\right] + \frac{f}{(f^2+a)(x+f)}$$

$$\int \frac{dx}{(x+f)^2(x^2+a)} = \frac{1}{(f^2+a)^2}\left[- af \log.\frac{(x+f)^2}{x^2+a} - a(f^2-a)\int\frac{dx}{x^2+a}\right] - \frac{f^2}{(f^2+a)(x+f)}$$

TAB. LV.

$$\int \frac{x^m dx}{X}$$

(*X* a product of binomial and trinomial factors.)

$$\int \frac{x^3 dx}{(x+f)^2(x^2+a)} = \frac{f^2(f^2+3a)}{(f^2+a)^2}\log.(x+f) - \frac{a(f^2-a)}{2(f^2+a)^2}\log.(x^2+a)$$
$$- \frac{2a^2 f}{(f^2+a)^2}\int\frac{dx}{x^2+a} + \frac{f^3}{(f^2+a)\,(x+f)}$$

$$\int \frac{dx}{(x+f)\,(x^2+ax+b)} = \frac{1}{f^2-af+b} \times$$
$$\left[\tfrac{1}{2}\log.\frac{(x+f)^2}{x^2+ax+b} + (f-\tfrac{1}{2}a)\int\frac{dx}{x^2+ax+b}\right]$$

$$\int \frac{x dx}{(x+f)\,(x^2+ax+b)} = \frac{1}{f^2-af+b} \times$$
$$\left[-\tfrac{1}{2}f\log.\frac{(x+f)^2}{x^2+ax+b} + (b-\tfrac{1}{2}af)\int\frac{dx}{x^2+ax+b}\right]$$

$$\int \frac{x^2 dx}{(x+f)\,(x^2+ax+b)} = \frac{1}{f^2-af+b} \times$$
$$\left[\begin{array}{l} f^2\log.(x+f) + \tfrac{1}{2}(b-af)\log.(x^2+ax+b) \\ +\tfrac{1}{2}(a^2f-ab-2bf)\displaystyle\int\frac{dx}{x^2+ax+b}\end{array}\right]$$

TABLE

Of some other general Formulæ.

$$a + bx = X$$

$$\int \frac{dx}{X^n} = -\frac{1}{(n-1)\,bX^{n-1}}$$

$$\int \frac{x^m dx}{X} = \frac{x^m}{mb} - \frac{a}{b}\int \frac{x^{m-1}dx}{X}$$

$$\int \frac{x^m dx}{X} = \frac{x^m}{mb} - \frac{ax^{m-1}}{(m-1)b^2} + \frac{a^2x^{m-2}}{(m-2)b^3} - \frac{a^3x^{m-3}}{(m-3)b^4} + \&c.$$

$$\pm \frac{a^{i-1}x^{m-i+1}}{(m-i+1)\,b^i} \mp \frac{a^i}{b^i}\int \frac{x^{m-i}dx}{X}$$

$$\int \frac{x^m dx}{X} = \frac{x^m}{mb} - \frac{ax^{m-1}}{(m-1)\,b^2} + \frac{a^2x^{m-2}}{(m-2)\,b^3} - \frac{a^3x^{m-3}}{(m-3)\,b^4} + \&c.$$

$$+ \frac{a^{m-1}x}{b^m} \mp \frac{a^m}{b^{m+1}} \log. X$$

$$\int \frac{x^m dx}{X^2} = \frac{x^m}{(m-1)\,bX} - \frac{ma}{(m-1)\,b}\int \frac{x^{m-1}dx}{X^2}$$

$$\int \frac{x^m dx}{X^2} = (Ax^m - Bx^{m-1} + Cx^{m-2} - Dx^{m-3} + Ex^{m-4} - \&c.$$

$$\pm Kx^{m-i+2} \mp Lx^{m-i+1})\,\frac{1}{X} \pm L\,(m-i+1)\,a\int \frac{x^{m-i}dx}{X^2}$$

$$A = \frac{1}{(m-1)\,b},\ B = \frac{ma}{(m-2)\,b}A,\ C = \frac{(m-1)\,a}{(m-3)\,b}B,$$

$$D = \frac{(m-2)\,a}{(m-4)\,b}C,\ E = \frac{(m-3)\,a}{(m-5)\,b}D,\ \&c.\ L = \frac{(m-i+2)\,a}{(m-i)\,b}K.$$

$$\int \frac{x^m dx}{X^3} = \frac{x^m}{(m-2)\,bX^2} - \frac{ma}{(m-2)\,b}\int \frac{x^{m-1}dx}{X^3}$$

$$\int \frac{x^m dx}{X^3} = (Ax^m - Bx^{m-1} + Cx^{m-2} - Dx^{m-3} + Ex^{m-4} - \&c.$$

$$\pm Kx^{m-i+2} \mp Lx^{m-i+1})\,\frac{1}{X^2} \pm L\,(m-i+1)\,a\int \frac{x^{m-i}dx}{X^3}$$

TABLE

Of some other general Formulæ.

$$a + bx = X$$

$$A = \frac{1}{(m-2)\,b},\ B = \frac{ma}{(m-3)\,b}A,\ C = \frac{(m-1)\,a}{(m-4)\,b}B,$$

$$D = \frac{(m-2)\,a}{(m-5)\,b}C,\ E\frac{(m-3)\,a}{(m-6)\,b}D, \text{ \&c., } L = \frac{(m-i+2)\,a}{(m-i-1)\,b}K.$$

$$\int\frac{x^m dx}{X^4} = \frac{x^m}{(m-3)\,bX^3} - \frac{ma}{(m-3)\,b}\int\frac{x^{m-1}dx}{X^4}$$

$$\int\frac{x^m dx}{X^4} = (Ax^m - Bx^{m-1} + Cx^{m-2} - Dx^{m-3} + Ex^{m-4} - \text{\&c.}$$

$$\pm Kx^{m-i+2} \mp Lx^{m-i+1})\frac{1}{X^3} \pm L(m-i+1)\,a\int\frac{x^{m-i}dx}{X^4}$$

$$A = \frac{1}{(m-3)\,b},\ B = \frac{ma}{(m-4)\,b}A,\ C = \frac{(m-1)\,a}{(m-5)\,b}B,$$

$$D = \frac{(m-2)\,a}{(m-6)\,b}C,\ E = \frac{(m-3)\,a}{(m-7)\,b}D, \text{ \&c. } L = \frac{(m-i+2)\,a}{[m-i-2)\,b}K.$$

$$\int\frac{x^m dx}{X^5} = \frac{x^m}{(m-4)\,bX^4} - \frac{ma}{(m-4)\,b}\int\frac{x^{m-1}dx}{X^5}$$

$$\int\frac{x^m dx}{X^5} = (Ax^m - Bx^{m-1} + Cx^{m-2} - Dx^{m-3} + Ex^{m-4} + \text{\&c.}$$

$$\pm Kx^{m-i+2} \mp Lx^{m-i+1})\frac{1}{X^4} \pm L(m-i+1)\,a\int\frac{x^{m-i}dx}{X^5}$$

$$A = \frac{1}{(m-4)\,b},\ B = \frac{ma}{(m-5)\,b}A,\ C = \frac{(m-1)\,a}{(m-6)\,b}B,$$

$$D = \frac{(m-2)\,a}{(m-7)\,b}C,\ E = \frac{(m-3)\,a}{(m-8)\,b}D, \text{ \&c., } L = \frac{(m-i+2)\,a}{(m-i-3)\,b}K.$$

$$\int\frac{x^m dx}{X^6} = \frac{x^m}{(m-5)\,bX^5} - \frac{ma}{(m-5)\,b}\int\frac{x^{m-1}dx}{X^6}$$

TABLE

of some other general Formulæ.

$$a + bx = X$$

$$\int \frac{x^m dx}{X^6} = (Ax^m - Bx^{m-1} + Cx^{m-2} - Dx^{m-3} + Ex^{m-4} - \&c.$$
$$\pm Kx^{m-i+2} \mp Lx^{m-i+1}) \frac{1}{X^5} \pm L(m-i+1)\, a \int \frac{x^{m-i} dx}{X^5}$$
$$A = \frac{1}{(m-5)\,b},\ B = \frac{ma}{(m-6)\,b} A,\ C = \frac{(m-1)\,a}{(m-7)\,b} B,$$
$$D = \frac{(m-2)\,a}{(m-8)\,b} C,\ E = \frac{(m-3)\,a}{(m-9)\,b} D, \&c.,\ L = \frac{(m-i+2)\,a}{(m-i-4)\,b} K.$$

$$\int \frac{dx}{x^m X} = -\frac{1}{(m-1)\,ax^{m-1}} - \frac{b}{a} \int \frac{dx}{x^{m-1} X}$$

$$\int \frac{dx}{x^m X} = -\frac{1}{(m-1)\,ax^{m-1}} + \frac{1}{(m-2)\,a^2 x^{m-2}} - \frac{b^2}{(m-3)\,a^3 x^{m-3}}$$
$$+ \frac{b^3}{(m-4)\,a^4 x^{m-4}} - \&c. \pm \frac{b^{i-1}}{(m-i)\,a^i x^{m-i}} + \frac{b^i}{a^i} \int \frac{dx}{x^{m-i} X}$$

$$\int \frac{dx}{x^m X} = -\frac{1}{(m-1)\,ax^{m-1}} + \frac{b}{(m-2)\,a^2 x^{m-2}} - \frac{b^2}{(m-3)\,a^3 x^{m-3}}$$
$$+ \frac{b^3}{(m-4)\,a^4 x^{m-3}} - \&c. \pm \frac{b^{m-2}}{a^{m-1} x} \mp \frac{b^{m-1}}{a^m} \log. \frac{X}{x}$$

$$\int \frac{dx}{x^m X} = -\frac{1}{(m-1)\,ax^{m-1} X} - \frac{mb}{(m-1)a} \int \frac{dx}{x^{m-1} X^2}$$

$$\int \frac{dx}{x^m X^2} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \&c.\right.$$
$$\left. \pm \frac{K}{x^{m-i+1}} \mp \frac{L}{x^{m-i}}\right) \frac{1}{X} \mp L(m-i+1)\,b \int \frac{dx}{x^{m-i} X^2}$$
$$A = -\frac{1}{(m-1)\,a},\ B = \frac{mb}{(m-2)\,a} A,\ C = \frac{(m-1)\,b}{(m-3)\,a} B,$$
$$D = \frac{(m-2)\,b}{(m-4)\,a} C,\ E = \frac{(m-3)\,b}{(m-5)\,a} D,\ \&c.,\ L = \frac{(m-i+2)\,b}{(m-i)\,a} K.$$

TABLE

Of some other general Formulæ.

$$a + bx = X$$

$$\int \frac{dx}{x^m X^3} = -\frac{1}{(m-1)\,ax^{m-1}X^2} - \frac{(m+1)\,b}{(m-1)\,a}\int \frac{dx}{x^{m-1}X^3}$$

$$\int \frac{dx}{x^m X^3} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \text{\&c.}\right.$$

$$\left. \pm \frac{K}{x^{m-i+1}} \mp \frac{L}{x^{m-i}}\right)\frac{1}{X^2} \mp L\,(m-i+2)\,b\int \frac{dx}{x^{m-i}X^3}$$

$$A = -\frac{1}{(m-1)\,a},\ B = \frac{(m+1)\,b}{(m-2)\,a}\,A,\ C = \frac{mb}{(m-3)\,a}\,B,$$

$$D = \frac{(m-1)\,b}{(m-4)\,a}\,C,\ E = \frac{(m-2)\,b}{(m-5)\,a}\,D,\ \text{\&c.},\ L = \frac{(m-i+3)\,b}{(m-i)\,a}\,K.$$

$$\int \frac{dx}{x^m X^4} = -\frac{1}{(m-1)\,ax^{m-1}X^3} - \frac{(m+2)\,b}{(m-1)\,a}\int \frac{dx}{x^{m-1}X^4}$$

$$\int \frac{dx}{x^m X^4} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}}\ \text{\&c.}\right.$$

$$\left. \pm \frac{K}{x^{m-i+1}} \mp \frac{L}{x^{m-i}}\right)\frac{1}{X^3} \pm L\,(m-i+3)\,b\int \frac{dx}{x^{m-i}X}$$

$$A = -\frac{1}{(m-1)\,a},\ B = \frac{(m+2)\,b}{(m-2)\,a}\,A,\ C = \frac{(m+1)\,b}{(m-3)\,a}\,B,$$

$$D = \frac{mb}{(m-4)\,a}\,C,\ E = \frac{(m-1)\,b}{(m-5)\,a}\,D,\ \text{\&c.},\ L = \frac{(m-i+4)\,b}{(m-i)\,a}\,K.$$

$$\int \frac{dx}{x^m X^5} = -\frac{1}{(m-1)\,ax^{m-1}X^4} - \frac{(m+3)\,b}{(m-1)\,a}\int \frac{dx}{x^{m-1}X^5}$$

$$\int \frac{dx}{x^m X^5} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \text{\&c.}\right.$$

$$\left. \pm \frac{K}{x^{m-i+1}} \mp \frac{L}{x^{m-i}}\right)\frac{1}{X^4} \mp L\,(m-i+4)\,b\int \frac{dx}{x^{m-i}X}$$

TABLE

Of some other general Formulæ.

$$a + bx = X$$

$$A = -\frac{1}{(m-1)\,a},\ B = \frac{(m+3)\,b}{(m-2)\,a}\,A,\ C = \frac{(m+2)\,b}{(m-3)\,a}\,B,$$

$$D = \frac{(m+1)\,b}{(m-4)\,a}\,C,\ E = \frac{mb}{(m-5)\,a}\,D,\ \&c.\ L = \frac{(m-i+5)\,b}{(m-i)\,a}\,K$$

$$\int \frac{dx}{x^m X^6} = -\frac{1}{(m-1)\,ax^{m-1}X^5} - \frac{(m+4)\,b}{(m-1)\,a}\int \frac{dx}{x^{m-1}X^6}$$

$$\int \frac{dx}{x^m X^6} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \&c.\right.$$

$$\left.\pm \frac{K}{x^{m-i+1}} \mp \frac{L}{x^{m-i}}\right)\frac{1}{X^5} \mp L\,(m-i+5)\,b\int \frac{dx}{x^{m-i}X^6}$$

$$A = -\frac{1}{(m-1)\,a},\ B = \frac{(m+4)\,b}{(m-2)\,a}\,A,\ C = \frac{(m+3)\,b}{(m-3)\,a}\,B,$$

$$D = \frac{(m+2)\,b}{(m-4)\,a}\,C,\ E = \frac{(m+1)\,b}{(m-5)\,a}\,D,\ \&c.\ L = \frac{(m-i+6)\,b}{(m-i)\,a}\,K.$$

$$a + bx^2 = X$$

$$\int \frac{dx}{X^p} = \frac{x}{(p-1)\,2aX^{p-1}} + \frac{2p-3}{(p-1)\,2a}\int \frac{dx}{X^{p-1}}$$

$$\int \frac{dx}{X^p} = \left(\frac{A}{X^{p-1}} + \frac{B}{X^{p-2}} + \frac{C}{X^{p-3}} + \frac{D}{X^{p-4}} + \frac{E}{X^{p-5}} + \&c.\right.$$

$$\left.+ \frac{K}{X^{p-i+1}} + \frac{L}{X^{p-i}}\right)x + L\,(2p-2i-1)\int \frac{dx}{X^{p-i}}$$

$$A = \frac{1}{(p-1)2a},\ B = \frac{2p-3}{(p-2)2a}\,A,\ C = \frac{2p-5}{(p-3)2a}\,B,$$

$$D = \frac{2p-7}{(p-4)\,2a}\,C,\ E = \frac{2p-9}{(p-5)2a}\,D,\ \&c.\ L = \frac{2p-2i+1}{(p-i)\,2a}\,K.$$

TABLE

Of some other general Formulæ.

$$a + bx^2 = X$$

$$\int \frac{x^m dx}{X} = \frac{x^{m-1}}{(m-1)b} - \frac{a}{b}\int \frac{x^{m-2}dx}{X}$$

$$\int \frac{x^m dx}{X} = (Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \&c.$$

$$\pm Kx^{m-2i+3} \mp Lx^{m-2i+1} \pm L(m-2i+1)\,a\int \frac{x^{m-2i}dx}{X}$$

$$A = \frac{1}{(m-1)b},\ B = \frac{(m-1)a}{(m-3)b}A,\ C = \frac{(m-3)a}{(m-5)b}B,$$

$$D = \frac{(m-5)a}{(m-7)b}C,\ E = \frac{(m-7)a}{(m-9)b}D,\ \&c.\ L = \frac{(m-2i+3)a}{(m-2i+1)b}K.$$

$$\int \frac{x^m dx}{X^2} = \frac{x^{m-1}dx}{(m-3)bX} - \frac{(m-1)a}{(m-3)b}\int \frac{x^{m-2}dx}{X^2}$$

$$\int \frac{x^m dx}{X^2} = (Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \&c.$$

$$\pm Kx^{m-2i+3} \mp Lx^{m-2i+1})\frac{1}{X} \pm L(m-2i+1)\,a\int \frac{x^{m-2i}dx}{X^2}$$

$$A = \frac{1}{(m-3)b},\ B = \frac{(m-1)a}{(m-5)b}A,\ C = \frac{(m-3)a}{(m-7)b}B,$$

$$D = \frac{(m-5)a}{(m-9)b}C,\ E = \frac{(m-7)a}{(m-11)b}D,\ \&c.\ L = \frac{(m-2i+3)a}{(m-2i-1)b}K.$$

$$\int \frac{x^m dx}{X^3} = \frac{x^{m-1}}{(m-5)b} - \frac{(m-1)a}{(m-5)b}\int \frac{x^{m-2}dx}{X^3}$$

$$\int \frac{x^m dx}{X^3} = (Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \&c.$$

$$\pm Kx^{m-2i+3} \mp Lx^{m-2i+1})\frac{1}{X^2} \pm L(m-2i+1)\,a\int \frac{x^{m-2i}dx}{X^3}$$

$$A = \frac{1}{(m-5)b},\ B = \frac{(m-1)a}{(m-7)b}A,\ C = \frac{(m-3)a}{(m-9)b}B,$$

$$D = \frac{(m-5)b}{(m-11)b}C,\ E = \frac{(m-7)a}{(m-13)b}D,\ \&c.\ L = \frac{(m-2i+3)a}{(m-2i-3)b}K.$$

TABLE

Of some other general Formulæ.

$$a + bx^2 = X$$

$$\int \frac{x^m dx}{X^4} = \frac{x^{m-1}}{(m-7)bX^3} - \frac{(m-1)a}{(m-7)b} \int \frac{x^{m-2}dx}{X^4}$$

$$\int \frac{x^m dx}{X^4} = (Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \&c.$$

$$\pm Kx^{m-2i+3} \mp Lx^{m-2i+1}) \frac{1}{X^3} \pm L(m-2i+1)a \int \frac{x^{m-2i}dx}{X^4}$$

$$A = \frac{1}{(m-7)b},\ B = \frac{(m-1)a}{(m-9)b} A,\ C = \frac{(m-3)a}{(m-11)b} B;$$

$$D = \frac{(m-5)a}{(m-13)b} C,\ E = \frac{(m-7)a}{(m-15)b} E,\ \&c.\ L = \frac{(m-2i+3)a}{(m-2i-5)b} K.$$

$$\int \frac{x^m dx}{X^5} = \frac{x^{m-1}}{(m-9)bX^4} - \frac{(m-1)a}{(m-9)b} \int \frac{x^{m-2}dx}{X^5}$$

$$\int \frac{x^m dx}{X^5} = (Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \&c.$$

$$\pm Kx^{m-2i+3} \mp Lx^{m-2i+1}) \frac{1}{X^4} \pm L(m-2i+1)a \int \frac{x^{m-2i}dx}{X^5}$$

$$A = \frac{1}{(m-9)b},\ B = \frac{(m-1)a}{(m-11)b} A,\ C = \frac{(m-3)a}{(m-13)b} B,$$

$$D = \frac{(m-5)a}{(m-15)b} C,\ E = \frac{(m-7)a}{(m-17)b} D,\ \&c.\ L = \frac{(m-2i+3)a}{(m-2i-7)b} K.$$

$$\int \frac{x^m dx}{X^6} = \frac{x^{m-1}}{(m-11)bX^5} - \frac{(m-1)a}{(m-11)b} \int \frac{x^{m-2}dx}{X^6}$$

$$\int \frac{x^m dx}{X^6} = (Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \&c.$$

$$\pm Kx^{m-2i+3} \mp Lx^{m-2i+1}) \frac{1}{X^5} \pm L(m-2i+1)a \int \frac{x^{m-2i}dx}{X^6}$$

$$A = \frac{1}{(m-11)b},\ B = \frac{(m-1)a}{(m-13)b} A,\ C = \frac{(m-3)a}{(m-15)b} B,$$

$$D = \frac{(m-5)a}{(m-17)b} C,\ E = \frac{(m-7)a}{(m-19)b} D,\ \&c.\ L = \frac{(m-2i+3)a}{(m-2i-9)b} K.$$

TABLE

Of some other general Formulæ.

$$a + bx^2 = X$$

$$\int\frac{dx}{x^m X} = -\frac{1}{(m-1)ax^{m-1}} - \frac{b}{a}\int\frac{dx}{x^{m-2}X}$$

$$\int\frac{dx}{x^m X} = \frac{A}{x^{m-1}} - \frac{B}{x^{m-3}} + \frac{C}{x^{m-5}} - \frac{D}{x^{m-7}} + \frac{E}{x^{m-9}} - \&c.$$

$$\pm\frac{K}{x^{m-2i+3}} \mp \frac{L}{x^{m-2i+1}} \mp L(m-2i+1)b\int\frac{dx}{x^{m-2i}X}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(m-1)b}{(m-3)a}A,\ C = \frac{(m-3)b}{(m-5)a}B,$$

$$D = \frac{(m-5)b}{(m-7)a}C,\ E = \frac{(m-7)b}{(m-9)a}D,\ \&c.,\ L = \frac{(m-2i+3)b}{(m-2i+1)a}K.$$

$$\int\frac{dx}{x^m X^2} = -\frac{1}{(m-1)ax^{m-1}X} - \frac{(m+1)b}{(m-1)a}\int\frac{dx}{x^{m-2}X^2}$$

$$\int\frac{dx}{x^m X^2} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-3}} + \frac{C}{x^{m-5}} - \frac{D}{x^{m-7}} + \frac{E}{x^{m-9}} - \&c.\right.$$

$$\left.\pm\frac{K}{x^{m-2i+3}} \mp \frac{L}{x^{m-2i+1}}\right)\frac{1}{X} \mp L(m-2i+3)b\int\frac{dx}{x^{m-2i}X^2}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(m+1)b}{(m-3)a}A,\ C = \frac{(m-1)b}{(m-5)a}B,$$

$$D = \frac{(m-3)b}{(m-7)a}C,\ E = \frac{(m-5)b}{(m-9)a}D,\ \&c.,\ L = \frac{(m-2i+5)b}{(m-2i+1)a}K.$$

$$\int\frac{dx}{x^m X^3} = -\frac{1}{(m-1)ax^{m-1}X^2} - \frac{(m+3)b}{(m-1)a}\int\frac{dx}{x^{m-2}X^3}$$

$$\int\frac{dx}{x^m X^3} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-3}} + \frac{C}{x^{m-5}} - \frac{D}{x^{m-7}} + \frac{E}{x^{m-9}} - \&c.\right.$$

$$\left.\pm\frac{K}{x^{m-2i+3}} \mp \frac{L}{x^{m-2i+1}}\right)\frac{1}{X^2} \mp L(m-2i+5)b\int\frac{dx}{x^{m-2i}X^3}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(m+3)b}{(m-3)a}A,\ C = \frac{(m+1)b}{(m-5)a}B,$$

$$D = \frac{(m-1)b}{(m-7)a}C,\ E = \frac{(m-3)b}{(m-9)a}D,\ \&c.\ L = \frac{(m-2i+7)b}{(m-2i+1)a}K.$$

TABLE

Of some other general Formulæ.

$$a + bx^2 = X$$

$$\int \frac{dx}{x^m X^4} = -\frac{1}{(m-1)ax^{m-1}X^3} - \frac{(m+5)b}{(m-1)a}\int \frac{dx}{x^{m-2}X^4}$$

$$\int \frac{dx}{x^m X^4} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-3}} + \frac{C}{x^{m-5}} - \frac{D}{x^{m-7}} + \frac{E}{x^{m-9}} - \&c.\right.$$

$$\left. \pm \frac{K}{x^{m-2i+3}} \mp \frac{L}{x^{m-2i+1}}\right)\frac{1}{X^3} \mp L(m-2i+7)b\int \frac{dx}{x^{m-2i}X^4}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(m+5)b}{(m-3)a}A,\ C = \frac{(m+3)b}{(m-5)a}B,$$

$$D = \frac{(m+1)b}{(m-7)a}C,\ E = \frac{(m-1)b}{(m-9)a}D,\ \&c.,\ L = \frac{(m-2i+9)b}{(m-2i+1)a}K.$$

$$\int \frac{dx}{x^m X^5} = -\frac{1}{(m-1)ax^{m-1}X^4} - \frac{(m+7)b}{(m-1)a}\int \frac{dx}{x^{m-2}X^5}$$

$$\int \frac{dx}{x^m X^5} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-3}} + \frac{C}{x^{m-5}} - \frac{D}{x^{m-7}} + \frac{E}{x^{m-9}} - \&c.\right.$$

$$\left. \pm \frac{K}{x^{m-2i+3}} \mp \frac{L}{x^{m-2i+1}}\right)\frac{1}{X^4} \mp L(m-2i+9)b\int \frac{dx}{x^{m-2i}X^5}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(m+7)b}{(m-3)a}A,\ C = \frac{(m+5)b}{(m-5)a}B,$$

$$D = \frac{(m+3)b}{(m-7)a}C,\ E = \frac{(m+1)b}{(m-9)a}D,\ \&c.,\ L = \frac{(m-2i+11)b}{(m-2i+1)a}K.$$

$$\int \frac{dx}{x^m X^6} = -\frac{1}{(m-1)ax^{m-1}X^5} - \frac{(m+9)b}{(m-1)a}\int \frac{dx}{x^{m-2}X^6}$$

$$\int \frac{dx}{x^m X^6} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-3}} + \frac{C}{x^{m-5}} - \frac{D}{x^{m-7}} + \frac{E}{x^{m-9}} - \&c.\right.$$

$$\left. \pm \frac{K}{x^{m-2i+3}} \mp \frac{L}{x^{m-2i+1}}\right)\frac{1}{X^5} \mp L(m-2i+11)b\int \frac{dx}{x^{m-2i}X^6}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(m+9)b}{(m-3)a}A,\ C = \frac{(m+7)b}{(m-5)a}B,$$

$$D = \frac{(m+5)b}{(m-7)a}C,\ E = \frac{(m+3)b}{(m-9)a}D,\ \&c.\ L = \frac{(m-2i+13)b}{(m-2i+1)a}K.$$

TABLE

Of some other general Formulæ.

$$a + bx + cx^2 = X,\ 4ac - b^2 = k$$

$$\int\frac{dx}{X^p} = \frac{2cx+b}{(p-1)kX^{p-1}} + \frac{(2p-3)2c}{(p-1)k}\int\frac{dx}{X^{p-1}}$$

$$\int\frac{dx}{X^p} = \left(\frac{A}{X^{p-1}} + \frac{B}{X^{p-2}} + \frac{C}{X^{p-3}} + \&c. + \frac{K}{X^{p-i+1}} + \frac{L}{X^{p-i}}\right)(2cx+b)$$

$$+ L(4p-2i-2)c\int\frac{dx}{X^{p-i}}$$

$$A = \frac{1}{(p-1)k},\ B = \frac{(4p-6)c}{(p-2)k}A,\ C = \frac{(4q-10)c}{(p-3)k}B,$$

$$D = \frac{(4p-14)c}{(p-4)k}C,\ E = \frac{(4p-18)c}{(p-5)k}D,\ \&c.\ L = \frac{(4p-4i+2)c}{(p-i)k}K.$$

$$\int\frac{x^m dx}{X} = \frac{x^{m-1}}{(m-1)c} - \frac{a}{c}\int\frac{x^{m-2}dx}{X} - \frac{b}{c}\int\frac{x^{m-1}dx}{X}$$

$$\int\frac{x^m dx}{X^2} = \frac{x^{m-1}}{(m-3)cX} - \frac{(m-1)a}{(m-3)c}\int\frac{x^{m-2}dx}{X^2} - \frac{(m-2)b}{(m-3)c}\int\frac{x^{m-1}dx}{X^2}$$

$$\int\frac{x^m dx}{X^3} = \frac{x^{m-1}}{(m-5)cX^2} - \frac{(m-1)a}{(m-5)c}\int\frac{x^{m-2}dx}{X^3} - \frac{(m-3)b}{(m-5)c}\int\frac{x^{m-1}dx}{X^3}$$

$$\int\frac{x^m dx}{X^4} = \frac{x^{m-1}}{(m-7)cX^3} - \frac{(m-1)a}{(m-7)c}\int\frac{x^{m-2}dx}{X^4} - \frac{(m-4)b}{(m-7)c}\int\frac{x^{m-1}dx}{X^4}$$

$$\int\frac{x^m dx}{X^5} = \frac{x^{m-1}}{(m-9)cX^4} - \frac{(m-1)a}{(m-9)c}\int\frac{x^{m-2}dx}{X^5} - \frac{(m-5)b}{(m-9)a}\int\frac{x^{m-1}dx}{X^5}$$

$$\int\frac{x^m dx}{X^6} = \frac{x^{m-1}}{(m-11)cX^5} - \frac{(m-1)a}{(m-11)c}\int\frac{x^{m-2}dx}{X^6} - \frac{(m-6)b}{(m-11)c}\int\frac{x^{m-1}dx}{X^6}$$

$$\int\frac{dx}{x^m X} = -\frac{1}{(m-1)ax^{m-1}} - \frac{b}{a}\int\frac{dx}{x^{m-1}X} - \frac{c}{a}\int\frac{dx}{x^{m-2}X}$$

$$\int\frac{dx}{x^m X^2} = -\frac{1}{(m-1)ax^{m-1}X} - \frac{mb}{(m-1)a}\int\frac{dx}{x^{m-1}X^2} - \frac{(m+1)b}{(m-1)a}\int\frac{dx}{x^{m-3}X^2}$$

$$\int\frac{dx}{x^m X^3} = -\frac{1}{(m-1)ax^{m-1}X^2} - \frac{(m+1)b}{(m-1)a}\int\frac{dx}{x^{m-1}X^3} - \frac{(m+3)c}{(m-1)a}\int\frac{dx}{x^{m-2}X^3}$$

TABLE

Of some other general Formulæ.

$$a + bx + cx^2 = X,\ 4ac - b^2 = k$$

$$\int \frac{dx}{x^m X^4} = -\frac{1}{(m-1)ax^{m-1}X^4} - \frac{(m+2)b}{(m-1)a}\int \frac{dx}{x^{m-1}X^4} - \frac{(m+5)c}{(m-1)a}\int \frac{dx}{x^{m-2}X^4}$$

$$\int \frac{dx}{x^m X^5} = -\frac{1}{(m-1)ax^{m-1}X^5} - \frac{(m+3)b}{(m-1)a}\int \frac{dx}{x^{m-1}X^5} - \frac{(m+7)c}{(m-1)a}\int \frac{dx}{x^{m-2}X^5}$$

$$\int \frac{dx}{x^m X^6} = -\frac{1}{(m-1)ax^{m-1}X^6} - \frac{(m+4)b}{(m-1)a}\int \frac{dx}{x^{m-1}X^6} - \frac{(m+9)c}{(m-1)a}\int \frac{dx}{x^{m-2}X^6}$$

$$a + bx^3 = X$$

$$\int \frac{x^m dx}{X} = \frac{x^{m-2}}{(m-2)b} - \frac{a}{b}\int \frac{x^{m-3}dx}{X}$$

$$\int \frac{x^m dx}{X^2} = \frac{x^{m-2}}{(m-5)bX} - \frac{(m-2)a}{(m-5)b}\int \frac{x^{m-3}dx}{X^2}$$

$$\int \frac{x^m dx}{X^3} = \frac{x^{m-2}}{(m-8)bX^2} - \frac{(m-2)a}{(m-8)b}\int \frac{x^{m-3}dx}{X^3}$$

$$\int \frac{dx}{x^m X} = -\frac{1}{(m-1)ax^{m-1}} - \frac{b}{a}\int \frac{dx}{x^{m-3}X}$$

$$\int \frac{dx}{x^m X^2} = -\frac{1}{(m-1)ax^{m-1}X} - \frac{(m+2)b}{(m-1)a}\int \frac{dx}{x^{m-3}X^2}$$

$$\int \frac{dx}{x^m X^3} = -\frac{1}{(m-1)ax^{m-1}X^2} - \frac{(m+5)b}{(m-1)a}\int \frac{dx}{x^{m-3}X^3}$$

$$a + bx^4 = X$$

$$\int \frac{x^m dx}{X} = \frac{x^{m-3}}{(m-3)b} - \frac{a}{b}\int \frac{x^{m-4}dx}{X}$$

$$\int \frac{x^m dx}{X} = \frac{x^{m-3}}{(m-7)bX} - \frac{(m-3)a}{(m-7)b}\int \frac{x^{m-4}dx}{X^2}$$

TABLE

Of some other general Formulæ.

$$a + bx^4 = X$$

$$\int \frac{x^m dx}{X^3} = \frac{x^{m-3}}{(m-11)bX^2} - \frac{(m-3)a}{(m-11)b} \int \frac{x^{m-4}dx}{X^3}$$

$$\int \frac{dx}{x^m X} = -\frac{1}{(m-1)ax^{m-1}} - \frac{b}{a} \int \frac{dx}{x^{m-4}X}$$

$$\int \frac{dx}{x^m X^2} = -\frac{1}{(m-1)ax^{m-1}X} - \frac{(m+3)b}{(m-1)a} \int \frac{dx}{x^{m-4}X^2}$$

$$\int \frac{dx}{x^m X^3} = -\frac{1}{(m-1)ax^{m-1}X^2} - \frac{(m+7)b}{(m-1)a} \int \frac{dx}{x^{m-4}X^3}$$

$$a + bx^5 = X$$

$$\int \frac{x^m dx}{X} = \frac{x^{m-4}}{(m-4)b} - \frac{a}{b} \int \frac{x^{m-5}dx}{X}$$

$$\int \frac{x^m dx}{X^2} = \frac{x^{m-4}}{(m-9)b} - \frac{(m-4)a}{(m-9)b} \int \frac{x^{m-5}dx}{X^2}$$

$$\int \frac{x^m dx}{X^3} = \frac{x^{m-4}}{(m-14)b} - \frac{(m-4)a}{(m-14)b} \int \frac{x^{m-5}dx}{X^3}$$

$$\int \frac{dx}{x^m X} = -\frac{1}{(m-1)ax^{m-1}} - \frac{b}{a} \int \frac{dx}{x^{m-5}X}$$

$$\int \frac{dx}{x^m X^2} = -\frac{1}{(m-1)ax^{m-1}X} - \frac{(m+4)b}{(m-1)a} \int \frac{dx}{x^{m-5}X^2}$$

$$\int \frac{dx}{x^m X^3} = -\frac{1}{(m-1)ax^{m-1}X^2} - \frac{(m+9)b}{(m-1)a} \int \frac{dx}{x^{m-5}X^3}$$

$$a + bx^6 = X$$

$$\int \frac{x^m dx}{X} = \frac{x^{m-5}}{(m-5)b} - \frac{a}{b} \int \frac{x^{m-6}dx}{X}$$

$$\int \frac{x^m dx}{X^2} = \frac{x^{m-5}}{(m-11)bX} - \frac{(m-5)a}{(m-11)b} \int \frac{x^{m-6}dx}{X^2}$$

TABLE

Of some other general Formulæ.

$$a + bx^6 = X$$

$$\int \frac{x^m dx}{X^3} = \frac{x^{m-5}}{(m-17)bX^2} - \frac{(m-5)a}{(m-17)b}\int \frac{x^{m-6}dx}{X^3}$$

$$\int \frac{dx}{x^m X} = -\frac{1}{(m-1)ax^{m-1}} - \frac{b}{a}\int \frac{dx}{x^{m-6}X}$$

$$\int \frac{dx}{x^m X^2} = -\frac{1}{(m-1)ax^{m-1}X} - \frac{(m+5)b}{(m-1)a}\int \frac{dx}{x^{m-6}X^2}$$

$$\int \frac{dx}{x^m X^3} = -\frac{1}{(m-1)ax^{m-1}X^2} - \frac{(m+11)b}{(m-1)a}\int \frac{dx}{x^{m-6}X^3}$$

$$a + bx^2 + cx^4 = X, \quad (p-1)(b^2-4ac)2a = k$$

$$\int \frac{dx}{X^p} = \frac{bcx^3 + (b^2-2ac)x}{kX^{p-1}} + \frac{(4p-7)bc}{k}\int \frac{x^2 dx}{X^{p-1}} + \frac{2(p-1)(b^2-4ac) + 2ac - b^2}{k}\int \frac{dx}{X^{p-1}}$$

$$\int \frac{x^m dx}{X^p} = \frac{bcx^{m+3} + (b^2-2ac)x^{m+1}}{kX^{p-1}} + \frac{(4p-m-5)bc}{k}\int \frac{x^{m+2}dx}{X^{p-1}} + \frac{2(p-1)(b^2-4ac) + (m+1)(2ac-b^2)}{k}\int \frac{x\,dx}{X^{p-1}}$$

$$\int \frac{x^m dx}{X^p} = \frac{x^{m-3}}{(m-4p+1)cX^{p-1}} - \frac{(m-3)a}{(m-4p+1)c}\int \frac{x^{m-4}dx}{X^p} - \frac{(m-2p-1)b}{(m-4p+1)c}\int \frac{x^{m-2}dx}{X^p}$$

$$\int \frac{dx}{x^m X^p} = -\frac{1}{(m-1)ax^{m-1}X^{p-1}} - \frac{(m+2p-3)b}{(m-1)a}\int \frac{dx}{x^{m-4}X^p} - \frac{m+4p-5}{(m-1)a}\int \frac{dx}{x^{m-4}X^p}$$

TABLE

Of some other general Formulæ.

$$a+bx^3+cx^6=X,\quad (p-1)(b^2-4ac)\,3a=k$$

$$\int\frac{dx}{X^p}=\frac{bcx^4+(b^2-2ac)x}{kX^{p-1}}+\frac{(6p-10)\,bc}{k}\int\frac{x^3dx}{X^{p-1}}$$
$$+\frac{3(p-1)(b^2-4ac)+2ac-b^2}{k}\int\frac{dx}{X^{p-1}}$$

$$\int\frac{x^m dx}{X^p}=\frac{bcx^{m+4}+(b^2-2ac)x^{m+1}}{kX^{p-1}}+\frac{(6p-m-10)\,bc}{k}\int\frac{x^{m+3}dx}{X^{p-1}}$$
$$+\frac{3(p-1)(b^2-4ac)+(m+1)(2ac-b^2)}{k}\int\frac{x^m dx}{X^{p-1}}$$

$$\int\frac{x^m dx}{X^p}=\frac{x^{m-5}}{(m-6p+1)\,cX^{p-1}}-\frac{(m-5)\,a}{(m-6p+1)\,c}\int\frac{x^{m-6}dx}{X^p}$$
$$-\frac{(m-3p-2)\,a}{(m-6p+1)\,c}\int\frac{x^{m-3}dx}{X^p}$$

$$\int\frac{dx}{x^mX^p}=-\frac{1}{(m-1)\,ax^{m-1}X^{p-1}}-\frac{(m+3p-4)\,b}{(m-1)\,a}\int\frac{dx}{x^{m-3}X^p}$$
$$-\frac{(m+6p-7)\,c}{(m-1)\,a}\int\frac{dx}{x^{m-6}X^p}$$

TABLE

of some more general Formulæ.

$$a + bx^n = X, \ \pi = 180°$$

When $m < n$ or $= 0$, generally we have

$$\int \frac{x^m dx}{X} = U + V,$$

where U is a logarithmic function, or $= 0$, and V the aggregate of terms of the following form:

$$\frac{1}{nbk^{n-m-1}} \left\{ \begin{array}{l} \cos. (n-m-1)\, \phi \log. (x^2 - 2kx \cos. \phi + k^2) \\ + 2 \sin. (n-m-1)\, \phi \text{ arc tang.} \dfrac{x \sin. \phi}{k - x \cos. \phi} \end{array} \right\}$$

A nearer Determination.

(1). *Let n be odd, and a, b, negative or positive.*

Then, making $k = \sqrt[n]{\frac{a}{b}}$, we have

$$U = \frac{1}{nb(-k)^{n-m-1}} \log. (x + k)$$

and V is an aggregate of terms of the above form, all of which may be obtained by putting in this expression for ϕ the $\frac{n-2}{2}$ successive values $\frac{2\pi}{n}, \frac{4\pi}{n} \ldots \frac{n-2}{n}\pi$.

(2). *Let n be even; a and b have different signs.*

Then, making $k = \sqrt[n]{-\frac{a}{b}}$, we have

$$U = \frac{1}{nbk^{n-m-1}} \log. (x-k) + \frac{1}{nb(-k)^{n-m-1}} \log. (x+k)$$

and V is an aggregate of terms of the above form, all of which may be obtained by putting for ϕ the $\frac{n-2}{2}$ successive values $\frac{2\pi}{n}$, $\frac{4\pi}{n} \ldots \frac{n-2}{n}\pi$.

(3.) *Let n be even; a and b have like signs.*

Then, making $k = \sqrt[n]{\frac{a}{b}}$, we have $U = 0$, and V an aggregate of are given by the above expression, when for ϕ we substitute the terms, which $\frac{n}{2}$ values $\frac{\pi}{n}, \frac{3\pi}{n}, \frac{5\pi}{n}, \frac{7\pi}{n} \ldots \frac{n-1}{n}\pi$.

TABLE

of some other general Formulæ.

$$a+bx^n+cx^{2n}=X,\ \pi=180°$$

The integral $\int\frac{x^m dx}{X}$, when the real expressions are required, has two forms, differing according as $4ac-b^2$ is a positive or a negative quantity.

I. *Let $4ac-b^2$ be positive, and $m<2n$.*

Let $k=\sqrt[n]{\frac{a}{c}}$, and α be an angle whose cosine $=-\frac{b}{2\sqrt{ac}}$, then $\int\frac{x^m dx}{X}$ is an aggregate of n terms of the following form:

$$\frac{1}{2nck^{2n-m-1}\sin.\alpha}\left\{\begin{array}{l}-\sin.(n-m-1)\varphi\log.(x^2-2kx\cos.\varphi+k^2)\\ +2\cos.(n-m-1)\varphi\ \text{arc tang.}\frac{x\sin.\varphi}{k-x\cos.\varphi}\end{array}\right\}$$

each of which may be obtained by substituting for φ its values $\frac{\alpha}{n}$, $\frac{2\pi+\alpha}{n}$, $\frac{4\pi+\alpha}{n}\ldots\frac{(2n-2)\pi+\alpha}{n}$ successively.

When $m>2n$, the integral $\int\frac{x^m dx}{X}$ is reducible to another in which $m<2n$.

II. *Let $4ac-b^2$ be negative.*

Make

$$\tfrac{1}{2}b-\tfrac{1}{2}\sqrt{(b^2-4ac)}=f$$
$$\tfrac{1}{2}b+\tfrac{1}{2}\sqrt{(b^2-4ac)}=g$$
$$\sqrt{(b^2-4ac)}=g-f=h.$$

Then

$$\int\frac{dx}{X}=\frac{c}{h}\left[\int\frac{dx}{cx^n+f}-\int\frac{dx}{cx^n+g}\right]$$
$$\int\frac{x^m dx}{X}=\frac{c}{h}\left[\int\frac{x^m dx}{cx^n+f}-\int\frac{x^m dx}{cx^n+g}\right]$$

TABLE

of some more general Formulæ.

$$\left\{\frac{Ax^h + Bx^{h-1} + Cx^{h-2} + Dx^{h-3} + \&c. + Kx + L = U}{ax^n + bx^{n-1} + cx^{n-2} + dx^{n-3} + \&c. + kx + l = V}\right\}$$

Let

$$\frac{dV}{dx} = nax^{n-1} + (n-1)\,bx^{n-2} + (n-2)\,cx^{n-3} + (n-3)\,dx^{n-4} + \ldots\ldots\ldots + k = Z.$$

Also let $r', r'', r''', r'''', \ldots\ldots r^{n\prime}$ be the n roots of the equation $V = 0$,

$U', U'', U''', U'''', \ldots\ldots U^{n\prime}$ the values of the function U, when these roots are substituted for x,

$Z', Z'', Z''', Z'''', \ldots\ldots Z^{n\prime}$ the values of the function Z, when these roots are substituted for x;

then, on the supposition that the roots r', r'', r''', &c. are different one from another, and $h < n$, we have generally

$$\int\frac{U\mathrm{d}x}{V} = \frac{U'}{Z'}\log.(x-r') + \frac{U''}{Z''}\log.(x-r'') + \frac{U'''}{Z'''}\log.(x-r''') + \frac{U''''}{Z''''}\log.(x-r'''') + \&c. + \frac{U^{n\prime}}{Z^{n\prime}}\log.(x-r^{n\prime})$$

$$\int\frac{x^m U\mathrm{d}x}{V} = \frac{U'}{Z'}\int\frac{x^m\mathrm{d}x}{x-r'} + \frac{U''}{Z''}\int\frac{x^m\mathrm{d}x}{x-r''} + \frac{U'''}{Z'''}\int\frac{x^m\mathrm{d}x}{x-r'''} + \frac{U''''}{Z''''}\int\frac{x^m\mathrm{d}x}{x-r''''} + \&c.\ \frac{U^{n\prime}}{Z^{n\prime}}\int\frac{x^m\mathrm{d}x}{x-r^{n\prime}}$$

$$\int\frac{U\mathrm{d}x}{x^m V} = \frac{U'}{Z'}\int\frac{\mathrm{d}x}{x^m(x-r')} + \frac{U''}{Z''}\int\frac{\mathrm{d}x}{x^m(x-r'')} + \frac{U'''}{Z'''}\int\frac{\mathrm{d}x}{x^m(x-r''')} + \frac{U''''}{Z''''}\int\frac{\mathrm{d}x}{x^m(x-r'''')} + \&c. + \frac{U^{n\prime}}{Z^{n\prime}}\int\frac{\mathrm{d}x}{x^m(x-r^{n\prime})}$$

and these formulæ are real, when the roots r', r'', r''', &c. are real.

Memorabilia in the preceding Table.

(1). The Formulæ, p. 74—87, are all derived from the general Formulæ of Reduction, p. 1—5. It is to be observed, however, that in p. 83, the first form is not immediately derivable from them. In this case, substituting in Form V, p. 5, $-p$ for p, $m=1$, $n=1$, we finally obtain

$$\int\frac{dx}{X^p}=\frac{Ax+Bx^2}{KX^{p-1}}+\frac{C}{K}\int\frac{dx}{X^{p-1}}+\frac{D}{K}\int\frac{xdx}{X^{p-1}}:$$

where $A=2ac-b^2$, $B=-bc$, $C=(p-1)\ (4ac-b^2)+b^2-2ac$, $D=(-2p+4)\,bc$, $K=(p-1)\,(4ac-b^2)\,a$. But we have

$$\int\frac{xdx}{X^{p-1}}=\frac{-1}{2c\,(p-2)X^{p-2}}-\frac{b}{2c}\int\frac{dx}{X^{p-1}}.$$

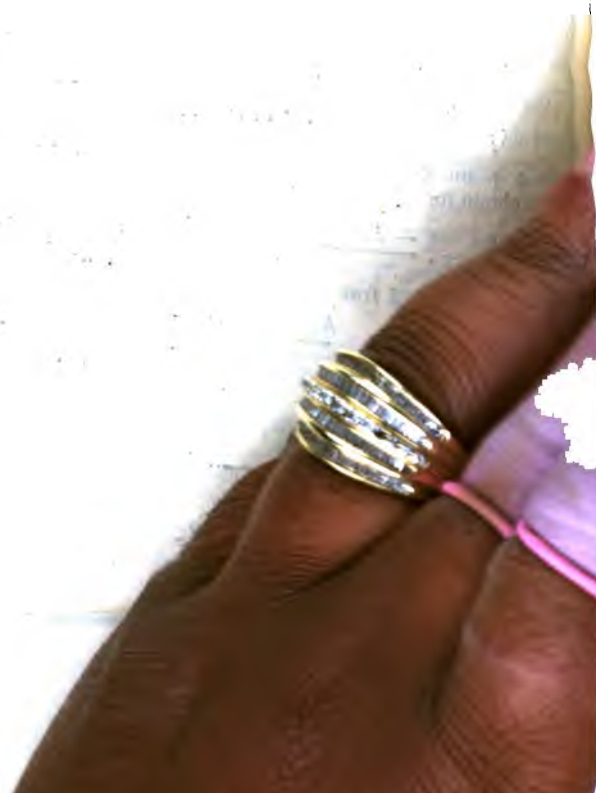
Substituting this value, we obtain, after reduction, the formula which has been given in p. 83.

(2.) Decompose $\frac{x^m}{a+bx^n}$, $\frac{x^m}{a+bx^n+cx^{2n}}$ into partial fraction, which is necessary to integrate $\frac{x^m dx}{a+bx^n}$, $\frac{x^m dx}{a+bx^n+cx^{2n}}$, since it is known that every trinomial factor of $x^n+\frac{a}{b}$ and $x^{2n}+\frac{b}{c}\,x^n+\frac{a}{c}$ (when b^2-4ac is negative) is of the form $x^2-2kx\cos.\varphi+k^2$, we need only substitute $\cos.\varphi+\sin.\varphi\sqrt{-1}$, and bear in mind that $(\cos.\varphi+\sin.\varphi\sqrt{-1})^n=\cos.n\varphi+\sin.n\varphi\sqrt{-1}$. We shall hence obtain for the first fraction, partial fractions of the form

$$\frac{2}{nbk^{n-m-2}}\cdot\frac{-k\cos.(n-m)\,\varphi+\cos.(n-m-1)\,\varphi\,.\,x}{x^2-2kx\cos.\varphi+k^2}$$

and for the second fraction

$$\frac{1}{nck^{2n-m-2}\sin.\alpha}\cdot\frac{k\sin.(n-m)\,\varphi-\sin.(n-m-1)\,\varphi\,.\,x}{x^2-2kx\cos.\varphi+k^2}$$



INTEGRAL TABLES

OF

IRRATIONAL DIFFERENTIALS.

In this [illegible]e Inte[illegible]ch Irrational Differentials as occur m[illegible]ly in[illegible]e given. The Author, moreover, [illegible] hi[illegible] to such Irrationals as i[illegible] partly for the above re[illegible]rtly [illegible]tegration of other [illegible] b[illegible] cases. The more [illegible]ds [illegible] inserted at the end.

TAB. I.

$$\int \frac{x^m dx}{\sqrt{(a+bx)}}$$

$$a + bx = X$$

$$\int \frac{dx}{\sqrt{X}} = \frac{2}{b}\sqrt{X}$$

$$\int \frac{x dx}{\sqrt{X}} = \left(\frac{1}{3}X - a\right)\frac{2\sqrt{X}}{b^2}$$

$$\int \frac{x^2 dx}{\sqrt{X}} = \left(\frac{1}{5}X^2 - \frac{2}{3}aX + a^2\right)\frac{2\sqrt{X}}{b^3}$$

$$\int \frac{x^3 dx}{\sqrt{X}} = \left(\frac{1}{7}X^3 - \frac{3}{5}aX^2 + a^2X - a^3\right)\frac{2\sqrt{X}}{b^4}$$

$$\int \frac{x^4 dx}{\sqrt{X}} = \left(\frac{1}{9}X^4 - \frac{4}{7}aX^3 + \frac{6}{5}a^2X^2 - \frac{4}{3}a^3X + a^4\right)\frac{2\sqrt{X}}{b^5}$$

$$\int \frac{x^5 dx}{\sqrt{X}} = \left(\frac{1}{11}X^5 - \frac{5}{9}aX^4 + \frac{10}{7}a^2X^3 - 2a^3X^2 + \frac{5}{3}a^4X - a^5\right)\frac{2\sqrt{X}}{b^6}$$

$$\int \frac{x^6 dx}{\sqrt{X}} = \left(\frac{1}{13}X^6 - \frac{6}{11}aX^5 + \frac{5}{3}a^2X^4 - \frac{20}{7}a^3X^3 + 3a^4X^2\right.$$
$$\left. - 2a^5X + a^6\right)\frac{2\sqrt{X}}{b^7}$$

$$\int \frac{x dx}{\sqrt{X}} = \left(\frac{1}{15}X^7 - \frac{7}{13}aX^6 + \frac{21}{11}a^2X^5 - \frac{35}{9}a^3X^4 + 5a^4X^3\right.$$
$$\left. - \frac{21}{5}a^5X^2 + \frac{7}{3}a^6X - a^7\right)\frac{2\sqrt{X}}{b^8}$$

$$\int \frac{x^8 dx}{\sqrt{X}} = \left(\frac{1}{17}X^8 - \frac{8}{15}aX^7 + \frac{28}{13}a^2X^6 + \frac{56}{11}a^3X^5 + \frac{70}{9}a^4X^4\right.$$
$$\left. - 8a^5X^3 + \frac{28}{5}a^6X^2 - \frac{8}{3}a^7X + a^8\right)\frac{2\sqrt{X}}{b^9}$$

$$\int \frac{x^9 dx}{\sqrt{X}} = \left(\frac{1}{19}X^9 - \frac{9}{17}aX^8 + \frac{12}{5}a^2X^7 - \frac{84}{13}a^3X^6 + \frac{126}{11}a^4X^5\right.$$
$$\left. - 14a^5X^4 + 12a^6X^3 - \frac{36}{5}a^7X^2 + 3a^8X - a^9\right)\frac{2\sqrt{X}}{b^{10}}$$

$$\int \frac{x^{10} dx}{\sqrt{X}} = \left(\frac{1}{21}X^{10} - \frac{10}{19}aX^9 + \frac{45}{17}a^2X^8 - 8a^3X^7 + \frac{210}{13}a^4X^6 - \frac{252}{11}a^5X^5\right.$$
$$\left. + \frac{70}{3}a^6X^4 - \frac{120}{7}a^7X^3 + 9a^8X^2 - \frac{10}{3}a^9X + a^{10}\right)\frac{2\sqrt{X}}{b^{11}}$$

TAB. II.

$$\int \frac{dx}{x^m \sqrt{(a+bx)}}$$

$$a + bx = X$$

$$\int \frac{dx}{x\sqrt{X}} = \int \frac{dx}{x\sqrt{X}}$$ [see the next page.]

$$\int \frac{dx}{x^2\sqrt{X}} = -\frac{\sqrt{X}}{ax} - \frac{b}{2a}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^3\sqrt{X}} = \left(-\frac{1}{2ax^2} + \frac{3b}{4a^2x}\right)\sqrt{X} + \frac{3b^2}{8a^2}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^4\sqrt{X}} = \left(-\frac{1}{3ax^3} + \frac{5b}{12a^2x^2} - \frac{5b^2}{8a^3x}\right)\sqrt{X} - \frac{5b^3}{16a^3}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^5\sqrt{X}} = \left(-\frac{1}{4ax^4} + \frac{7b}{24a^2x^3} - \frac{35b^2}{96a^3x^2} + \frac{35b^3}{64a^4x}\right)\sqrt{X} + \frac{35b^4}{128a^4}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^6\sqrt{X}} = \left(-\frac{1}{5ax^5} + \frac{9b}{40a^2x^4} - \frac{21b^2}{80a^3x^3} + \frac{21b^3}{64a^4x^2} - \frac{63b^4}{128a^5x}\right)\sqrt{X} - \frac{63b^5}{256a^5}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^7\sqrt{X}} = \left(-\frac{1}{6ax^6} + \frac{11b}{60a^2x^5} - \frac{33b^2}{160a^3x^4} + \frac{77b^3}{320a^4x^3} - \frac{77b^4}{256a^5x^2} + \frac{231b^5}{512a^6x}\right)\sqrt{X} + \frac{231b^6}{1024a^6}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^8\sqrt{X}} = \left(-\frac{1}{7ax^7} + \frac{13b}{84a^2x^6} - \frac{143b^2}{840a^3x^5} + \frac{429b^3}{2240a^4x^4} - \frac{143b^4}{640a^5x^3} + \frac{143b^5}{512a^6x^2} - \frac{429b^6}{1024a^7x}\right)\sqrt{X} - \frac{429b^7}{2048a^7}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^9\sqrt{X}} = \left(-\frac{1}{8ax^8} + \frac{15b}{112a^2x^7} - \frac{65b^2}{448a^3x^6} + \frac{143b^3}{896a^4x^5} - \frac{1287b^4}{7168a^5x^4} + \frac{429b^5}{2048a^6x^3} - \frac{2145b^6}{8192a^7x^2} + \frac{6435b^7}{16384a^8x}\right)\sqrt{X} + \frac{6435b^8}{32768a^8}\int \frac{dx}{x\sqrt{X}}$$

Note on the preceding Table.

In general

$$\int \frac{dx}{x\sqrt{(a+bx)}} = \frac{1}{\sqrt{a}} \log \frac{\sqrt{(a+bx)}-\sqrt{a}}{\sqrt{(a+bx)}+\sqrt{a}} + \text{const.}$$

$$\text{or} \int \frac{dx}{x\sqrt{(a+bx)}} = \frac{2}{\sqrt{-a}} \text{arc tang} \frac{\sqrt{(a+bx)}}{\sqrt{-a}} + \text{const.}$$

The first expression is real when a is positive; the second, when a is negative.

I. a positive.

$$\int \frac{dx}{x\sqrt{(a+bx)}} + \text{const.} = \frac{1}{\sqrt{a}} \log \frac{\sqrt{(a+bx)}-\sqrt{a}}{\sqrt{(a+bx)}+\sqrt{a}}$$

$$= \frac{2}{\sqrt{a}} \log \frac{\sqrt{(a+bx)}-\sqrt{a}}{\sqrt{x}} = -\frac{2}{\sqrt{a}} \log \frac{\sqrt{x}}{\sqrt{(a+bx)}-\sqrt{a}},$$

and in these expressions a can be taken either positive or negative. The integral $\int \frac{dx}{x\sqrt{a+bx}}$ being then infinite, cannot begin from $x=0$.

II. a negative

$$\int \frac{dx}{x\sqrt{(bx-a)}} = \frac{2}{\sqrt{a}} \text{arc tang} \sqrt{\frac{bx-a}{a}} = \frac{2}{\sqrt{a}} \text{arc cot} \sqrt{\frac{a}{bx-a}}$$

$$= \frac{2}{\sqrt{a}} \text{arc sec} \sqrt{\frac{bx}{a}} = \frac{2}{\sqrt{a}} \text{arc cosec} \sqrt{\frac{bx}{bx-a}} = \frac{2}{\sqrt{a}} \text{arc cos} \sqrt{\frac{a}{bx}}$$

$$= \frac{2}{\sqrt{a}} \text{arc sin} \sqrt{\frac{bx-a}{bx}} = \frac{1}{\sqrt{a}} \text{arc cos} \frac{2a-bx}{bx}$$

$$= \frac{1}{\sqrt{a}} \text{arc sin vers} \frac{2(bx-a)}{bx}.$$

In the integral $\int \frac{dx}{x\sqrt{(bx-a)}}$, b cannot be negative; moreover, since this integral begins from $x = \frac{a}{b}$, it cannot vanish for any smaller value. By substituting for this value of x, the above integrals vanish.

TAB. III.

$$\int \frac{x^m dx}{(a+bx)^{\frac{3}{2}}}$$

$$a + bx = X$$

$$\int \frac{dx}{X^{\frac{3}{2}}} = -\frac{2}{b\sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{3}{2}}} = \left(X + a\right)\frac{2}{b^2\sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{3}{2}}} = \left(\frac{1}{3}X^2 - 2aX - a^2\right)\frac{2}{b^3\sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{3}{2}}} = \left(\frac{1}{5}X^3 - aX^2 + 3a^2X + a^3\right)\frac{2}{b^4\sqrt{X}}$$

$$\int \frac{x^4 dx}{X^{\frac{3}{2}}} = \left(\frac{1}{7}X^4 - \frac{4}{5}aX^3 + 2a^2X^2 - 4a^3X - a^4\right)\frac{2}{b^5\sqrt{X}}$$

$$\int \frac{x^5 dx}{X^{\frac{3}{2}}} = \left(\frac{1}{9}X^5 - \frac{5}{7}aX^4 + 2a^2X^3 - \frac{10}{3}a^3X^2 + 5a^4X + a^5\right)\frac{2}{b^6\sqrt{X}}$$

$$\int \frac{x^6 dx}{X^{\frac{3}{2}}} = \left(\frac{1}{11}X^6 - \frac{2}{3}aX^5 + \frac{15}{7}a^2X^4 - 4a^3X^3 + 5a^4X^2 - 6a^5X - a^6\right)\frac{2}{b^7\sqrt{X}}$$

$$\int \frac{x^7 dx}{X^{\frac{3}{2}}} = \left(\frac{1}{13}X^7 - \frac{7}{11}aX^6 + \frac{7}{3}a^2X^5 - 5a^3X^4 + 7a^4X^3 - 7a^5X^2 + 7a^6X + a^7\right)\frac{2}{b^8\sqrt{X}}$$

$$\int \frac{x^8 dx}{X^{\frac{3}{2}}} = \left(\frac{1}{15}X^8 - \frac{8}{13}aX^7 + \frac{28}{11}a^2X^6 - \frac{56}{9}a^3X^5 + 10a^4X^4 - \frac{56}{5}a^5X^3 + \frac{28}{3}a^6X^2 - 8a^7X - a^8\right)\frac{2}{b^9\sqrt{X}}$$

$$\int \frac{x^9 dx}{X^{\frac{3}{2}}} = \left(\frac{1}{17}X^9 - \frac{3}{5}aX^8 + \frac{36}{13}a^2X^7 - \frac{84}{11}a^3X^6 + 14a^4X^5 - 18a^5X^4 + \frac{84}{5}a^6X^3 - 12a^7X^2 + 9a^8X + a^9\right)\frac{2}{b^{10}\sqrt{X}}$$

$$\int \frac{x^{10} dx}{X^{\frac{3}{2}}} = \left(\frac{1}{19}X^{10} - \frac{10}{17}aX^9 + 3a^2X^8 - \frac{120}{13}a^3X^7 + \frac{210}{11}a^4X^6 - 28a^5X^5 + 30a^6X^4 - 24a^7X^3 + 15a^8X^2 - 10a^9X - a^{10}\right)\frac{2}{b^{11}\sqrt{X}}$$

TAB. IV.

$$\int \frac{dx}{x^m (a+bx)^{\frac{3}{2}}}$$

$$a + bx = X$$

$$\int \frac{dx}{xX^{\frac{3}{2}}} = \frac{2}{a\sqrt{X}} + \frac{1}{a}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{3}{2}}} = \left(-\frac{1}{ax} - \frac{3b}{a^2}\right)\frac{1}{\sqrt{X}} - \frac{3b}{2a^2}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^3X^{\frac{3}{2}}} = \left(-\frac{1}{2ax^2} + \frac{5b}{4a^2x} + \frac{15b^2}{4a^3}\right)\frac{1}{\sqrt{X}} + \frac{15b^2}{8a^3}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^4X^{\frac{3}{2}}} = \left(-\frac{1}{3ax^3} + \frac{7b}{12a^2x^2} - \frac{35b^2}{24a^3x} - \frac{35b^3}{8a^4}\right)\frac{1}{\sqrt{X}} - \frac{35b^3}{16a^4}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^5X^{\frac{3}{2}}} = \left(-\frac{1}{4ax^4} + \frac{3b}{8a^2x^3} - \frac{21b^2}{32a^3x^2} + \frac{105b^3}{64a^4x} + \frac{315b^4}{64a^5}\right)\frac{1}{\sqrt{X}}$$
$$+ \frac{315b^4}{128a^5}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^6X^{\frac{3}{2}}} = \left(-\frac{1}{5ax^5} + \frac{11b}{40a^2x^4} - \frac{33b^2}{80a^3x^3} + \frac{231b^3}{320a^4x^2} - \frac{231b^4}{128a^5x}\right.$$
$$\left. - \frac{693b^5}{128a^6}\right)\frac{1}{\sqrt{X}} - \frac{693b^5}{256a^6}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^7X^{\frac{3}{2}}} = \left(-\frac{1}{6ax^6} + \frac{13b}{60a^2x^5} - \frac{143b^2}{480a^3x^4} + \frac{143b^3}{320a^4x^3} - \frac{1001b^4}{1280a^5x^2}\right.$$
$$\left. + \frac{1001b^5}{512a^6x} + \frac{3003b^6}{512a^7}\right)\frac{1}{\sqrt{X}} + \frac{3003b^6}{1024a^7}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^8X^{\frac{3}{2}}} = \left(-\frac{1}{7ax^7} + \frac{5b}{28a^2x^6} - \frac{13b^2}{56a^3x^5} + \frac{143b^3}{448a^4x^4} - \frac{429b^4}{896a^5x^3}\right.$$
$$\left. + \frac{429b^5}{512a^6x^2} - \frac{2145b^6}{1024a^7x} - \frac{6435b^7}{1024a^8}\right)\frac{1}{\sqrt{X}} - \frac{6435b^7}{2048a^8}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^9X^{\frac{3}{2}}} = \left(-\frac{1}{8ax^8} + \frac{17b}{112a^2x^7} - \frac{85b^2}{448a^3x^6} + \frac{221b^3}{896a^4x^5} - \frac{2431b^4}{7168a^5x^4}\right.$$
$$\left. + \frac{7293b^5}{14336a^6x^3} - \frac{7293b^6}{8192a^7x^2} + \frac{36465b^7}{16384a^8x} + \frac{109395b^8}{16384a^9}\right)\frac{1}{\sqrt{X}}$$
$$+ \frac{109395b^8}{32768a^{10}}\int \frac{dx}{x\sqrt{X}}$$

TAB. V.

$$\int \frac{x^m dx}{(a+bx)^{\frac{5}{2}}}$$

$$a + bx = X$$

$$\int \frac{dx}{X^{\frac{5}{2}}} = -\frac{2}{3bX\sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{5}{2}}} = \left(-X + \frac{1}{3}a\right)\frac{2}{b^2X\sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{5}{2}}} = \left(X^2 + 2aX - \frac{1}{3}a^2\right)\frac{2}{b^3X\sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{5}{2}}} = \left(\frac{1}{3}X^3 - 3aX^2 - 3a^2X + \frac{1}{3}a^3\right)\frac{2}{b^4X\sqrt{X}}$$

$$\int \frac{x^4 dx}{X^{\frac{5}{2}}} = \left(\frac{1}{5}X^4 - \frac{4}{3}aX^3 + 6a^2X^2 + 4a^3X - \frac{1}{3}a^4\right)\frac{2}{b^5X\sqrt{X}}$$

$$\int \frac{x^5 dx}{X^{\frac{5}{2}}} = \left(\frac{1}{7}X^5 - aX^4 + \frac{10}{3}a^2X^3 - 10a^3X^2 - 5a^4X + \frac{1}{3}a^5\right)\frac{2}{b^6X\sqrt{X}}$$

$$\int \frac{x^6 dx}{X^{\frac{5}{2}}} = \left(\frac{1}{9}X^6 - \frac{6}{7}aX^5 + 3a^2X^4 - \frac{20}{3}a^3X^3 + 15a^4X^2 + 6a^5X - \frac{1}{3}a^6\right) \times \frac{2}{b^7X\sqrt{X}}$$

$$\int \frac{x^7 dx}{X^{\frac{5}{2}}} = \left(\frac{1}{11}X^7 - \frac{7}{9}aX^6 + 3a^2X^5 - 7a^3X^4 + \frac{35}{3}a^4X^3 - 21a^5X^2 - 7a^6X + \frac{1}{3}a^7\right)\frac{2}{b^8X\sqrt{X}}$$

$$\int \frac{x^8 dx}{X^{\frac{5}{2}}} = \left(\frac{1}{13}X^8 - \frac{8}{11}aX^7 + \frac{28}{9}a^2X^6 - 8a^3X^5 + 14a^4X^4 - \frac{56}{3}a^5X^3 + 28a^6X^2 + 8a^7X - \frac{1}{3}a^8\right)\frac{2}{b^9X\sqrt{X}}$$

$$\int \frac{x^9 dx}{X^{\frac{5}{2}}} = \left(\frac{1}{15}X^9 - \frac{9}{13}aX^8 + \frac{36}{11}a^2X^7 - \frac{28}{3}a^3X^6 + 18a^4X^5 - \frac{126}{5}a^5X^4 + 28a^6X^3 - 36a^7X^2 - 9a^8X + \frac{1}{3}a^9\right)\frac{2}{b^{10}X\sqrt{X}}$$

INTEGRAL TABLES

OF

IRRATIONAL DIFFERENTIALS.

In this Section the Integrals of such Irrational Differentials as occur most frequently in practice, are given. The Author, moreover, has restricted himself, for the most part, to such Irrationals as involve the square root of quantities only, partly for the above reason, and partly because the complete integration of other Irrationals can be effected but in very few cases. The more general Methods and Formulæ, however, are inserted at the end.

TAB. IV.

$$\int \frac{dx}{x^m (a+bx)^{\frac{3}{2}}}$$

$$a + bx = X$$

$$\int \frac{dx}{x X^{\frac{3}{2}}} = \frac{2}{a\sqrt{X}} + \frac{1}{a}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2 X^{\frac{3}{2}}} = \left(-\frac{1}{ax} - \frac{3b}{a^2}\right)\frac{1}{\sqrt{X}} - \frac{3b}{2a^2}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^3 X^{\frac{3}{2}}} = \left(-\frac{1}{2ax^2} + \frac{5b}{4a^2x} + \frac{15b^2}{4a^3}\right)\frac{1}{\sqrt{X}} + \frac{15b^2}{8a^3}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^4 X^{\frac{3}{2}}} = \left(-\frac{1}{3ax^3} + \frac{7b}{12a^2x^2} - \frac{35b^2}{24a^3x} - \frac{35b^3}{8a^4}\right)\frac{1}{\sqrt{X}} - \frac{35b^3}{16a^4}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^5 X^{\frac{3}{2}}} = \left(-\frac{1}{4ax^4} + \frac{3b}{8a^2x^3} - \frac{21b^2}{32a^3x^2} + \frac{105b^3}{64a^4x} + \frac{315b^4}{64a^5}\right)\frac{1}{\sqrt{X}} + \frac{315b^4}{128a^5}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^6 X^{\frac{3}{2}}} = \left(-\frac{1}{5ax^5} + \frac{11b}{40a^2x^4} - \frac{33b^2}{80a^3x^3} + \frac{231b^3}{320a^4x^2} - \frac{231b^4}{128a^5x} - \frac{693b^5}{128a^6}\right)\frac{1}{\sqrt{X}} - \frac{693b^5}{256a^6}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^7 X^{\frac{3}{2}}} = \left(-\frac{1}{6ax^6} + \frac{13b}{60a^2x^5} - \frac{143b^2}{480a^3x^4} + \frac{143b^3}{320a^4x^3} - \frac{1001b^4}{1280a^5x^2} + \frac{1001b^5}{512a^6x} + \frac{3003b^6}{512a^7}\right)\frac{1}{\sqrt{X}} + \frac{3003b^6}{1024a^7}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^8 X^{\frac{3}{2}}} = \left(-\frac{1}{7ax^7} + \frac{5b}{28a^2x^6} - \frac{13b^2}{56a^3x^5} + \frac{143b^3}{448a^4x^4} - \frac{429b^4}{896a^5x^3} + \frac{429b^5}{512a^6x^2} - \frac{2145b^6}{1024a^7x} - \frac{6435b^7}{1024a^8}\right)\frac{1}{\sqrt{X}} - \frac{6435b^7}{2048a^8}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^9 X^{\frac{3}{2}}} = \left(-\frac{1}{8ax^8} + \frac{17b}{112a^2x^7} - \frac{85b^2}{448a^3x^6} + \frac{221b^3}{896a^4x^5} - \frac{2431b^4}{7168a^5x^4} + \frac{7293b^5}{14336a^6x^3} - \frac{7293b^6}{8192a^7x^2} + \frac{36465b^7}{16384a^8x} + \frac{109395b^8}{16384a^9}\right)\frac{1}{\sqrt{X}} + \frac{109395b^8}{32768a^{10}}\int \frac{dx}{x\sqrt{X}}$$

TAB. V.

$$\int \frac{x^m dx}{(a+bx)^{\frac{5}{2}}}$$

$$a + bx = X$$

$$\int \frac{dx}{X^{\frac{5}{2}}} = -\frac{2}{3bX\sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{5}{2}}} = \left(-X + \frac{1}{3}a\right)\frac{2}{b^2X\sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{5}{2}}} = \left(X^2 + 2aX - \frac{1}{3}a^2\right)\frac{2}{b^3X\sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{5}{2}}} = \left(\frac{1}{3}X^3 - 3aX^2 - 3a^2X + \frac{1}{3}a^3\right)\frac{2}{b^4X\sqrt{X}}$$

$$\int \frac{x^4 dx}{X^{\frac{5}{2}}} = \left(\frac{1}{5}X^4 - \frac{4}{3}aX^3 + 6a^2X^2 + 4a^3X - \frac{1}{3}a^4\right)\frac{2}{b^5X\sqrt{X}}$$

$$\int \frac{x^5 dx}{X^{\frac{5}{2}}} = \left(\frac{1}{7}X^5 - aX^4 + \frac{10}{3}a^2X^3 - 10a^3X^2 - 5a^4X + \frac{1}{3}a^5\right)\frac{2}{b^6X\sqrt{X}}$$

$$\int \frac{x^6 dx}{X^{\frac{5}{2}}} = \left(\frac{1}{9}X^6 - \frac{6}{7}aX^5 + 3a^2X^4 - \frac{20}{3}a^3X^3 + 15a^4X^2 + 6a^5X - \frac{1}{3}a^6\right) \times \frac{2}{b^7X\sqrt{X}}$$

$$\int \frac{x^7 dx}{X^{\frac{5}{2}}} = \left(\frac{1}{11}X^7 - \frac{7}{9}aX^6 + 3a^2X^5 - 7a^3X^4 + \frac{35}{3}a^4X^3 - 21a^5X^2 - 7a^6X + \frac{1}{3}a^7\right)\frac{2}{b^8X\sqrt{X}}$$

$$\int \frac{x^8 dx}{X^{\frac{5}{2}}} = \left(\frac{1}{13}X^8 - \frac{8}{11}aX^7 + \frac{28}{9}a^2X^6 - 8a^3X^5 + 14a^4X^4 - \frac{56}{3}a^5X^3 + 28a^6X^2 + 8a^7X - \frac{1}{3}a^8\right)\frac{2}{b^9X\sqrt{X}}$$

$$\int \frac{x^9 dx}{X^{\frac{5}{2}}} = \left(\frac{1}{15}X^9 - \frac{9}{13}aX^8 + \frac{36}{11}a^2X^7 - \frac{28}{3}a^3X^6 + 18a^4X^5 - \frac{126}{5}a^5X^4 + 28a^6X^3 - 36a^7X^2 - 9a^8X + \frac{1}{3}a^9\right)\frac{2}{b^{10}X\sqrt{X}}$$

TAB. VI.

$$\int \frac{dx}{x^m(a+bx^2)^{\frac{5}{2}}}$$

$$a + bx = X$$

$$\int \frac{dx}{xX^{\frac{5}{2}}} = \left(\frac{8}{3a} + \frac{2bx}{a^2}\right)\frac{1}{X\sqrt{X}} + \frac{1}{a^2}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{5}{2}}} = \left(-\frac{1}{ax} - \frac{20b}{3a^2} - \frac{5b^2x}{a^3}\right)\frac{1}{X\sqrt{X}} - \frac{5b}{2a^3}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^3X^{\frac{5}{2}}} = \left(-\frac{1}{2ax^2} + \frac{7b}{4a^2x} + \frac{35b^2}{3a^3} + \frac{35b^3x}{4a^4}\right)\frac{1}{X\sqrt{X}} + \frac{35b^2}{8a^4}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^4X^{\frac{5}{2}}} = \left(-\frac{1}{3ax^3} + \frac{3b}{4a^2x^2} - \frac{21b^2}{8a^3x} - \frac{35b^3}{2a^4} - \frac{105b^4x}{8a^5}\right)\frac{1}{X\sqrt{X}} - \frac{105b^3}{16a^5}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^5X^{\frac{5}{2}}} = \left(-\frac{1}{4ax^4} + \frac{11b}{24a^2x^3} - \frac{33b^2}{32a^3x^2} + \frac{231b^3}{64a^4x} + \frac{385b^4}{16a^5} + \frac{1155b^5x}{64a^6}\right)\frac{1}{X\sqrt{X}} + \frac{1155b^4}{128a^6}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^6X^{\frac{5}{2}}} = \left(-\frac{1}{5ax^5} + \frac{13b}{40a^2x^4} - \frac{143b^2}{240a^3x^3} + \frac{429b^3}{320a^4x^2} - \frac{3003b^4}{640a^5x} - \frac{1001b^5}{32a^6} - \frac{3003b^6x}{128a^7}\right)\frac{1}{X\sqrt{X}} - \frac{3003b^5}{256a^7}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^7X^{\frac{5}{2}}} = \left(-\frac{1}{6ax^6} + \frac{b}{4a^2x^5} - \frac{13b^2}{32a^3x^4} + \frac{143b^3}{192a^4x^3} - \frac{429b^4}{256a^5x^2} + \frac{3003b^5}{512a^6x} + \frac{5005b^6}{128a^7} + \frac{15015b^7x}{512a^8}\right)\frac{1}{X\sqrt{X}} + \frac{15015b^6}{1024a^8}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^8X^{\frac{5}{2}}} = \left(-\frac{1}{7ax^7} + \frac{17b}{84a^2x^6} - \frac{17b^2}{56a^3x^5} + \frac{221b^3}{448a^4x^4} - \frac{2431b^4}{2688a^5x^3} + \frac{7293b^5}{3584a^6x^2} - \frac{7293b^6}{1024a^7x} - \frac{12155b^7}{256a^8} - \frac{36465b^8x}{1024a^9}\right)\frac{1}{X\sqrt{X}} - \frac{36465b^7}{2048a^9}\int \frac{dx}{x\sqrt{X}}$$

TAB. VII.

$$\int \frac{x^m dx}{(a+bx)^{\frac{7}{2}}}$$

$$a + bx = X$$

$$\int \frac{dx}{X^{\frac{7}{2}}} = -\frac{2}{5bX^2\sqrt{X}}$$

$$\int \frac{x\,dx}{X^{\frac{7}{2}}} = \left(-\frac{1}{3}X + \frac{1}{5}a\right)\frac{2}{b^2X^2\sqrt{X}}$$

$$\int \frac{x^2dx}{X^{\frac{7}{2}}} = \left(-X^2 + \frac{2}{3}aX - \frac{1}{5}a^2\right)\frac{2}{b^3X^2\sqrt{X}}$$

$$\int \frac{x^3dx}{X^{\frac{7}{2}}} = \left(X^3 + 3aX^2 - a^2X + \frac{1}{5}a^3\right)\frac{2}{b^4X^2\sqrt{X}}$$

$$\int \frac{x^4dx}{X^{\frac{7}{2}}} = \left(\frac{1}{3}X^4 - 4aX^3 - 6a^2X^2 + \frac{4}{3}a^3X - \frac{1}{5}a^4\right)\frac{2}{b^5X^2\sqrt{X}}$$

$$\int \frac{x^5dx}{X^{\frac{7}{2}}} = \left(\frac{1}{5}X^5 - \frac{5}{3}aX^4 + 10a^2X^3 + 10a^3X^2 - \frac{5}{3}a^4X + \frac{1}{5}a^5\right)\frac{2}{b^6X\cdot\sqrt{X}}$$

$$\int \frac{x^6dx}{X^{\frac{7}{2}}} = \left(\frac{1}{7}X^6 - \frac{6}{5}aX^5 + 5a^2X^4 - 20a^3X^3 - 15a^4X^2 + 2a^5X - \frac{1}{5}a^6\right)\frac{2}{b^7X^2\sqrt{X}}$$

$$\int \frac{x^7dx}{X^{\frac{7}{2}}} = \left(\frac{1}{9}X^7 - aX^6 + \frac{21}{5}a^2X^5 - \frac{35}{3}a^3X^4 + 35a^4X^3 + 21a^5X^2 - \frac{7}{3}a^6X + \frac{1}{5}a^7\right)\frac{2}{b^8X^2\sqrt{X}}$$

$$\int \frac{x^8dx}{X^{\frac{7}{2}}} = \left(\frac{1}{11}X^8 - \frac{8}{9}aX^7 + 4a^2X^6 - \frac{56}{5}a^3X^5 + \frac{70}{3}a^4X^4 - 56a^5X^3 - 28a^6X^2 + \frac{8}{3}a^7X - \frac{1}{5}a^8\right)\frac{2}{b^9X^2\sqrt{X}}$$

$$\int \frac{x^9dx}{X^{\frac{7}{2}}} = \left(\frac{1}{13}X^9 - \frac{9}{11}aX^8 + 4a^2X^7 - 12a^3X^6 + \frac{126}{5}a^4X^5 - 42a^5X^4 + 84a^6X^3 + 36a^7X^2 - 3a^8X + \frac{1}{5}a^9\right)\frac{2}{b^{10}X^2\sqrt{X}}$$

TAB. VIII.

$$\int \frac{dx}{x^m (a+bx)^{\frac{7}{2}}}$$

$$a + bx = X$$

$$\int \frac{dx}{xX^{\frac{7}{2}}} = \left(\frac{46}{15a} + \frac{14bx}{3a^2} + \frac{2b^2x^2}{a^3}\right)\frac{1}{X^2\sqrt{X}} + \frac{1}{a^3}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{7}{2}}} = -\frac{1}{axX^2\sqrt{X}} - \frac{7b}{2a}\int \frac{dx}{xX^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{7}{2}}} = \left(-\frac{1}{2ax^2} + \frac{9b}{4a^2x}\right)\frac{1}{X^2\sqrt{X}} + \frac{63b^2}{8a^2}\int \frac{dx}{xX^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{7}{2}}} = \left(-\frac{1}{3ax^3} + \frac{11b}{12a^2x^2} - \frac{33b^2}{8a^3x}\right)\frac{1}{X^2\sqrt{X}} - \frac{231b^3}{16a^3}\int \frac{dx}{xX^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{7}{2}}} = \left(-\frac{1}{4ax^4} + \frac{13b}{24a^2x^3} - \frac{143b^2}{96a^3x^2} + \frac{429b^3}{64a^4x}\right)\frac{1}{X^2\sqrt{X}} + \frac{3003b^4}{128a^4}\int \frac{dx}{xX^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^6X^{\frac{7}{2}}} = \left(-\frac{1}{5ax^5} + \frac{3b}{8a^2x^4} - \frac{13b^2}{16a^3x^3} + \frac{143b^3}{64a^4x^2} - \frac{1287b^4}{128a^5x}\right)\frac{1}{X^2\sqrt{X}} - \frac{9009b^5}{256a^5}\int \frac{dx}{xX^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^7X^{\frac{7}{2}}} = \left(-\frac{1}{6ax^6} + \frac{17b}{60a^2x^5} - \frac{17b^2}{32a^3x^4} + \frac{221b^3}{192a^4x^3} - \frac{2431b^4}{768a^5x^2} + \frac{7293b^5}{512a^6x}\right)\frac{1}{X^2\sqrt{X}} + \frac{51051b^6}{1024a^6}\int \frac{dx}{xX^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^8X^{\frac{7}{2}}} = \left(-\frac{1}{7ax^7} + \frac{19b}{84a^2x^6} - \frac{323b^2}{840a^3x^5} + \frac{323b^3}{448a^4x^4} - \frac{4199b^4}{2688a^5x^3} + \frac{46189b^5}{10752a^6x^2} - \frac{138567b^6}{7168a^7x}\right)\frac{1}{X^2\sqrt{X}} - \frac{138567b^7}{2048a^7}\int \frac{dx}{xX^{\frac{7}{2}}}$$

TAB. IX.

$$\int \frac{x^m dx}{(a+bx)^{\frac{9}{2}}},\ \int \frac{dx}{x^m (a+bx)^{\frac{9}{2}}}$$

$$a + bx = X$$

$$\int \frac{dx}{X^{\frac{9}{2}}} = -\frac{2}{7bX^3\sqrt{X}}$$

$$\int \frac{xdx}{X^{\frac{9}{2}}} = \left(-\frac{1}{5}X + \frac{1}{7}a\right)\frac{2}{b^2X^3\sqrt{X}}$$

$$\int \frac{x^2dx}{X^{\frac{9}{2}}} = \left(-\frac{1}{3}X^2 + \frac{2}{5}aX - \frac{1}{7}a^2\right)\frac{2}{b^3X^3\sqrt{X}}$$

$$\int \frac{x^3dx}{X^{\frac{9}{2}}} = \left(-X^3 + aX^2 - \frac{3}{5}a^2X + \frac{1}{7}a^3\right)\frac{2}{b^4X^3\sqrt{X}}$$

$$\int \frac{x^4dx}{X^{\frac{9}{2}}} = \left(X^4 + 4aX^3 - 2a^2X^2 + \frac{4}{5}a^3X - \frac{1}{7}a^4\right)\frac{2}{b^5X^3\sqrt{X}}$$

$$\int \frac{x^5dx}{X^{\frac{9}{2}}} = \left(\frac{1}{3}X^5 - 5aX^4 - 10a^2X^3 + \frac{10}{3}a^3X^2 - a^4X + \frac{1}{7}a^5\right)\frac{2}{b^6X^3\sqrt{X}}$$

$$\int \frac{x^6dx}{X^{\frac{9}{2}}} = \left(\frac{1}{5}X^6 - 2aX^5 + 15a^2X^4 + 20a^3X^3 - 5a^4X^2 + \frac{6}{5}a^5X - \frac{1}{7}a^6\right)\frac{1}{b^7X^3\sqrt{X}}$$

$$\int \frac{dx}{xX^{\frac{9}{2}}} = \left(\frac{352}{105a} + \frac{116bx}{15a^2} + \frac{20b^2x^2}{3a^3} + \frac{2b^3x^3}{a^4}\right)\frac{1}{X^3\sqrt{X}} + \frac{1}{a^4}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{9}{2}}} = -\frac{1}{axX^3\sqrt{X}} - \frac{9b}{2a}\int \frac{dx}{xX^{\frac{9}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{9}{2}}} = \left(-\frac{1}{2ax^2} + \frac{11b}{4a^2x}\right)\frac{1}{X^3\sqrt{X}} + \frac{99b^2}{8a^2}\int \frac{dx}{xX^{\frac{9}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{9}{2}}} = \left(-\frac{1}{3ax^3} + \frac{13b}{12a^2x^2} - \frac{143b^2}{24a^3x}\right)\frac{1}{X^3\sqrt{X}} - \frac{429b^3}{16a^3}\int \frac{dx}{xX^{\frac{9}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{9}{2}}} = \left(-\frac{1}{4ax^4} + \frac{5b}{8a^2x^3} - \frac{65b^2}{32a^3x^2} + \frac{715b^3}{64a^4x}\right)\frac{1}{X^3\sqrt{X}} + \frac{6435b^4}{128a^4}\int \frac{dx}{xX^{\frac{9}{2}}}$$

TAB. X.

$$\int x^m dx\sqrt{(a+bx)}$$

$$a + bx = X$$

$$\int dx\sqrt{X} = \frac{2X\sqrt{X}}{3b}$$

$$\int x dx\sqrt{X} = \left(\frac{1}{5}X - \frac{1}{3}a\right)\frac{2X\sqrt{X}}{b^2}$$

$$\int x^2 dx\sqrt{X} = \left(\frac{1}{7}X^2 - \frac{2}{5}aX + \frac{1}{3}a^2\right)\frac{2X\sqrt{X}}{b^3}$$

$$\int x^3 dx\sqrt{X} = \left(\frac{1}{9}X^3 - \frac{3}{7}aX^2 + \frac{3}{5}a^2X - \frac{1}{3}a^3\right)\frac{2X\sqrt{X}}{b^4}$$

$$\int x^4 dx\sqrt{X} = \left(\frac{1}{11}X^4 - \frac{4}{9}aX^3 + \frac{6}{7}a^2X^2 - \frac{4}{5}a^3X + \frac{1}{3}a^4\right)\frac{2X\sqrt{X}}{b^5}$$

$$\int x^5 dx\sqrt{X} = \left(\frac{1}{13}X^5 - \frac{5}{11}aX^4 + \frac{10}{9}a^2X^3 - \frac{10}{7}a^3X^2 + a^4X - \frac{1}{3}a^5\right)\frac{2X\sqrt{X}}{b^6}$$

$$\begin{aligned}\int x^6 dx\sqrt{X} = \Big(\frac{1}{15}X^6 - \frac{6}{13}aX^5 + \frac{15}{11}a^2X^4 - \frac{20}{9}a^3X^3 + \frac{15}{7}a^4X^2 \\ - \frac{6}{5}a^5X + \frac{1}{3}a^6\Big)\frac{2X\sqrt{X}}{b^7}\end{aligned}$$

$$\begin{aligned}\int x^7 dx\sqrt{X} = \Big(\frac{1}{17}X^7 - \frac{7}{15}aX^6 + \frac{21}{13}a^2X^5 - \frac{35}{11}a^3X^4 + \frac{35}{9}a^4X^3 \\ - 3a^5X^2 + \frac{7}{5}a^6X - \frac{1}{3}a^7\Big)\frac{2X\sqrt{X}}{b^8}\end{aligned}$$

$$\begin{aligned}\int x^8 dx\sqrt{X} = \Big(\frac{1}{19}X^8 - \frac{8}{17}aX^7 + \frac{28}{15}a^2X^6 - \frac{56}{13}a^3X^5 + \frac{70}{11}a^4X^4 \\ - \frac{56}{9}a^5X^3 + 4a^6X^2 - \frac{8}{5}a^7X + \frac{1}{3}a^8\Big)\frac{2X\sqrt{X}}{b^9}\end{aligned}$$

$$\begin{aligned}\int x^9 dx\sqrt{X} = \Big(\frac{1}{21}X^9 - \frac{9}{19}aX^8 + \frac{36}{17}a^2X^7 - \frac{28}{5}a^3X^6 + \frac{126}{13}a^4X^5 \\ - \frac{126}{11}a^5X^4 + \frac{28}{3}a^6X^3 - \frac{36}{7}a^7X^2 + \frac{9}{5}a^8X - \frac{1}{3}a^9\Big)\frac{2X\sqrt{X}}{b^{10}}\end{aligned}$$

TAB. XI

$$\int \frac{dx\sqrt{(a+bx)}}{x^m}$$

$$a + bx = X$$

$$\int \frac{dx\sqrt{X}}{x} = 2\sqrt{X} + a\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx\sqrt{X}}{x^2} = -\frac{\sqrt{X}}{x} + \frac{b}{2}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx\sqrt{X}}{x^3} = -\frac{X\sqrt{X}}{2ax^2} + \frac{b\sqrt{X}}{4ax} - \frac{b^2}{8a}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx\sqrt{X}}{x^4} = \left(-\frac{1}{3ax^3} + \frac{b}{4a^2x^2}\right) X\sqrt{X} - \frac{b^2\sqrt{X}}{8a^2x} + \frac{b^3}{16a^2}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx\sqrt{X}}{x^5} = \left(-\frac{1}{4ax^4} + \frac{5b}{24a^2x^3} - \frac{5b^2}{32a^3x^2}\right) X\sqrt{X} + \frac{5b^3\sqrt{X}}{64a^3x} - \frac{5b^4}{128a^3}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx\sqrt{X}}{x^6} = \left(-\frac{1}{5ax^5} + \frac{7b}{40a^2x^4} - \frac{7b^2}{48a^3x^3} + \frac{7b^3}{64a^4x^2}\right) X\sqrt{X} - \frac{7b^4\sqrt{X}}{128a^4x} + \frac{7b^5}{256a^4}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx\sqrt{X}}{x^7} = \left(-\frac{1}{6ax^6} + \frac{3b}{20a^2x^5} - \frac{21b^2}{160a^3x^4} + \frac{7b^3}{64a^4x^3} - \frac{21b^4}{256a^5x^2}\right) X\sqrt{X} + \frac{21b^5\sqrt{X}}{512a^5x} - \frac{21b^6}{1024a^5}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx\sqrt{X}}{x^8} = -\frac{X\sqrt{X}}{7ax^7} - \frac{11b}{14a}\int \frac{dx\sqrt{X}}{x^7}$$

$$\int \frac{dx\sqrt{X}}{x^9} = \left(-\frac{1}{8ax^8} + \frac{13b}{112a^2x^7}\right) X\sqrt{X} + \frac{143b^3}{224a^3}\int \frac{dx\sqrt{X}}{x^7}$$

$$\int \frac{dx\sqrt{X}}{x^{10}} = \left(-\frac{1}{9ax^9} + \frac{5b}{48a^2x^8} - \frac{65b^2}{672a^3x^7}\right) X\sqrt{X} - \frac{715b^3}{1344a^3}\int \frac{dx\sqrt{X}}{x^7}$$

TAB. XII.

$$\int x^m dx (a+bx)^{\frac{3}{2}}$$

$$a + bx = X$$

$$\int dx X^{\frac{3}{2}} = \frac{2X^2\sqrt{X}}{5b}$$

$$\int x dx X^{\frac{3}{2}} = \left(\frac{1}{7}X - \frac{1}{5}a\right)\frac{2X^2\sqrt{X}}{b^2}$$

$$\int x^2 dx X^{\frac{3}{2}} = \left(\frac{1}{9}X^2 - \frac{2}{7}aX + \frac{1}{5}a^2\right)\frac{2X^2\sqrt{X}}{b^3}$$

$$\int x^3 dx X^{\frac{3}{2}} = \left(\frac{1}{11}X^3 - \frac{1}{3}aX^2 + \frac{3}{7}a^2X - \frac{1}{5}a^3\right)\frac{2X^2\sqrt{X}}{b^4}$$

$$\int x^4 dx X^{\frac{3}{2}} = \left(\frac{1}{13}X^4 - \frac{4}{11}aX^3 + \frac{2}{3}a^2X^2 - \frac{4}{7}a^3X + \frac{1}{5}a^4\right)\frac{2X^2\sqrt{X}}{b^5}$$

$$\int x^5 dx X^{\frac{3}{2}} = \left(\frac{1}{15}X^5 - \frac{5}{13}aX^4 + \frac{10}{11}a^2X^3 - \frac{10}{9}a^3X^2 + \frac{5}{7}a^4X - \frac{1}{5}a^5\right)\frac{2X^2\sqrt{X}}{b^6}$$

$$\int x^6 dx X^{\frac{3}{2}} = \left(\frac{1}{17}X^6 - \frac{2}{5}a^2X^5 + \frac{15}{13}a^3X^4 - \frac{20}{11}a^4X^3 + \frac{5}{3}a^5X\right.$$
$$\left. - \frac{6}{7}a^6X + \frac{1}{5}a^7\right)\frac{2X^2\sqrt{X}}{b^7}$$

$$\int x^7 dx X^{\frac{3}{2}} = \left(\frac{1}{19}X^7 - \frac{7}{17}aX^6 + \frac{7}{5}a^2X^5 - \frac{35}{13}a^3X^4 + \frac{35}{11}a^4X\right.$$
$$\left. - \frac{7}{3}a^5X^2 + a^6X - \frac{1}{5}a^7\right)\frac{2X^2\sqrt{X}}{b^8}$$

$$\int x^8 dx X^{\frac{3}{2}} = \left(\frac{1}{21}X^8 - \frac{8}{19}aX^7 + \frac{28}{17}a^2X^6 - \frac{56}{15}a^3X^5 + \frac{70}{13}a^4X\right.$$
$$\left. - \frac{56}{11}a^5X^3 + \frac{28}{9}a^6X^2 - \frac{8}{7}a^7X + \frac{1}{5}a^8\right)\frac{2X^2\sqrt{X}}{b^9}$$

$$\int x^9 dx X^{\frac{3}{2}} = \left(\frac{1}{23}X^9 - \frac{3}{7}aX^8 + \frac{36}{19}a^2X^7 - \frac{84}{17}a^3X^6 + \frac{42}{5}a^4X\right.$$
$$\left. - \frac{126}{13}a^5X^4 + \frac{84}{11}a^6X^3 - 4a^7X^2 + \frac{9}{7}a^8X - \frac{1}{5}a^9\right)\frac{2X^2\sqrt{X}}{b^{10}}$$

TAB. XIII.

$$\int \frac{dx(a+bx)^{\frac{3}{2}}}{x^m}$$

$$a + bx = X$$

$$\int \frac{dxX^{\frac{3}{2}}}{x} = \left(\frac{1}{3}X + a\right) 2\sqrt{X} + a^2 \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx\,X^{\frac{3}{2}}}{x^2} = -\frac{X^2\sqrt{X}}{ax} + \frac{3b}{3a}\int \frac{dxX^{\frac{3}{2}}}{x}$$

$$\int \frac{dxX^{\frac{3}{2}}}{x^3} = \left(-\frac{1}{2ax^2} - \frac{b}{4a^2x}\right) X^2\sqrt{X} + \frac{3b^2}{8a^2}\int \frac{dxX^{\frac{3}{2}}}{x}$$

$$\int \frac{dxX^{\frac{3}{2}}}{x^4} = \left(-\frac{1}{3ax^3} + \frac{b}{12a^2x^2} + \frac{b^2}{24a^3x}\right) X^2\sqrt{X} - \frac{b^3}{16a^3}\int \frac{dxX^{\frac{3}{2}}}{x}$$

$$\int \frac{dxX^{\frac{3}{2}}}{x^5} = \left(-\frac{1}{4ax^4} + \frac{b}{8a^2x^3} - \frac{b^2}{32a^3x^2} - \frac{b^3}{64a^4x}\right) X^2\sqrt{X}$$
$$+ \frac{3b^4}{128a^4}\int \frac{dxX^{\frac{3}{2}}}{x}$$

$$\int \frac{dxX^{\frac{3}{2}}}{x^6} = \left(-\frac{1}{5ax^5} + \frac{b}{8a^2x^4} - \frac{b^2}{16a^3x^3} + \frac{b^3}{64a^4x^2} + \frac{b^4}{128a^5x}\right) X^2\sqrt{X}$$
$$- \frac{3b^5}{256a^5}\int \frac{dxX^{\frac{3}{2}}}{x}$$

$$\int \frac{dxX^{\frac{3}{2}}}{x^7} = \left(-\frac{1}{6ax^6} + \frac{7b}{60a^2x^5} - \frac{7b^2}{96a^3x^4} + \frac{7b^3}{192a^4x^3} - \frac{7b^4}{768a^5x^2}\right.$$
$$\left. - \frac{7b^5}{1536a^6x}\right) X^2\sqrt{X} + \frac{7b^6}{1024a^6}\int \frac{dxX^{\frac{3}{2}}}{x}$$

$$\int \frac{dxX^{\frac{3}{2}}}{x^8} = -\frac{X^2\sqrt{X}}{7ax^7} - \frac{9b}{14a}\int \frac{dxX^{\frac{3}{2}}}{x^7}$$

$$\int \frac{dxX^{\frac{3}{2}}}{x^9} = \left(-\frac{1}{8ax^8} + \frac{11b}{112a^2x^7}\right) X^2\sqrt{X} + \frac{99b^2}{224a^2}\int \frac{dxX^{\frac{3}{2}}}{x^7}$$

$$\int \frac{dxX^{\frac{3}{2}}}{x^{10}} = \left(-\frac{1}{9ax^9} + \frac{13b}{144a^2x^8} - \frac{143b}{2016a^3x^7}\right) X^2\sqrt{X} - \frac{143b^3}{448a^3}\int \frac{dxX^{\frac{3}{2}}}{x^7}$$

TAB. XIV.

$$\int x^m dx(a+bx)^{\frac{5}{2}}$$

$$a + bx = X$$

$$\int dx X^{\frac{5}{2}} = \frac{2X^3\sqrt{X}}{7b}$$

$$\int x dx X^{\frac{5}{2}} = \left(\frac{1}{9}X - \frac{1}{7}a\right)\frac{2X^3\sqrt{X}}{b^2}$$

$$\int x^2 dx X^{\frac{5}{2}} = \left(\frac{1}{11}X^2 - \frac{2}{9}aX + \frac{1}{7}a^2\right)\frac{2X^3\sqrt{X}}{b^3}$$

$$\int x^3 dx X^{\frac{5}{2}} = \left(\frac{1}{13}X^3 - \frac{3}{11}aX^2 + \frac{1}{3}a^2X - \frac{1}{7}a^3\right)\frac{2X^3\sqrt{X}}{b^4}$$

$$\int x^4 dx X^{\frac{5}{2}} = \left(\frac{1}{15}X^4 - \frac{4}{13}aX^3 + \frac{6}{11}a^2X^2 - \frac{4}{9}a^3X + \frac{1}{7}a^4\right)\frac{2X^3\sqrt{X}}{b^5}$$

$$\int x^5 dx X^{\frac{5}{2}} = \left(\frac{1}{17}X^5 - \frac{1}{3}aX^4 + \frac{10}{13}a^2X^3 - \frac{10}{11}a^3X^2 + \frac{5}{9}a^4X - \frac{1}{7}a^5\right)\frac{2X^3\sqrt{X}}{b^6}$$

$$\int x^6 dx X^{\frac{5}{2}} = \left(\frac{1}{19}X^6 - \frac{6}{17}aX^5 + a^2X^4 - \frac{20}{13}a^3X^3 + \frac{15}{11}a^4X^2 - \frac{2}{3}a^5X + \frac{1}{7}a^6\right)\frac{2X^3\sqrt{X}}{b^7}$$

$$\int x^7 dx X^{\frac{5}{2}} = \left(\frac{1}{21}X^7 - \frac{7}{19}aX^6 + \frac{21}{17}a^2X^5 - \frac{7}{3}a^3X^4 + \frac{35}{13}a^4X^3 - \frac{21}{11}a^5X^2 + \frac{7}{9}a^6X - \frac{1}{7}a^7\right)\frac{2X^3\sqrt{X}}{b^8}$$

$$\int x^8 dx X^{\frac{5}{2}} = \left(\frac{1}{23}X^8 - \frac{8}{21}aX^7 + \frac{28}{19}a^2X^6 - \frac{56}{17}a^3X^5 + \frac{14}{3}a^4X^4 - \frac{56}{13}a^5X^3 + \frac{28}{11}a^6X^2 - \frac{8}{9}a^7X + \frac{1}{7}a^8\right)\frac{2X^3\sqrt{X}}{b^9}$$

$$\int x^9 dx X^{\frac{5}{2}} = \left(\frac{1}{25}X^9 - \frac{9}{23}aX^8 + \frac{12}{7}a^2X^7 - \frac{84}{19}a^3X^6 + \frac{126}{17}a^4X^5 - \frac{42}{5}a^5X^4 + \frac{84}{13}a^6X^3 - \frac{36}{11}a^7X^2 + a^8X - \frac{1}{7}a^9\right)\frac{2X^3\sqrt{X}}{b^{10}}$$

TAB. XV.

$$\int \frac{dx(a+bx)^{\frac{5}{2}}}{x^m}$$

$$a + bx = X$$

$$\int \frac{dx X^{\frac{5}{2}}}{x} = \left(\frac{1}{5}X^2 + \frac{1}{3}aX + a^2\right) 2\sqrt{X} + a^3 \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^2} = -\frac{X^3\sqrt{X}}{ax} + \frac{5b}{2a}\int \frac{dx X^{\frac{5}{2}}}{x}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^3} = \left(-\frac{1}{2ax^2} - \frac{3b}{4a^2x}\right) X^3\sqrt{X} + \frac{15b^2}{8a^2}\int \frac{dx X^{\frac{5}{2}}}{x}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^4} = \left(-\frac{1}{3ax^3} - \frac{b}{12a^2x^2} - \frac{b^2}{8a^3x}\right) X^3\sqrt{X} + \frac{5b^3}{16a^3}\int \frac{dx X^{\frac{5}{2}}}{x}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^5} = \left(-\frac{1}{4ax^4} + \frac{b}{24a^2x^3} + \frac{b^2}{96a^3x^2} + \frac{b^3}{64a^4x}\right) X^3\sqrt{X} - \frac{5b^4}{128a^4}\int \frac{dx X^{\frac{5}{2}}}{x}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^6} = \left(-\frac{1}{5ax^5} + \frac{3b}{40a^2x^4} - \frac{b^2}{80a^3x^3} - \frac{b^3}{320a^4x^2} - \frac{3b^4}{640a^5x}\right) X^3\sqrt{X} + \frac{3b^5}{256a^5}\int \frac{dx X^{\frac{5}{2}}}{x}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^7} = \left(-\frac{1}{6ax^6} + \frac{1}{12a^2x^7} - \frac{b^2}{32a^3x^4} + \frac{b^3}{192a^4x^3} + \frac{b^4}{768a^5x^2} + \frac{b^5}{512a^6x}\right) X^3\sqrt{X} - \frac{5b^6}{1024a^6}\int \frac{dx X^{\frac{5}{2}}}{x}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^8} = -\frac{X^3\sqrt{X}}{7ax^7} - \frac{b}{2a}\int \frac{dx X^{\frac{5}{2}}}{x^7}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^9} = \left(-\frac{1}{8ax^8} + \frac{9b}{112a^2x^7}\right) X^3\sqrt{X} + \frac{9b^2}{32a^2}\int \frac{dx X^{\frac{5}{2}}}{x^7}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^{10}} = \left(-\frac{1}{9ax^9} + \frac{11b}{144a^2x^8} - \frac{11b^2}{224a^3x^7}\right) X^3\sqrt{X} - \frac{11b^3}{64a^3}\int \frac{dx X^{\frac{5}{2}}}{x^7}$$

TAB. XVI.

$$\int x^m dx(a+bx)^{\frac{7}{2}}$$

$$a + bx = X$$

$$\int dx\sqrt{X^7} = \frac{2X^4\sqrt{X}}{9b}$$

$$\int x dx\sqrt{X^7} = \left(\frac{1}{11}X - \frac{1}{9}a\right)\frac{2X^4\sqrt{X}}{b^2}$$

$$\int x^2 dx X^{\frac{7}{2}} = \left(\frac{1}{13}X^2 - \frac{2}{11}aX + \frac{1}{9}a^2\right)\frac{2X^4\sqrt{X}}{b^3}$$

$$\int x^3 dx X^{\frac{7}{2}} = \left(\frac{1}{15}X^3 - \frac{3}{13}aX^2 + \frac{3}{11}a^2X - \frac{1}{9}a^3\right)\frac{2X^4\sqrt{X}}{b^4}$$

$$\int x^4 dx X^{\frac{7}{2}} = \left(\frac{1}{17}X^4 - \frac{4}{15}aX^3 + \frac{6}{13}a^2X^2 - \frac{4}{11}a^3X + \frac{1}{9}a^4\right)\frac{2X^4\sqrt{X}}{b^5}$$

$$\int x^5 dx X^{\frac{7}{2}} = \left(\frac{1}{19}X^5 - \frac{5}{17}aX^4 + \frac{2}{3}a^2X^3 - \frac{10}{13}a^3X^2 + \frac{5}{11}a^4X - \frac{1}{9}a^5\right)\frac{2X^4\sqrt{X}}{b^6}$$

$$\int x^6 dx X^{\frac{7}{2}} = \left(\frac{1}{21}X^6 - \frac{6}{19}aX^5 + \frac{15}{17}a^2X^4 - \frac{4}{3}a^3X^3 + \frac{15}{13}a^4X^2\right.$$
$$\left. - \frac{6}{11}a^5X + \frac{1}{9}a^6\right)\frac{2X^4\sqrt{X}}{b^7}$$

$$\int x^7 dx X^{\frac{7}{2}} = \left(\frac{1}{23}X^7 - \frac{1}{3}aX^6 + \frac{21}{19}a^2X^5 - \frac{35}{17}a^3X^4 + \frac{7}{3}a^4X^3\right.$$
$$\left. - \frac{21}{13}a^5X^2 + \frac{7}{11}a^6X - \frac{1}{9}a^7\right)\frac{2X^4\sqrt{X}}{b^8}$$

$$\int x^8 dx X^{\frac{7}{2}} = \left(\frac{1}{25}X^8 - \frac{8}{23}aX^7 + \frac{4}{3}a^2X^6 - \frac{56}{19}a^3X^5 + \frac{70}{17}a^4X^4\right.$$
$$\left. - \frac{56}{15}a^5X^3 + \frac{28}{13}a^6X^2 - \frac{8}{11}a^7X + \frac{1}{9}a^8\right)\frac{2X^4\sqrt{X}}{b^9}$$

$$\int x^9 dx X^{\frac{7}{2}} = \left(\frac{1}{27}X^9 - \frac{9}{25}aX^8 + \frac{36}{23}a^2X^7 - 4a^3X^6 + \frac{126}{19}a^4X^5\right.$$
$$\left. - \frac{126}{17}a^5X^4 + \frac{28}{5}a^6X^3 - \frac{36}{13}a^7X^2 + \frac{9}{11}a^8X - \frac{1}{9}a^9\right)\frac{2X^4\sqrt{X}}{b^{10}}$$

TAB. XVII.

$$\int\frac{dx(a+bx)^{\frac{7}{2}}}{x^m}$$

$$a+bx=X$$

$$\int\frac{dxX^{\frac{7}{2}}}{x}=\left(\frac{1}{7}X^3+\frac{1}{5}aX^2+\frac{1}{3}a^2X+a^3\right)2\sqrt{X}+a^4\int\frac{dx}{x\sqrt{X}}$$

$$\int\frac{dxX^{\frac{7}{2}}}{x^2}=-\frac{X^4\sqrt{X}}{ax}+\frac{7b}{2a}\int\frac{dxX^{\frac{7}{2}}}{x}$$

$$\int\frac{dxX^{\frac{7}{2}}}{x^3}=\left(-\frac{1}{2ax^2}-\frac{5b}{4a^2x}\right)X^4\sqrt{X}+\frac{35b^2}{8a^2}\int\frac{dxX^{\frac{7}{2}}}{x}$$

$$\int\frac{dxX^{\frac{7}{2}}}{x^4}=\left(-\frac{1}{3ax^3}-\frac{b}{4a^2x^2}-\frac{5b^2}{8a^3x}\right)X^4\sqrt{X}+\frac{35b^3}{16a^3}\int\frac{dxX^{\frac{7}{2}}}{x}$$

$$\int\frac{dxX^{\frac{7}{2}}}{x^5}=\left(-\frac{1}{4ax^4}-\frac{b}{24a^2x^3}-\frac{b^2}{32a^3x^2}-\frac{5b^3}{64a^4x}\right)X^4\sqrt{X}$$
$$+\frac{35b^4}{128a^4}\int\frac{dxX^{\frac{7}{2}}}{x}$$

$$\int\frac{dxX^{\frac{7}{2}}}{x^6}=\left(-\frac{1}{5ax^5}+\frac{b}{40a^2x^4}+\frac{b^2}{240a^3x^3}+\frac{b^3}{320a^4x^2}+\frac{b^4}{128a^5x}\right)X^4\sqrt{X}$$
$$-\frac{7b^5}{256a^5}\int\frac{dxX^{\frac{7}{2}}}{x}$$

$$\int\frac{dxX^{\frac{7}{2}}}{x^7}=\left(-\frac{1}{6ax^6}+\frac{b}{20a^2x^5}-\frac{b^2}{160a^3x^4}-\frac{b^3}{960a^4x^3}-\frac{b^4}{1280a^5x^2}\right.$$
$$\left.-\frac{b^5}{512a^6x}\right)X^4\sqrt{X}+\frac{7b^6}{1024a^6}\int\frac{dxX^{\frac{7}{2}}}{x}$$

$$\int\frac{dxX^{\frac{7}{2}}}{x^8}=-\frac{X^4\sqrt{X}}{7ax^7}-\frac{5b}{14a}\int\frac{dxX^{\frac{7}{2}}}{x^7}$$

$$\int\frac{dxX^{\frac{7}{2}}}{x^9}=\left(-\frac{1}{8ax^8}+\frac{b}{16a^2x^7}\right)X^4\sqrt{X}+\frac{5b^2}{32a^2}\int\frac{dxX^{\frac{7}{2}}}{x^7}$$

$$\int\frac{dxX^{\frac{7}{2}}}{x^{10}}=\left(-\frac{1}{9ax^9}+\frac{b}{16a^2x^8}-\frac{b^2}{32a^3x^7}\right)X^4\sqrt{X}-\frac{5b^3}{64a^3}\int\frac{dxX^{\frac{7}{2}}}{x^7}$$

TAB. XVIII.

$$\int x^m dx(a+bx)^{\frac{9}{2}}, \quad \int \frac{dx(a+bx)^{\frac{9}{2}}}{x^m}$$

$$a + bx = X$$

$$\int dx X^{\frac{9}{2}} = \frac{2X^5\sqrt{X}}{11b}$$

$$\int x dx X^{\frac{9}{2}} = \left(\frac{1}{13}X - \frac{1}{11}a\right)\frac{2X^5\sqrt{X}}{b^2}$$

$$\int x^2 dx X^{\frac{9}{2}} = \left(\frac{1}{15}X^2 - \frac{2}{13}aX + \frac{1}{11}a^2\right)\frac{2X^5\sqrt{X}}{b^3}$$

$$\int x^3 dx X^{\frac{9}{2}} = \left(\frac{1}{17}X^3 - \frac{1}{5}aX^2 + \frac{3}{13}a^2X - \frac{1}{11}a^3\right)\frac{2X^5\sqrt{X}}{b^4}$$

$$\int x^4 dx X^{\frac{9}{2}} = \left(\frac{1}{19}X^4 - \frac{4}{17}aX^3 + \frac{2}{5}a^2X^2 - \frac{4}{13}a^3X + \frac{1}{11}a^4\right)\frac{2X^5\sqrt{X}}{b^5}$$

$$\int x^5 dx X^{\frac{9}{2}} = \left(\frac{1}{21}X^5 - \frac{5}{19}aX^4 + \frac{10}{17}a^2X^3 - \frac{2}{3}a^3X^2 + \frac{5}{13}a^4X - \frac{1}{11}a^5\right)\frac{2X^5\sqrt{X}}{b^6}$$

$$\int x^6 dx X^{\frac{9}{2}} = \left(\frac{1}{23}X^6 - \frac{2}{7}aX^5 + \frac{15}{19}a^2X^4 - \frac{20}{17}a^3X^3 + a^4X^2 - \frac{6}{13}a^5X + \frac{1}{11}a^6\right)\frac{2X^5\sqrt{X}}{b^7}$$

$$\int \frac{dx X^{\frac{9}{2}}}{x} = \left(\frac{1}{9}X^4 + \frac{1}{7}aX^3 + \frac{1}{5}a^2X^2 + \frac{1}{3}a^3X + a^4\right)2\sqrt{X} + a^5\int\frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx X^{\frac{9}{2}}}{x^2} = -\frac{X^5\sqrt{X}}{ax} + \frac{9b}{2a}\int\frac{dx X^{\frac{9}{2}}}{x}$$

$$\int \frac{dx X^{\frac{9}{2}}}{x^3} = \left(-\frac{1}{2ax^2} - \frac{7b}{4a^2x}\right)X^5\sqrt{X} + \frac{63b^2}{8a^2}\int\frac{dx X^{\frac{9}{2}}}{x}$$

$$\int \frac{dx X^{\frac{9}{2}}}{x^4} = \left(-\frac{1}{3ax^3} - \frac{5b}{12a^2x^2} - \frac{35b^2}{24a^3x}\right)X^5\sqrt{X} + \frac{105b^3}{16a^3}\int\frac{dx X^{\frac{9}{2}}}{x}$$

$$\int \frac{dx X^{\frac{9}{2}}}{x^5} = \left(-\frac{1}{4ax^4} - \frac{b}{8a^2x^3} - \frac{5b^2}{32a^3x^2} - \frac{35b^3}{64a^4x}\right)X^5\sqrt{X} + \frac{315b^4}{128a^4}\int\frac{dx X^{\frac{9}{2}}}{x}$$

TAB. XIX.

$$\int \frac{x^m dx}{\sqrt[3]{(a+bx)}}, \quad \int \frac{x^m dx}{\sqrt[3]{(a+bx)^2}}$$

$$a + bx = X$$

$$\int \frac{dx}{\sqrt[3]{X}} = \frac{3\sqrt[3]{X^2}}{2b}$$

$$\int \frac{x\,dx}{\sqrt[3]{X}} = \left(\frac{1}{5}X - \frac{1}{2}a\right)\frac{3\sqrt[3]{X^2}}{b^2}$$

$$\int \frac{x^2 dx}{\sqrt[3]{X}} = \left(\frac{1}{8}X^2 - \frac{2}{5}aX + \frac{1}{2}a^2\right)\frac{3\sqrt[3]{X^2}}{b^3}$$

$$\int \frac{x^3 dx}{\sqrt[3]{X}} = \left(\frac{1}{11}X^3 - \frac{3}{8}aX^2 + \frac{3}{5}a^2X - \frac{1}{2}a^3\right)\frac{3\sqrt[3]{X^2}}{b^4}$$

$$\int \frac{x^4 dx}{\sqrt[3]{X}} = \left(\frac{1}{14}X^4 - \frac{4}{11}aX^3 + \frac{3}{4}a^2X^2 - \frac{4}{5}a^3X + \frac{1}{2}a^4\right)\frac{3\sqrt[3]{X^2}}{b^5}$$

$$\int \frac{x^5 dx}{\sqrt[3]{X}} = \left(\frac{1}{17}X^5 - \frac{5}{14}aX^4 + \frac{10}{11}a^2X^3 - \frac{5}{4}a^3X^2 + a^4X - \frac{1}{2}a^5\right)\frac{3\sqrt[3]{X^2}}{b^6}$$

$$\int \frac{dx}{\sqrt[3]{X^2}} = \frac{3\sqrt[3]{X}}{b}$$

$$\int \frac{x\,dx}{\sqrt[3]{X^2}} = \left(\frac{1}{4}X - a\right)\frac{3\sqrt[3]{X}}{b^2}$$

$$\int \frac{x^2 dx}{\sqrt[3]{X^2}} = \left(\frac{1}{7}X^2 - \frac{1}{2}aX + a^2\right)\frac{3\sqrt[3]{X}}{b^3}$$

$$\int \frac{x^3 dx}{\sqrt[3]{X^2}} = \left(\frac{1}{10}X^3 - \frac{3}{7}aX^2 + \frac{3}{4}a^2X - a^3\right)\frac{3\sqrt[3]{X}}{b^4}$$

$$\int \frac{x^4 dx}{\sqrt[3]{X^2}} = \left(\frac{1}{13}X^4 - \frac{2}{5}aX^3 + \frac{6}{7}a^2X^2 - a^3X + a^4\right)\frac{3\sqrt[3]{X}}{b^5}$$

$$\int \frac{x^5 dx}{\sqrt[3]{X^2}} = \left(\frac{1}{16}X^5 - \frac{5}{13}aX^4 + a^2X^3 - \frac{10}{7}a^3X^2 + \frac{5}{4}a^4X - a^5\right)\frac{3\sqrt[3]{X}}{b^6}$$

TAB. XX.

$$\int \frac{dx}{x^m \sqrt[3]{(a+bx)}}, \int \frac{dx}{x^m \sqrt[3]{(a+bx)^2}}.$$

$$a + bx = X$$

$$\int \frac{dx}{x\sqrt[3]{X}} = \frac{1}{\sqrt[3]{a}} \left[\frac{3}{2} \log. \frac{\sqrt[3]{X} - \sqrt[3]{a}}{\sqrt[3]{x}} + \sqrt{3}. \text{arc tang.} \frac{\sqrt{3}\,.\,\sqrt[3]{X}}{\sqrt[3]{X} + 2\sqrt[3]{a}}\right]$$

$$\int \frac{dx}{x^2\sqrt[3]{X}} = -\frac{\sqrt[3]{X^2}}{ax} - \frac{b}{3a}\int \frac{dx}{x\sqrt[3]{X}}$$

$$\int \frac{dx}{x^3\sqrt[3]{X}} = \left(-\frac{1}{2ax^2} + \frac{2b}{3a^2x}\right)\sqrt[3]{X^2} + \frac{2b^2}{9a^2}\int \frac{dx}{x\sqrt[3]{X}}$$

$$\int \frac{dx}{x^4\sqrt[3]{X}} = \left(-\frac{1}{3ax^3} + \frac{7b}{18a^2x^2} - \frac{14b^2}{27a^3x}\right)\sqrt[3]{X^2} - \frac{14b^3}{81a^3}\int \frac{dx}{x\sqrt[3]{X}}$$

$$\int \frac{dx}{x^5\sqrt[3]{X}} = \left(-\frac{1}{4ax^4} + \frac{5b}{18a^2x^3} - \frac{35b^2}{108a^3x^2} + \frac{35b^3}{81a^4x}\right)\sqrt[3]{X^2} + \frac{35b^4}{243a^4}\int \frac{dx}{x\sqrt[3]{X}}$$

$$\int \frac{dx}{x\sqrt[3]{X^2}} = \frac{1}{\sqrt[3]{a^2}} \left[\frac{3}{2} \log. \frac{\sqrt[3]{X} - \sqrt[3]{a}}{\sqrt[3]{x}} - \sqrt{3}\,.\,\text{arc tang.} \frac{\sqrt{3}\,.\,\sqrt[3]{X}}{\sqrt[3]{X} + 2\sqrt[3]{a}}\right]$$

$$\int \frac{dx}{x^2\sqrt[3]{X^2}} = -\frac{\sqrt[3]{X}}{ax} - \frac{2b}{3a}\int \frac{dx}{x\sqrt[3]{X^2}}$$

$$\int \frac{dx}{x^3\sqrt[3]{X^2}} = \left(-\frac{1}{2ax^2} + \frac{5b}{6a^2x}\right)\sqrt[3]{X} + \frac{5b^2}{9a^2}\int \frac{dx}{x\sqrt[3]{X^2}}$$

$$\int \frac{dx}{x^4\sqrt[3]{X^2}} = \left(-\frac{1}{3ax^3} + \frac{4b}{9a^2x^2} - \frac{20b^2}{27a^3x}\right)\sqrt[3]{X} - \frac{40b^3}{81a^3}\int \frac{dx}{x\sqrt[3]{X^2}}$$

$$\int \frac{dx}{x^5\sqrt[3]{X^2}} = \left(-\frac{1}{4ax^4} + \frac{11b}{36a^2x^3} - \frac{11b^2}{27a^3x^2} + \frac{55b^3}{81a^4x}\right)\sqrt[3]{X} + \frac{110b^4}{243a^4}\int \frac{dx}{x\sqrt[3]{X^2}}$$

TAB. XXI.

$$\int x^m dx\, \sqrt[3]{(a+bx)}, \quad \int x^m dx\, \sqrt[3]{(a+bx)^2}$$

$$a + bx = X$$

$$\int dx \sqrt[3]{X} = \frac{3X\sqrt[3]{X}}{4b}$$

$$\int x dx \sqrt[3]{X} = \left(\frac{1}{7}X - \frac{1}{4}a\right)\frac{3X\sqrt[3]{X}}{b^2}$$

$$\int x^2 dx \sqrt[3]{X} = \left(\frac{1}{10}X^2 - \frac{2}{7}aX + \frac{1}{4}a^2\right)\frac{3X\sqrt[3]{X}}{b^3}$$

$$\int x^3 dx \sqrt[3]{X} = \left(\frac{1}{13}X^3 - \frac{3}{10}aX^2 + \frac{3}{7}a^2X - \frac{1}{4}a^3\right)\frac{3X\sqrt[3]{X}}{b^4}$$

$$\int x^4 dx \sqrt[3]{X} = \left(\frac{1}{16}X^4 - \frac{4}{13}aX^3 + \frac{3}{5}a^2X^2 - \frac{4}{7}a^3X + \frac{1}{4}a^4\right)\frac{3X\sqrt[3]{X}}{b^5}$$

$$\int x^5 dx \sqrt[3]{X} = \left(\frac{1}{19}X^5 - \frac{5}{16}aX^4 + \frac{10}{13}a^2X^3 - a^3X^2 + \frac{5}{7}a^4X - \frac{1}{5}a^5\right)\frac{2X\sqrt[3]{X}}{b^6}$$

$$\int dx \sqrt[3]{X^2} = \frac{3X\sqrt[3]{X^2}}{5b}$$

$$\int x dx \sqrt[3]{X^2} = \left(\frac{1}{8}X - \frac{1}{5}a\right)\frac{3X\sqrt[3]{X^2}}{b^2}$$

$$\int x^2 dx \sqrt[3]{X^2} = \left(\frac{1}{11}X^2 - \frac{1}{4}aX + \frac{1}{5}a^2\right)\frac{3X\sqrt[3]{X^2}}{b^3}$$

$$\int x^3 dx \sqrt[3]{X^2} = \left(\frac{1}{14}X^3 - \frac{3}{11}aX^2 + \frac{3}{8}a^2X - \frac{1}{5}a^3\right)\frac{3X\sqrt[3]{X^2}}{b^4}$$

$$\int x^4 dx \sqrt[3]{X^2} = \left(\frac{1}{17}X^4 - \frac{2}{7}aX^3 + \frac{6}{11}a^2X^2 - \frac{1}{2}a^3X + \frac{1}{5}a^4\right)\frac{3X\sqrt[3]{X^2}}{b^5}$$

$$\int x^5 dx \sqrt[3]{X^2} = \left(\frac{1}{20}X^5 - \frac{5}{17}aX^4 + \frac{5}{7}a^2X^3 - \frac{10}{11}a^3X^2 + \frac{5}{8}a^4X - \frac{1}{5}a^5\right)\frac{3X\sqrt[3]{X^2}}{b^6}$$

TAB. XXII.

$$\int\frac{dx\sqrt[3]{(a+bx)}}{x^m},\ \int\frac{dx\sqrt[3]{(a+bx)^2}}{x^m}$$

$$a+bx=X$$

$$\int\frac{dx\sqrt[3]{X}}{x}=3\sqrt[3]{X}+a\int\frac{dx}{x\sqrt[3]{X^2}}$$

$$\int\frac{dx\sqrt[3]{X}}{x^2}=-\frac{X\sqrt[3]{X}}{ax}+\frac{b}{3a}\int\frac{dx\sqrt[3]{X}}{x}$$

$$\int\frac{dx\sqrt[3]{X}}{x^3}=\left(-\frac{1}{2ax^2}+\frac{b}{3a^2x}\right)X\sqrt[3]{X}-\frac{b^2}{9a^2}\int\frac{dx\sqrt[3]{X}}{x}$$

$$\int\frac{dx\sqrt[3]{X}}{x^4}=\left(-\frac{1}{3ax^3}+\frac{5b}{18a^2x^2}-\frac{5b^2}{27a^3x}\right)X\sqrt[3]{X}+\frac{5b^3}{81a^3}\int\frac{dx\sqrt[3]{X}}{x}$$

$$\int\frac{dx\sqrt[3]{X}}{x^5}=\left(-\frac{1}{4ax^4}+\frac{2b}{9a^2x^3}-\frac{5b^2}{27a^3x^2}+\frac{10b^3}{81a^4x}\right)X\sqrt[3]{X}-\frac{10b^4}{243a^4}\int\frac{dx\sqrt[3]{X}}{x}$$

$$\int\frac{dx\sqrt[3]{X^2}}{x}=\frac{3}{2}\sqrt[3]{X^2}+a\int\frac{dx}{x\sqrt[3]{X}}$$

$$\int\frac{dx\sqrt[3]{X^2}}{x^2}=-\frac{X\sqrt[3]{X^2}}{ax}+\frac{2b}{3a}\int\frac{dx\sqrt[3]{X^2}}{x}$$

$$\int\frac{dx\sqrt[3]{X^2}}{x^3}=\left(-\frac{1}{2ax^2}+\frac{b}{6a^2x}\right)X\sqrt[3]{X^2}-\frac{b^2}{9a^2}\int\frac{dx\sqrt[3]{X^2}}{x}$$

$$\int\frac{dx\sqrt[3]{X^2}}{x^4}=\left(-\frac{1}{3ax^3}+\frac{2b}{9a^2x^2}-\frac{2b^2}{27a^3x}\right)X\sqrt[3]{X}+\frac{4b^3}{81a^3}\int\frac{dx\sqrt[3]{X^2}}{x}$$

$$\int\frac{dx\sqrt[3]{X^2}}{x^5}=\left(-\frac{1}{4ax^4}+\frac{7b}{36a^2x^3}-\frac{7b^2}{54a^3x^2}+\frac{7b^3}{162a^4x}\right)X\sqrt[3]{X^2}-\frac{7b^4}{243a^4}\int\frac{dx\sqrt[3]{X^2}}{x}$$

TAB. XXIII.

$$\int \frac{dx}{(a+bx^2)^{\frac{n}{2}}}$$

$$a + bx^2 = X$$

$$\int \frac{dx}{X^{\frac{1}{2}}} = \int \frac{dx}{\sqrt{X}}$$ [see the following page.]

$$\int \frac{dx}{X^{\frac{3}{2}}} = \frac{x}{a\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{5}{2}}} = \left(\frac{1}{3aX} + \frac{2}{3a^2}\right)\frac{x}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{7}{2}}} = \left(\frac{1}{5aX^2} + \frac{4}{15a^2X} + \frac{1}{15a^3}\right)\frac{x}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{9}{2}}} = \left(\frac{1}{7aX^3} + \frac{6}{35a^2X^2} + \frac{8}{35a^3X} + \frac{16}{35a^4}\right)\frac{x}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{11}{2}}} = \left(\frac{1}{9aX^4} + \frac{8}{63a^2X^3} + \frac{16}{105a^3X^2} + \frac{64}{315a^4X} + \frac{128}{315a^5}\right)\frac{x}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{13}{2}}} = \left(\frac{1}{11aX^5} + \frac{10}{99a^2X^4} + \frac{80}{693a^3X^3} + \frac{32}{231a^4X^2} + \frac{128}{693a^5X} + \frac{256}{693a^6}\right)\frac{x}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{15}{2}}} = \left(\frac{1}{13aX^6} + \frac{12}{143a^2X^5} + \frac{40}{429a^3X^4} + \frac{320}{3003a^4X^3} + \frac{128}{1001a^5X^2} + \frac{512}{3003a^6X} + \frac{1024}{3003a^7}\right)\frac{x}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{17}{2}}} = \left(\frac{1}{15aX^7} + \frac{14}{195a^2X^6} + \frac{56}{715a^3X^5} + \frac{112}{1287a^4X^4} + \frac{128}{1287a^5X^3} + \frac{256}{2145a^6X^2} + \frac{1024}{6435a^7X} + \frac{2048}{6435a^8}\right)\frac{x}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{19}{2}}} = \left(\frac{1}{17aX^8} + \frac{16}{255a^2X^7} + \frac{224}{3315a^3X^6} + \frac{896}{12155a^4X^5} + \frac{1792}{21879a^5X^4} + \frac{2048}{21879a^6X^3} + \frac{4096}{36465a^7X^2} + \frac{16384}{109395a^8X} + \frac{32768}{109395a^9}\right)\frac{x}{\sqrt{X}}$$

Note on the preceding Table.

In general

$$\int\frac{dx}{\sqrt{(a+bx^2)}}=\frac{1}{\sqrt{b}}\log\left[x\sqrt{b}+\sqrt{(a+bx^2)}\right]+\text{const.}$$

or $$\int\frac{dx}{\sqrt{(a+bx^2)}}=\frac{1}{\sqrt{-b}}\text{ arc sin } x\sqrt{-\frac{b}{a}}+\text{const.}$$

The first expression is real when b is positive; the second when b is negative. Both a and b cannot be negative at the same time. Hence, we have

$$\text{I. }\int\frac{dx}{\sqrt{(\pm a+bx^2)}}=\frac{1}{\sqrt{b}}\log\left[x\sqrt{b}+\sqrt{(\pm a+bx^2)}\right]+\text{const.}$$

$$\text{II. }\int\frac{dx}{\sqrt{(a-bx^2)}}=\frac{1}{\sqrt{b}}\text{ arc sin } x\sqrt{\frac{b}{a}}=\frac{1}{\sqrt{b}}\text{ arc cos }\sqrt{\frac{a-bx^2}{a}}$$

$$=\frac{1}{2\sqrt{b}}\text{ arc cos }\frac{a-2bx^2}{a}=\pm\frac{1}{\sqrt{b}}\text{ arc tang }\frac{x\sqrt{b}}{\sqrt{(a-bx^2)}}$$

$$=\frac{1}{\sqrt{b}}\text{ arc cot }\frac{\sqrt{(a-bx^2)}}{x\sqrt{b}}=\frac{1}{\sqrt{b}}\text{ arc sec }\sqrt{\frac{a}{a-bx^2}}$$

$$=\frac{1}{\sqrt{b}}\text{ arc cosec }\sqrt{\frac{a}{bx^2}}=\frac{1}{2\sqrt{b}}\text{ arc sin vers }\frac{2bx^2}{a}$$

All these circular arcs vanish when $x=0$

Particular cases are

$$\int\frac{dx}{\sqrt{(1+x^2)}}=\log\left[x+\sqrt{(1+x^2)}\right]+\text{const.}$$

$$\int\frac{dx}{\sqrt{(x^2-1)}}=\log\left[x+\sqrt{(x^2-1)}\right]+\text{const.}$$

$$\int\frac{dx}{\sqrt{(1-x^2)}}=\text{arc sin } x=\text{arc cos }\sqrt{(1-x^2)}=\tfrac{1}{2}\text{ arc cos }(1-2x^2)$$

$$=\pm\text{ arc tang }\frac{x}{\sqrt{(1-x^2)}}=\text{arc cot }\frac{\sqrt{(1-x^2)}}{x}=\text{arc sec }\frac{1}{\sqrt{(1-x^2)}}$$

$$=\text{arc cosec }\frac{1}{x}=\tfrac{1}{2}\text{ arc sin vers } 2x^2.$$

The integral $\int\frac{dx}{\sqrt{(\pm a+bx^2)}}$ can only vanish on the supposition that $x=0$, when the upper sign is taken, and in this case

$$\int\frac{dx}{\sqrt{(\mp a+bx^2)}}=\frac{1}{\sqrt{b}}\log\left(x\sqrt{\frac{b}{a}}+\sqrt{\frac{a+bx^2}{a}}\right)$$

TAB. XXIV.

$$\int \frac{x^m dx}{\sqrt{(a+bx^2)}}$$

$$a + bx^2 = X$$

$$\int \frac{dx}{\sqrt{X}} = \int \frac{dx}{\sqrt{X}} \text{ (see the preceding page)}$$

$$\int \frac{x dx}{\sqrt{X}} = \frac{\sqrt{X}}{b}$$

$$\int \frac{x^2 dx}{\sqrt{X}} = \frac{x\sqrt{X}}{2b} - \frac{a}{2b} \int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^3 dx}{\sqrt{X}} = \left(\frac{x^2}{3b} - \frac{2a}{3b^2}\right)\sqrt{X}$$

$$\int \frac{x^4 dx}{\sqrt{X}} = \left(\frac{x^3}{4b} - \frac{3ax}{8b^2}\right)\sqrt{X} + \frac{3a^2}{8b^2}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^5 dx}{\sqrt{X}} = \left(\frac{x^4}{5b} - \frac{4ax^2}{15b^2} + \frac{8a^2}{15b^3}\right)\sqrt{X}$$

$$\int \frac{x^6 dx}{\sqrt{X}} = \left(\frac{x^5}{6b} - \frac{5ax^3}{24b^2} + \frac{5a^2x}{16b^3}\right)\sqrt{X} - \frac{5a^3}{16b^3}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^7 dx}{\sqrt{X}} = \left(\frac{x^6}{7b} - \frac{6ax^4}{35b^2} + \frac{8a^2x^2}{35b^3} - \frac{16a^3}{35b^4}\right)\sqrt{X}$$

$$\int \frac{x^8 dx}{\sqrt{X}} = \left(\frac{x^7}{8b} - \frac{7ax^5}{48b^2} + \frac{35a^2x^3}{192b^3} - \frac{35a^3x}{128b^4}\right)\sqrt{X} + \frac{35a^4}{128b^4}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^9 dx}{\sqrt{X}} = \left(\frac{x^8}{9b} - \frac{8ax^6}{63b^2} + \frac{16a^2x^4}{105b^3} - \frac{64a^3x^2}{315b^4} + \frac{128a^4}{315b^5}\right)\sqrt{X}$$

$$\int \frac{x^{10} dx}{\sqrt{X}} = \left(\frac{x^9}{10b} - \frac{9ax^7}{80b^2} + \frac{21a^2x^5}{160b^3} - \frac{21a^3x^3}{128b^4} + \frac{63a^4x}{256b^5}\right)\sqrt{X} - \frac{63a^5}{256b^5}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^{11} dx}{\sqrt{X}} = \left(\frac{x^{10}}{11b} - \frac{10ax^8}{99b^2} + \frac{80a^2x^6}{693b^3} - \frac{32a^3x^4}{231b^4} + \frac{128a^4x^2}{693b^5} - \frac{256a^5}{693b^6}\right)\sqrt{X}$$

$$\int \frac{x^{12} dx}{\sqrt{X}} = \left(\frac{x^{11}}{12b} - \frac{11ax^9}{120b^2} + \frac{33a^2x^7}{320b^3} - \frac{77a^3x^5}{640b^4} + \frac{77a^4x}{512b^5} - \frac{231a^5x}{1024b^6}\right)\sqrt{X} + \frac{231a^6}{1024b^6}\int \frac{dx}{\sqrt{X}}$$

TAB. XXV.

$$\int \frac{x^m dx}{\sqrt{(1-x^2)}}$$

$$1 - x^2 = X$$

$$\int \frac{dx}{\sqrt{X}} = \text{arc sin. } x \text{ [see the preceding page.]}$$

$$\int \frac{x dx}{\sqrt{X}} = -\sqrt{X}$$

$$\int \frac{x^2 dx}{\sqrt{X}} = -\frac{1}{2}x\sqrt{X} + \frac{1}{2}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^3 dx}{\sqrt{X}} = -\left(\frac{1}{3}x^2 + \frac{2}{3}\right)\sqrt{X}$$

$$\int \frac{x^4 dx}{\sqrt{X}} = -\left(\frac{1}{4}x^3 + \frac{3}{8}x\right)\sqrt{X} + \frac{3}{8}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^5 dx}{\sqrt{X}} = -\left(\frac{1}{5}x^4 + \frac{4}{15}x^2 + \frac{8}{15}\right)\sqrt{X}$$

$$\int \frac{x^6 dx}{\sqrt{X}} = -\left(\frac{1}{6}x^5 + \frac{5}{24}x^3 + \frac{5}{16}x\right)\sqrt{X} + \frac{5}{16}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^7 dx}{\sqrt{X}} = -\left(\frac{1}{7}x^6 + \frac{6}{35}x^4 + \frac{8}{35}x^2 + \frac{16}{35}\right)\sqrt{X}$$

$$\int \frac{x^8 dx}{\sqrt{X}} = -\left(\frac{1}{8}x^7 + \frac{7}{48}x^5 + \frac{35}{192}x^3 + \frac{35}{128}x\right)\sqrt{X} + \frac{35}{128}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^9 dx}{\sqrt{X}} = -\left(\frac{1}{9}x^8 + \frac{8}{63}x^6 + \frac{16}{105}x^4 + \frac{64}{315}x^2 + \frac{128}{315}\right)\sqrt{X}.$$

$$\int \frac{x^{10} dx}{\sqrt{X}} = -\left(\frac{1}{10}x^9 + \frac{9}{80}x^7 + \frac{21}{160}x^5 + \frac{21}{128}x^3 + \frac{63}{256}x\right)\sqrt{X} + \frac{63}{256}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^{11} dx}{\sqrt{X}} = -\left(\frac{1}{11}x^{10} + \frac{10}{99}x^8 + \frac{80}{693}x^6 + \frac{32}{231}x^4 + \frac{128}{693}x^2 + \frac{256}{693}\right)\sqrt{X}$$

$$\int \frac{x^{12} dx}{\sqrt{X}} = -\left(\frac{1}{12}x^{11} + \frac{11}{120}x^9 + \frac{33}{320}x^7 + \frac{77}{640}x^5 + \frac{77}{512}x^3 + \frac{231}{1024}x\right)\sqrt{X}$$

$$+ \frac{231}{1024}\int \frac{dx}{\sqrt{X}}$$

TAB. XXVI.

$$\int \frac{dx}{x^m \sqrt{(a + bx^2)}}$$

$$a + bx^2 = X$$

$$\int \frac{dx}{x\sqrt{X}} = \int \frac{dx}{x\sqrt{X}}$$ [see the following page.]

$$\int \frac{dx}{x^2\sqrt{X}} = -\frac{\sqrt{X}}{ax}$$

$$\int \frac{dx}{x^3\sqrt{X}} = -\frac{\sqrt{X}}{2ax^2} - \frac{b}{2a}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^4\sqrt{X}} = \left(-\frac{1}{3ax^3} + \frac{2b}{3a^2x}\right)\sqrt{X}$$

$$\int \frac{dx}{x^5\sqrt{X}} = \left(-\frac{1}{4ax^4} + \frac{3b}{8a^2x^2}\right)\sqrt{X} + \frac{3b^2}{8a^2}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^6\sqrt{X}} = \left(-\frac{1}{5ax^5} + \frac{4b}{15a^2x^3} - \frac{8b^2}{15a^3x}\right)\sqrt{X}$$

$$\int \frac{dx}{x^7\sqrt{X}} = \left(-\frac{1}{6ax^6} + \frac{5b}{24a^2x^4} - \frac{5b^2}{16a^3x^2}\right)\sqrt{X} - \frac{5b^3}{16a^3}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^8\sqrt{X}} = \left(-\frac{1}{7ax^7} + \frac{6b}{35a^4x^5} - \frac{8b^2}{35a^3x^3} + \frac{16b^3}{35a^4x}\right)\sqrt{X}$$

$$\int \frac{dx}{x^9\sqrt{X}} = \left(-\frac{1}{8ax^8} + \frac{7b}{48a^2x^6} - \frac{35b^2}{192a^3x^4} + \frac{35b^3}{128a^4x^2}\right)\sqrt{X} + \frac{35b^3}{128a^4}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^{10}\sqrt{X}} = \left(-\frac{1}{9ax^9} + \frac{8b}{63a^2x^7} - \frac{16b^2}{105a^3x^5} + \frac{64b^3}{315a^4x^3} - \frac{128b^4}{315a^5x}\right)\sqrt{X}$$

$$\int \frac{dx}{x^{11}\sqrt{X}} = \left(-\frac{1}{10ax^{10}} + \frac{9b}{80a^2x^8} - \frac{21b^2}{160a^3x^6} + \frac{21b^3}{128a^4x^4} - \frac{63b^4}{256a^5x^2}\right)\sqrt{X} - \frac{63b^4}{256a^5}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^{12}\sqrt{X}} = \left(-\frac{1}{11ax^{11}} + \frac{10b}{99a^2x^9} - \frac{80b^2}{693a^3x^7} + \frac{32b^3}{231a^4x^5} - \frac{128b^4}{693a^5x^3} + \frac{256b^5}{693a^6x}\right)\sqrt{X}$$

Note on the preceding Table.

In general

$$\int \frac{dx}{x\sqrt{(a+bx^2)}} = \frac{1}{2\sqrt{a}} \log \frac{\sqrt{(a+bx^2)}-\sqrt{a}}{\sqrt{(a+bx^2)}+\sqrt{a}} + \text{const.}$$

$$\text{or} \int \frac{dx}{x\sqrt{(a+bx^2)}} = \frac{1}{\sqrt{-a}} \text{ arc sec } x\sqrt{\left(-\frac{b}{a}\right)} + \text{const.}$$

the first of which is real, when a is positive; the second, when a is negative: a and b cannot both be negative at the same time.

$$\text{I.} \int \frac{dx}{x\sqrt{(+a+bx^2)}} = \frac{1}{2\sqrt{a}} \log \frac{\sqrt{(a+bx^2)}-\sqrt{a}}{\sqrt{(a+bx^2)}+\sqrt{a}} + \text{const.}$$

$$= \frac{1}{\sqrt{a}} \log \frac{\sqrt{(a+bx^2)}-\sqrt{a}}{x} + \text{const.}$$

where $\sqrt{a}$ may be positive or negative. This integral cannot vanish when $x=0$.

$$\text{II.} \int \frac{dx}{x\sqrt{(-a+bx^2)}} = \frac{1}{\sqrt{a}} \text{ arc sec } x\sqrt{\frac{b}{a}} = \frac{1}{\sqrt{a}} \text{ arc tang } \sqrt{\frac{bx^2-a}{a}}$$

$$= \frac{1}{\sqrt{a}} \text{ arc cot } \sqrt{\frac{a}{bx^2-a}} = \frac{1}{\sqrt{a}} \text{ arc cosec } \frac{x\sqrt{b}}{\sqrt{(bx^2-a)}}$$

$$= \frac{1}{\sqrt{a}} \text{ arc cos } \frac{\sqrt{a}}{x\sqrt{b}} = \frac{1}{2\sqrt{a}} \text{ arc cos } \frac{2a-bx^2}{bx^2}$$

$$= \frac{1}{\sqrt{a}} \text{ arc sin } \frac{\sqrt{(bx^2-a)}}{x\sqrt{b}} = \frac{1}{2\sqrt{a}} \text{ arc sin vers } \frac{2(bx^2-a)}{bx^2}$$

All these integrals vanish, when $x = \sqrt{\frac{a}{b}}$; when $x=0$ they cannot vanish.

Particular Cases are

$$\int \frac{dx}{x\sqrt{(1+x^2)}} = \log \frac{\sqrt{(1+x^2)}-1}{x} + \text{const.}$$

$$\int \frac{dx}{x\sqrt{(1-x^2)}} = \log \frac{\sqrt{(1-x^2)}-1}{x} + \text{const.} = \log \frac{1-\sqrt{(1-x^2)}}{x} + \text{const.}$$

$$\int \frac{dx}{x\sqrt{(x^2-1)}} = \text{arc sec } x = \text{arc tang } \sqrt{(x^2-1)} = \text{arc cot } \sqrt{\frac{1}{x^2-1}}$$

$$= \text{arc cosec } \frac{x}{\sqrt{(x^2-1)}} = \text{arc cos } \frac{1}{x} = \tfrac{1}{2} \text{ arc cos } \frac{2-x^2}{x^2}$$

$$= \text{arc sin } \frac{\sqrt{(x^2-1)}}{x} = \tfrac{1}{2} \text{ arc sin vers } \frac{2(x^2-1)}{x^2}.$$

TAB. XXVII.

$$\int \frac{x^m \mathrm{d}x}{(a+bx^2)^{\frac{3}{2}}}$$

$$a+bx^2 = X$$

$$\int \frac{\mathrm{d}x}{X^{\frac{3}{2}}} = \frac{x}{a\sqrt{X}}$$

$$\int \frac{x\mathrm{d}x}{X^{\frac{3}{2}}} = -\frac{1}{b\sqrt{X}}$$

$$\int \frac{x^2\mathrm{d}x}{X^{\frac{3}{2}}} = -\frac{x}{b\sqrt{X}} + \frac{1}{b}\int \frac{\mathrm{d}x}{\sqrt{X}}$$

$$\int \frac{x^3\mathrm{d}x}{X^{\frac{3}{2}}} = \left(\frac{x^2}{b} + \frac{2a}{b^2}\right)\frac{1}{\sqrt{X}}$$

$$\int \frac{x^4\mathrm{d}x}{X^{\frac{3}{2}}} = \left(\frac{x^3}{2b} + \frac{3ax}{2b^2}\right)\frac{1}{\sqrt{X}} - \frac{3a}{2b^2}\int \frac{\mathrm{d}x}{\sqrt{X}}$$

$$\int \frac{x^5\mathrm{d}x}{X^{\frac{3}{2}}} = \left(\frac{x^4}{3b} - \frac{4ax^2}{3b^2} - \frac{8a^2}{3b^3}\right)\frac{1}{\sqrt{X}}$$

$$\int \frac{x^6\mathrm{d}x}{X^{\frac{3}{2}}} = \left(\frac{x^5}{4b} - \frac{5ax^3}{8b^2} - \frac{15a^2x}{8b^3}\right)\frac{1}{\sqrt{X}} + \frac{15a^2}{8b^3}\int \frac{\mathrm{d}x}{\sqrt{X}}$$

$$\int \frac{x^7\mathrm{d}x}{X^{\frac{3}{2}}} = \left(\frac{x^6}{5b} - \frac{2ax^4}{5b^2} + \frac{8a^2x^2}{5b^3} + \frac{16a^3}{5b^4}\right)\frac{1}{\sqrt{X}}$$

$$\int \frac{x^8\mathrm{d}x}{X^{\frac{3}{2}}} = \left(\frac{x^7}{6b} - \frac{7ax^5}{24b^2} + \frac{35a^2x^3}{48b^3} + \frac{35a^3x}{16b^4}\right)\frac{1}{\sqrt{X}} - \frac{35a^3}{16b^4}\int \frac{\mathrm{d}x}{\sqrt{X}}$$

$$\int \frac{x^9\mathrm{d}x}{X^{\frac{3}{2}}} = \left(\frac{x^8}{7b} - \frac{8ax^6}{35b^2} + \frac{16a^2x^4}{35b^3} - \frac{64a^3x^2}{35b^4} - \frac{128a^4}{35b^5}\right)\frac{\mathrm{d}x}{\sqrt{X}}$$

$$\int \frac{x^{10}\mathrm{d}x}{X^{\frac{3}{2}}} = \left(\frac{x^9}{8b} - \frac{3ax^7}{16b^2} + \frac{21a^2x^5}{64b^3} - \frac{105a^3x^3}{128b^4} - \frac{315a^4x}{128b^5}\right)\frac{1}{\sqrt{X}} + \frac{315a^4}{128b^5}\int \frac{\mathrm{d}x}{\sqrt{X}}$$

$$\int \frac{x^{11}\mathrm{d}x}{X^{\frac{3}{2}}} = \left(\frac{x^{10}}{9b} - \frac{10ax^8}{63b^2} + \frac{16a^2x^6}{63b^3} - \frac{32a^3x^4}{63b^4} + \frac{128a^4x^2}{63b^5} + \frac{256a^5}{63b^6}\right)\frac{1}{\sqrt{X}}$$

TAB. XXVIII.

$$\int \frac{dx}{x^m(a+bx^2)^{\frac{3}{2}}}$$

$$a + bx^2 = X$$

$$\int \frac{dx}{xX^{\frac{3}{2}}} = \frac{1}{a\sqrt{X}} + \frac{1}{a}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{3}{2}}} = \left(-\frac{1}{ax} - \frac{2bx}{a^2}\right)\frac{1}{\sqrt{X}}$$

$$\int \frac{dx}{x^3X^{\frac{3}{2}}} = \left(-\frac{1}{2ax^2} - \frac{3b}{2a^2}\right)\frac{1}{\sqrt{X}} - \frac{3b}{2a^2}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^4X^{\frac{3}{2}}} = \left(-\frac{1}{3ax^3} + \frac{4b}{3a^2x} + \frac{8b^2x}{3a^3}\right)\frac{1}{\sqrt{X}}$$

$$\int \frac{dx}{x^5X^{\frac{3}{2}}} = \left(-\frac{1}{4ax^4} + \frac{5b}{8a^2x^2} + \frac{15b^2}{8a^3}\right)\frac{1}{\sqrt{X}} + \frac{15b^2}{8a^3}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^6X^{\frac{3}{2}}} = \left(-\frac{1}{5ax^5} + \frac{2b}{5a^2x^3} - \frac{8b^2}{5a^3x} - \frac{16b^3x}{5a^4}\right)\frac{1}{\sqrt{X}}$$

$$\int \frac{dx}{x^7X^{\frac{3}{2}}} = \left(-\frac{1}{6ax^6} + \frac{7b}{24a^2x^4} - \frac{35b^2}{48a^3x^2} - \frac{35b^3}{16a^4}\right)\frac{1}{\sqrt{X}} - \frac{35b^3}{16a^4}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^8X^{\frac{3}{2}}} = \left(-\frac{1}{7ax^7} + \frac{8b}{35a^2x^5} - \frac{16b^2}{35a^3x^3} + \frac{64b^3}{35a^4x} + \frac{128b^4x}{35a^5}\right)\frac{1}{\sqrt{X}}$$

$$\int \frac{dx}{x^9X^{\frac{3}{2}}} = \left(-\frac{1}{8ax^8} + \frac{3b}{16a^2x^6} - \frac{21b^2}{64a^3x^4} + \frac{105b^3}{128a^4x^2} + \frac{315b^4}{128a^5}\right)\frac{1}{\sqrt{X}} + \frac{315b^4}{128a^5}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^{10}X^{\frac{3}{2}}} = \left(-\frac{1}{9ax^9} + \frac{10b}{63a^2x^7} - \frac{16b^2}{63a^3x^5} + \frac{32b^3}{63a^4x^3} - \frac{128b^4}{63a^5x} - \frac{256b^5x}{63a^6}\right)\frac{1}{\sqrt{X}}$$

$$\int \frac{dx}{x^{11}X^{\frac{3}{2}}} = \left(-\frac{1}{10ax^{10}} + \frac{11b}{80a^2x^8} - \frac{33b^2}{160a^3x^6} + \frac{231b^3}{640a^4x^4} - \frac{231b^4}{256a^5x^2} - \frac{693b^5}{256a^6}\right)\frac{1}{\sqrt{X}} - \frac{693b^4}{256a^6}\int \frac{dx}{x\sqrt{X}}$$

TAB. XXIX.

$$\int \frac{x^m \mathrm{d}x}{(a+bx^2)^{\frac{5}{2}}}$$

$$a + bx^2 = X$$

$$\int \frac{\mathrm{d}x}{X^{\frac{5}{2}}} = \left(\frac{2bx^3}{3a^2} + \frac{x}{a}\right)\frac{1}{X\sqrt{X}}$$

$$\int \frac{x\mathrm{d}x}{X^{\frac{5}{2}}} = -\frac{1}{3bX\sqrt{X}}$$

$$\int \frac{x^2\mathrm{d}x}{X^{\frac{5}{2}}} = \frac{x^3}{3aX\sqrt{X}}$$

$$\int \frac{x^3\mathrm{d}x}{X^{\frac{5}{2}}} = \left(-\frac{x^2}{b} - \frac{2a}{3b^2}\right)\frac{1}{X\sqrt{X}}$$

$$\int \frac{x^4\mathrm{d}x}{X^{\frac{5}{2}}} = \left(-\frac{4x^3}{3b} - \frac{ax}{b^2}\right)\frac{1}{X\sqrt{X}} + \frac{1}{b^2}\int \frac{\mathrm{d}x}{\sqrt{X}}$$

$$\int \frac{x^5\mathrm{d}x}{X^{\frac{5}{2}}} = \left(\frac{x^4}{b} + \frac{4ax^2}{b^2} + \frac{8a^2}{3b^3}\right)\frac{1}{X\sqrt{X}}$$

$$\int \frac{x^6\mathrm{d}x}{X^{\frac{5}{2}}} = \left(\frac{x^5}{2b} + \frac{10ax^3}{3b^2} + \frac{5a^2x}{2b^3}\right)\frac{1}{X\sqrt{X}} - \frac{5a}{2b^3}\int \frac{\mathrm{d}x}{\sqrt{X}}$$

$$\int \frac{x^7\mathrm{d}x}{X^{\frac{5}{2}}} = \left(\frac{x^6}{3b} - \frac{2ax^4}{b^2} - \frac{8a^2x^2}{b^3} - \frac{16a^3}{3b^4}\right)\frac{1}{X\sqrt{X}}$$

$$\int \frac{x^8\mathrm{d}x}{X^{\frac{5}{2}}} = \left(\frac{x^7}{4b} - \frac{7ax^5}{8b^2} - \frac{35a^2x^3}{6b^3} - \frac{35a^3x}{8b^4}\right)\frac{1}{X\sqrt{X}} + \frac{35a^2}{8b^4}\int \frac{\mathrm{d}x}{\sqrt{X}}$$

$$\int \frac{x^9\mathrm{d}x}{X^{\frac{5}{2}}} = \left(\frac{x^8}{5b} - \frac{8ax^6}{15b^2} + \frac{16a^2x^4}{5b^3} + \frac{64a^3x^2}{5b^4} + \frac{128a^4}{15b^5}\right)\frac{1}{X\sqrt{X}}$$

$$\int \frac{x^{10}\mathrm{d}x}{X^{\frac{5}{2}}} = \left(\frac{x^9}{6b} - \frac{3ax^7}{8b^2} + \frac{21a^2x^5}{16b^3} + \frac{35a^3x^3}{4b^4} + \frac{105a^4x}{16b^5}\right)\frac{1}{X\sqrt{X}} - \frac{105a^3}{16b^5}\int \frac{\mathrm{d}x}{\sqrt{X}}$$

$$\int \frac{x^{11}\mathrm{d}x}{X^{\frac{5}{2}}} = \left(\frac{x^{10}}{7b} - \frac{2ax^8}{7b^2} + \frac{16a^2x^6}{21b^3} - \frac{32a^3x^4}{7b^4} - \frac{128a^4x^2}{7b^5} - \frac{256a^5}{21b^6}\right)\frac{1}{X\sqrt{X}}$$

TAB. XXX.

$$\int \frac{dx}{x^m(a+bx^2)^{\frac{5}{2}}}$$

$$a + bx^2 = X$$

$$\int \frac{dx}{xX^{\frac{5}{2}}} = \left(\frac{4}{3a} + \frac{bx^2}{a^2}\right)\frac{1}{X\sqrt{X}} + \frac{1}{a^2}\int \frac{dx}{X\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{5}{2}}} = -\frac{1}{axX\sqrt{X}} - \frac{4b}{a}\int \frac{dx}{X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{5}{2}}} = -\frac{1}{2ax^2X\sqrt{X}} - \frac{5b}{2a}\int \frac{dx}{xX^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{5}{2}}} = \left(-\frac{1}{3ax^3} + \frac{2b}{a^2x}\right)\frac{1}{X\sqrt{X}} + \frac{8b^2}{a^2}\int \frac{dx}{X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{5}{2}}} = \left(-\frac{1}{4ax^4} + \frac{7b}{8a^2x^2}\right)\frac{1}{X\sqrt{X}} + \frac{35b^2}{8a^2}\int \frac{dx}{xX^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^6X^{\frac{5}{2}}} = \left(-\frac{1}{5ax^5} + \frac{8b}{15a^2x^3} - \frac{16b^2}{5a^3x}\right)\frac{1}{X\sqrt{X}} - \frac{64b^3}{5a^3}\int \frac{dx}{X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^7X^{\frac{5}{2}}} = \left(-\frac{1}{6ax^6} + \frac{3b}{8a^2x^4} - \frac{21b^2}{16a^3x^2}\right)\frac{1}{X\sqrt{X}} - \frac{105b^3}{16a^3}\int \frac{dx}{xX^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^8X^{\frac{5}{2}}} = \left(-\frac{1}{7ax^7} + \frac{2b}{7a^2x^5} - \frac{16b^2}{21a^3x^3} + \frac{32b^3}{7a^4x}\right)\frac{1}{X\sqrt{X}} + \frac{128b^4}{7a^4}\int \frac{dx}{X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^9X^{\frac{5}{2}}} = \left(-\frac{1}{8ax^8} + \frac{11b}{48a^2x^6} - \frac{33b^2}{64a^3x^4} + \frac{231b^3}{128a^4x^2}\right)\frac{1}{X\sqrt{X}} + \frac{1155b^4}{128a^4}\int \frac{dx}{xX^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^{10}X^{\frac{5}{2}}} = \left(-\frac{1}{9ax^9} + \frac{4b}{21a^2x^7} - \frac{8b^2}{21a^3x^5} + \frac{64b^3}{63a^4x^3} - \frac{128b^4}{21a^5x}\right)\frac{1}{X\sqrt{X}} - \frac{512b^5}{21a^5}\int \frac{dx}{X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^{11}X^{\frac{5}{2}}} = \left(-\frac{1}{10ax^{10}} + \frac{13b}{80a^2x^8} - \frac{143b^2}{480a^3x^6} + \frac{429b^3}{640a^4x^4} - \frac{3003b^4}{1280a^5x^2}\right)\frac{1}{X\sqrt{X}} - \frac{3003b^5}{256a^5}\int \frac{dx}{xX^{\frac{5}{2}}}$$

TAB. XXXI

$$\int \frac{x^m dx}{(a+bx^2)^{\frac{7}{2}}}$$

$$a + bx^2 = X$$

$$\int \frac{dx}{X^{\frac{7}{2}}} = \left(\frac{8b^2x^5}{15a^3} + \frac{4bx^3}{3a^2} + \frac{x}{a}\right)\frac{1}{X^2\sqrt{X}}$$

$$\int \frac{x\,dx}{X^{\frac{7}{2}}} = -\frac{1}{5bX^2\sqrt{X}}$$

$$\int \frac{x^2dx}{X^{\frac{7}{2}}} = \left(\frac{2bx^5}{15a^2} + \frac{x^3}{3a}\right)\frac{1}{X^2\sqrt{X}}$$

$$\int \frac{x^3dx}{X^{\frac{7}{2}}} = \left(-\frac{x^2}{3b} - \frac{2a}{15b^2}\right)\frac{1}{X^2\sqrt{X}}$$

$$\int \frac{x^4dx}{X^{\frac{7}{2}}} = \frac{x^5}{5aX^2\sqrt{X}}$$

$$\int \frac{x^5dx}{X^{\frac{7}{2}}} = \left(-\frac{x^4}{b} - \frac{4ax^2}{3b^2} - \frac{8a^2}{15b^3}\right)\frac{1}{X^2\sqrt{X}}$$

$$\int \frac{x^6dx}{X^{\frac{7}{2}}} = \left(-\frac{23x^5}{15b} - \frac{7ax^3}{3b^2} - \frac{a^2x}{b^3}\right)\frac{1}{X^2\sqrt{X}} + \frac{1}{b^3}\int\frac{dx}{\sqrt{X}}$$

$$\int \frac{x^7dx}{X^{\frac{7}{2}}} = \left(\frac{x^6}{b} + \frac{6ax^4}{b^2} + \frac{8a^2x^2}{b^3} + \frac{16a^3}{5b^4}\right)\frac{1}{X^2\sqrt{X}}$$

$$\int \frac{x^8dx}{X^{\frac{7}{2}}} = \left(\frac{x^7}{2b} + \frac{161ax^5}{30b^2} + \frac{49a^2x^3}{6b^3} + \frac{7a^3x}{2b^4}\right)\frac{1}{X^2\sqrt{X}} - \frac{7a}{2b^4}\int\frac{dx}{\sqrt{X}}$$

$$\int \frac{x^9dx}{X^{\frac{7}{2}}} = \left(\frac{x^8}{3b} - \frac{8ax^6}{3b^2} - \frac{16a^2x^4}{b^3} - \frac{64a^3x^2}{3b^4} - \frac{128a^4}{15b^5}\right)\frac{1}{X^2\sqrt{X}}$$

$$\int \frac{x^{10}dx}{X^{\frac{7}{2}}} = \left(\frac{x^9}{4b} - \frac{9ax^7}{8b^2} - \frac{483a^2x^5}{40b^3} - \frac{147a^3x^3}{8b^4} - \frac{63a^4x}{8b^5}\right)\frac{1}{X^2\sqrt{X}} + \frac{63a^4}{8b^5}\int\frac{dx}{\sqrt{X}}$$

$$\int \frac{x^{11}dx}{X^{\frac{7}{2}}} = \left(\frac{x^{10}}{5b} - \frac{2ax^8}{3b^2} + \frac{16a^2x^6}{3b^3} + \frac{32a^3x^4}{b^4} + \frac{128a^4x^2}{3b^5} + \frac{256a^5}{15b^6}\right)\frac{1}{X^2\sqrt{X}}$$

TAB. XXXII.

$$\int \frac{dx}{x^m(a+bx^2)^{\frac{7}{2}}}$$

$$a + bx^2 = X$$

$$\int \frac{dx}{xX^{\frac{7}{2}}} = \left(\frac{23}{15a} + \frac{7bx^2}{3a^2} + \frac{b^2x^4}{a^3}\right)\frac{1}{X^2\sqrt{X}} + \frac{1}{a^3}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{7}{2}}} = -\frac{1}{axX^2\sqrt{X}} - \frac{6b}{a}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{7}{2}}} = -\frac{1}{2ax^2X^2\sqrt{X}} - \frac{7b}{2a}\int \frac{dx}{xX^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{7}{2}}} = \left(-\frac{1}{3ax^3} + \frac{8b}{3a^2x}\right)\frac{1}{X^2\sqrt{X}} + \frac{16b^2}{a^2}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{7}{2}}} = \left(-\frac{1}{4ax^4} + \frac{9b}{8a^2x^2}\right)\frac{1}{X^2\sqrt{X}} + \frac{63b^2}{8a^2}\int \frac{dx}{xX^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^6X^{\frac{7}{2}}} = \left(-\frac{1}{5ax^5} + \frac{2b}{3a^2x^3} - \frac{16b^2}{3a^3x}\right)\frac{1}{X^2\sqrt{X}} - \frac{32b^3}{a^3}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^7X^{\frac{7}{2}}} = \left(-\frac{1}{6ax^6} + \frac{11b}{24a^2x^4} - \frac{33b^2}{16a^3x^2}\right)\frac{1}{X^2\sqrt{X}} - \frac{231b^3}{16a^3}\int \frac{dx}{xX^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^8X^{\frac{7}{2}}} = \left(-\frac{1}{7ax^7} + \frac{12b}{35a^2x^5} - \frac{8b^2}{7a^3x^3} + \frac{64b^3}{7a^4x}\right)\frac{1}{X^2\sqrt{X}} + \frac{384b^4}{7a^4}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^9X^{\frac{7}{2}}} = \left(-\frac{1}{8ax^8} + \frac{13b}{48a^2x^6} - \frac{143b^2}{192a^3x^4} + \frac{429b^3}{128a^4x^2}\right)\frac{1}{X^2\sqrt{X}} + \frac{3003b^4}{128a^4}\int \frac{dx}{xX^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^{10}X^{\frac{7}{2}}} = \left(-\frac{1}{9ax^9} + \frac{2b}{9a^2x^7} - \frac{8b^2}{15a^3x^5} + \frac{16b^3}{9a^4x^3} - \frac{128b^4}{9a^5x}\right)\frac{1}{X^2\sqrt{X}} - \frac{256b^5}{3a^5}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^{11}X^{\frac{7}{2}}} = \left(-\frac{1}{10ax^{10}} + \frac{3b}{16a^2x^8} - \frac{13b^2}{32a^3x^6} + \frac{143b^3}{128a^4x^4} - \frac{1287b^4}{256a^5x^2}\right)\frac{1}{X^2\sqrt{X}} - \frac{9009b^5}{256a^5}\int \frac{dx}{xX^{\frac{7}{2}}}$$

TAB. XXXIII.

$$\int\frac{x^m\,dx}{(a+bx^2)^{\frac{9}{2}}},\quad\int\frac{dx}{x^m\,(a+bx^2)^{\frac{9}{2}}}$$

$$a+bx^2=X$$

$$\int\frac{dx}{X^{\frac{9}{2}}}=\left(\frac{16b^3x^7}{35a^4}+\frac{8b^2x^5}{5a^3}+\frac{2bx^3}{a^2}+\frac{x}{a}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int\frac{x\,dx}{X^{\frac{9}{2}}}=-\frac{1}{7bX^3\sqrt{X}}$$

$$\int\frac{x^2\,dx}{X^{\frac{9}{2}}}=\left(\frac{8b^2x^7}{105a^3}+\frac{4bx^5}{15a^2}+\frac{x^3}{3a}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int\frac{x^3\,dx}{X^{\frac{9}{2}}}=\left(-\frac{x^2}{5b}-\frac{2a}{35b^2}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int\frac{x^4\,dx}{X^{\frac{9}{2}}}=\left(\frac{2bx^7}{35a^2}+\frac{x^5}{5a}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int\frac{x^5\,dx}{X^{\frac{9}{2}}}=\left(-\frac{x^4}{3b}-\frac{4ax^2}{15b^2}-\frac{8a^2}{105b^3}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int\frac{x^6\,dx}{X^{\frac{9}{2}}}=\frac{x^7}{7aX^3\sqrt{X}}$$

$$\int\frac{x^7\,dx}{X^{\frac{9}{2}}}=\left(-\frac{x^6}{b}-\frac{2ax^4}{b^2}-\frac{8a^2x^2}{5b^3}-\frac{16a^3}{35b^4}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int\frac{dx}{xX^{\frac{9}{2}}}=\left(\frac{176}{105a}+\frac{58bx^2}{15a^2}+\frac{10b^2x^4}{3a^3}+\frac{b^3x^6}{a^4}\right)\frac{1}{X^3\sqrt{X}}+\frac{1}{a^4}\int\frac{dx}{x\sqrt{X}}$$

$$\int\frac{dx}{x^2X^{\frac{9}{2}}}=-\frac{1}{axX^3\sqrt{X}}-\frac{8b}{a}\int\frac{dx}{X^{\frac{9}{2}}}$$

$$\int\frac{dx}{x^3X^{\frac{9}{2}}}=-\frac{1}{2ax^2X^3\sqrt{X}}-\frac{9b}{2a}\int\frac{dx}{xX^{\frac{9}{2}}}$$

$$\int\frac{dx}{x^4X^{\frac{9}{2}}}=\left(-\frac{1}{3ax^3}+\frac{10b}{3a^2x}\right)\frac{1}{X^3\sqrt{X}}+\frac{80b^2}{3a^2}\int\frac{dx}{X^{\frac{9}{2}}}$$

$$\int\frac{dx}{x^5X^{\frac{9}{2}}}=\left(-\frac{1}{4ax^4}+\frac{11b}{8a^2x^2}\right)\frac{1}{X^3\sqrt{X}}+\frac{99b^2}{8a^2}\int\frac{dx}{xX^{\frac{9}{2}}}$$

$$\int\frac{dx}{x^6X^{\frac{9}{2}}}=\left(-\frac{1}{5ax^5}+\frac{4b}{5a^2x^3}-\frac{8b^2}{a^3x}\right)\frac{1}{X^3\sqrt{X}}-\frac{64b^3}{a^3}\int\frac{dx}{X^{\frac{9}{2}}}$$

$$\int\frac{dx}{x^7X^{\frac{9}{2}}}=\left(-\frac{1}{6ax^6}+\frac{13b}{24a^2x^4}-\frac{143b^2}{48a^3x^2}\right)\frac{1}{X^3\sqrt{X}}-\frac{429b^3}{16a^3}\int\frac{dx}{xX^{\frac{9}{2}}}$$

TAB. XXXII.

$$\int \frac{dx}{x^m (a+bx^2)^{\frac{7}{2}}}$$

$$a + bx^2 = X$$

$$\int \frac{dx}{xX^{\frac{7}{2}}} = \left(\frac{23}{15a} + \frac{7bx^2}{3a^2} + \frac{b^2x^4}{a^3}\right)\frac{1}{X^2\sqrt{X}} + \frac{1}{a^3}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{7}{2}}} = -\frac{1}{axX^2\sqrt{X}} - \frac{6b}{a}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{7}{2}}} = -\frac{1}{2ax^2X^2\sqrt{X}} - \frac{7b}{2a}\int \frac{dx}{xX^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{7}{2}}} = \left(-\frac{1}{3ax^3} + \frac{8b}{3a^2x}\right)\frac{1}{X^2\sqrt{X}} + \frac{16b^2}{a^2}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{7}{2}}} = \left(-\frac{1}{4ax^4} + \frac{9b}{8a^2x^2}\right)\frac{1}{X^2\sqrt{X}} + \frac{63b^2}{8a^2}\int \frac{dx}{xX^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^6X^{\frac{7}{2}}} = \left(-\frac{1}{5ax^5} + \frac{2b}{3a^2x^3} - \frac{16b^2}{3a^3x}\right)\frac{1}{X^2\sqrt{X}} - \frac{32b^3}{a^3}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^7X^{\frac{7}{2}}} = \left(-\frac{1}{6ax^6} + \frac{11b}{24a^2x^4} - \frac{33b^2}{16a^3x^2}\right)\frac{1}{X^2\sqrt{X}} - \frac{231b^3}{16a^3}\int \frac{dx}{xX^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^8X^{\frac{7}{2}}} = \left(-\frac{1}{7ax^7} + \frac{12b}{35a^2x^5} - \frac{8b^2}{7a^3x^3} + \frac{64b^3}{7a^4x}\right)\frac{1}{X^2\sqrt{X}} + \frac{384b^4}{7a^4}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^9X^{\frac{7}{2}}} = \left(-\frac{1}{8ax^8} + \frac{13b}{48a^2x^6} - \frac{143b^2}{192a^3x^4} + \frac{429b^3}{128a^4x^2}\right)\frac{1}{X^2\sqrt{X}} + \frac{3003b^4}{128a^4}\int \frac{dx}{xX^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^{10}X^{\frac{7}{2}}} = \left(-\frac{1}{9ax^9} + \frac{2b}{9a^2x^7} - \frac{8b^2}{15a^3x^5} + \frac{16b^3}{9a^4x^3} - \frac{128b^4}{9a^5x}\right)\frac{1}{X^2\sqrt{X}} - \frac{256b^5}{3a^5}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^{11}X^{\frac{7}{2}}} = \left(-\frac{1}{10ax^{10}} + \frac{3b}{16a^2x^8} - \frac{13b^2}{32a^3x^6} + \frac{143b^3}{128a^4x^4} - \frac{1287b^4}{256a^5x^2}\right)\frac{1}{X^2\sqrt{X}} - \frac{9009b^5}{256a^5}\int \frac{dx}{xX^{\frac{7}{2}}}$$

TAB. XXXIII.

$$\int \frac{x^m dx}{(a+bx^2)^{\frac{9}{2}}},\quad \int \frac{dx}{x^m (a+bx^2)^{\frac{9}{2}}}$$

$$a + bx^2 = X$$

$$\int \frac{dx}{X^{\frac{9}{2}}} = \left(\frac{16b^3x^7}{35a^4} + \frac{8b^2x^5}{5a^3} + \frac{2bx^3}{a^2} + \frac{x}{a}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{9}{2}}} = -\frac{1}{7bX^3\sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{9}{2}}} = \left(\frac{8b^2x^7}{105a^3} + \frac{4bx^5}{15a^2} + \frac{x^3}{3a}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{9}{2}}} = \left(-\frac{x^2}{5b} - \frac{2a}{35b^2}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int \frac{x^4 dx}{X^{\frac{9}{2}}} = \left(\frac{2bx^7}{35a^2} + \frac{x^5}{5a}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int \frac{x^5 dx}{X^{\frac{9}{2}}} = \left(-\frac{x^4}{3b} - \frac{4ax^2}{15b^2} - \frac{8a^2}{105b^3}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int \frac{x^6 dx}{X^{\frac{9}{2}}} = \frac{x^7}{7aX^3\sqrt{X}}$$

$$\int \frac{x^7 dx}{X^{\frac{9}{2}}} = \left(-\frac{x^6}{b} - \frac{2ax^4}{b^2} - \frac{8a^2x^2}{5b^3} - \frac{16a^3}{35b^4}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int \frac{dx}{xX^{\frac{9}{2}}} = \left(\frac{176}{105a} + \frac{58bx^2}{15a^2} + \frac{10b^2x^4}{3a^3} + \frac{b^3x^6}{a^4}\right)\frac{1}{X^3\sqrt{X}} + \frac{1}{a^4}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{9}{2}}} = -\frac{1}{axX^3\sqrt{X}} - \frac{8b}{a}\int \frac{dx}{X^{\frac{9}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{9}{2}}} = -\frac{1}{2ax^2X^3\sqrt{X}} - \frac{9b}{2a}\int \frac{dx}{xX^{\frac{9}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{9}{2}}} = \left(-\frac{1}{3ax^3} + \frac{10b}{3a^2x}\right)\frac{1}{X^3\sqrt{X}} + \frac{80b^2}{3a^2}\int \frac{dx}{X^{\frac{9}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{9}{2}}} = \left(-\frac{1}{4ax^4} + \frac{11b}{8a^2x^2}\right)\frac{1}{X^3\sqrt{X}} + \frac{99b^2}{8a^2}\int \frac{dx}{xX^{\frac{9}{2}}}$$

$$\int \frac{dx}{x^6X^{\frac{9}{2}}} = \left(-\frac{1}{5ax^5} + \frac{4b}{5a^2x^3} - \frac{8b^2}{a^3x}\right)\frac{1}{X^3\sqrt{X}} - \frac{64b^3}{a^3}\int \frac{dx}{X^{\frac{9}{2}}}$$

$$\int \frac{dx}{x^7X^{\frac{9}{2}}} = \left(-\frac{1}{6ax^6} + \frac{13b}{24a^2x^4} - \frac{143b^2}{48a^3x^2}\right)\frac{1}{X^3\sqrt{X}} - \frac{429b^3}{16a^3}\int \frac{dx}{xX^{\frac{9}{2}}}$$

TAB. XXXII.

$$\int \frac{dx}{x^m(a+bx^2)^{\frac{7}{2}}}$$

$$a + bx^2 = X$$

$$\int \frac{dx}{xX^{\frac{7}{2}}} = \left(\frac{23}{15a} + \frac{7bx^2}{3a^2} + \frac{b^2x^4}{a^3}\right)\frac{1}{X^2\sqrt{X}} + \frac{1}{a^3}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{7}{2}}} = -\frac{1}{axX^2\sqrt{X}} - \frac{6b}{a}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{7}{2}}} = -\frac{1}{2ax^2X^2\sqrt{X}} - \frac{7b}{2a}\int \frac{dx}{xX^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{7}{2}}} = \left(-\frac{1}{3ax^3} + \frac{8b}{3a^2x}\right)\frac{1}{X^2\sqrt{X}} + \frac{16b^2}{a^2}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{7}{2}}} = \left(-\frac{1}{4ax^4} + \frac{9b}{8a^2x^2}\right)\frac{1}{X^2\sqrt{X}} + \frac{63b^2}{8a^2}\int \frac{dx}{xX^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^6X^{\frac{7}{2}}} = \left(-\frac{1}{5ax^5} + \frac{2b}{3a^2x^3} - \frac{16b^2}{3a^3x}\right)\frac{1}{X^2\sqrt{X}} - \frac{32b^3}{a^3}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^7X^{\frac{7}{2}}} = \left(-\frac{1}{6ax^6} + \frac{11b}{24a^2x^4} - \frac{33b^2}{16a^3x^2}\right)\frac{1}{X^2\sqrt{X}} - \frac{231b^3}{16a^3}\int \frac{dx}{xX^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^8X^{\frac{7}{2}}} = \left(-\frac{1}{7ax^7} + \frac{12b}{35a^2x^5} - \frac{8b^2}{7a^3x^3} + \frac{64b^3}{7a^4x}\right)\frac{1}{X^2\sqrt{X}} + \frac{384b^4}{7a^4}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^9X^{\frac{7}{2}}} = \left(-\frac{1}{8ax^8} + \frac{13b}{48a^2x^6} - \frac{143b^2}{192a^3x^4} + \frac{429b^3}{128a^4x^2}\right)\frac{1}{X^2\sqrt{X}} + \frac{3003b^4}{128a^4}\int \frac{dx}{xX^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^{10}X^{\frac{7}{2}}} = \left(-\frac{1}{9ax^9} + \frac{2b}{9a^2x^7} - \frac{8b^2}{15a^3x^5} + \frac{16b^3}{9a^4x^3} - \frac{128b^4}{9a^5x}\right)\frac{1}{X^2\sqrt{X}} - \frac{256b^5}{3a^5}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^{11}X^{\frac{7}{2}}} = \left(-\frac{1}{10ax^{10}} + \frac{3b}{16a^2x^8} - \frac{13b^2}{32a^3x^6} + \frac{143b^3}{128a^4x^4} - \frac{1287b^4}{256a^5x^2}\right)\frac{1}{X^2\sqrt{X}} - \frac{9009b^5}{256a^5}\int \frac{dx}{xX^{\frac{7}{2}}}$$

TAB. XXXIII.

$$\int\frac{x^m\,dx}{(a+bx^2)^{\frac{9}{2}}},\ \int\frac{dx}{x^m\,(a+bx^2)^{\frac{9}{2}}}$$

$$a+bx^2=X$$

$$\int\frac{dx}{X^{\frac{9}{2}}}=\left(\frac{16b^3x^7}{35a^4}+\frac{8b^2x^5}{5a^3}+\frac{2bx^3}{a^2}+\frac{x}{a}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int\frac{x\,dx}{X^{\frac{9}{2}}}=-\frac{1}{7bX^3\sqrt{X}}$$

$$\int\frac{x^2dx}{X^{\frac{9}{2}}}=\left(\frac{8b^2x^7}{105a^3}+\frac{4bx^5}{15a^2}+\frac{x^3}{3a}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int\frac{x^3dx}{X^{\frac{9}{2}}}=\left(-\frac{x^2}{5b}-\frac{2a}{35b^2}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int\frac{x^4dx}{X^{\frac{9}{2}}}=\left(\frac{2bx^7}{35a^2}+\frac{x^5}{5a}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int\frac{x^5dx}{X^{\frac{9}{2}}}=\left(-\frac{x^4}{3b}-\frac{4ax^2}{15b^2}-\frac{8a^2}{105b^3}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int\frac{x^6dx}{X^{\frac{9}{2}}}=\frac{x^7}{7aX^3\sqrt{X}}$$

$$\int\frac{x^7dx}{X^{\frac{9}{2}}}=\left(-\frac{x^6}{b}-\frac{2ax^4}{b^2}-\frac{8a^2x^2}{5b^3}-\frac{16a^3}{35b^4}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int\frac{dx}{xX^{\frac{9}{2}}}=\left(\frac{176}{105a}+\frac{58bx^2}{15a^2}+\frac{10b^2x^4}{3a^3}+\frac{b^3x^6}{a^4}\right)\frac{1}{X^3\sqrt{X}}+\frac{1}{a^4}\int\frac{dx}{x\sqrt{X}}$$

$$\int\frac{dx}{x^2X^{\frac{9}{2}}}=-\frac{1}{axX^3\sqrt{X}}-\frac{8b}{a}\int\frac{dx}{X^{\frac{9}{2}}}$$

$$\int\frac{dx}{x^3X^{\frac{9}{2}}}=-\frac{1}{2ax^2X^3\sqrt{X}}-\frac{9b}{2a}\int\frac{dx}{xX^{\frac{9}{2}}}$$

$$\int\frac{dx}{x^4X^{\frac{9}{2}}}=\left(-\frac{1}{3ax^3}+\frac{10b}{3a^2x}\right)\frac{1}{X^3\sqrt{X}}+\frac{80b^2}{3a^2}\int\frac{dx}{X^{\frac{9}{2}}}$$

$$\int\frac{dx}{x^5X^{\frac{9}{2}}}=\left(-\frac{1}{4ax^4}+\frac{11b}{8a^2x^2}\right)\frac{1}{X^3\sqrt{X}}+\frac{99b^2}{8a^2}\int\frac{dx}{xX^{\frac{9}{2}}}$$

$$\int\frac{dx}{x^6X^{\frac{9}{2}}}=\left(-\frac{1}{5ax^5}+\frac{4b}{5a^2x^3}-\frac{8b^2}{a^3x}\right)\frac{1}{X^3\sqrt{X}}-\frac{64b^3}{a^3}\int\frac{dx}{X^{\frac{9}{2}}}$$

$$\int\frac{dx}{x^7X^{\frac{9}{2}}}=\left(-\frac{1}{6ax^6}+\frac{13b}{24a^2x^4}-\frac{143b^2}{48a^3x^2}\right)\frac{1}{X^3\sqrt{X}}-\frac{429b^3}{16a^3}\int\frac{dx}{xX^{\frac{9}{2}}}$$

TAB. XXXIV.

$$\int x^m \mathrm{d}x \sqrt{(a+bx^2)}$$

$$a + bx^2 = X$$

$$\int \mathrm{d}x\sqrt{X} = \frac{x\sqrt{X}}{2} + \frac{a}{2}\int\frac{\mathrm{d}x}{\sqrt{X}}$$

$$\int x\,\mathrm{d}x\sqrt{X} = \frac{X\sqrt{X}}{3b}$$

$$\int x^2\mathrm{d}x\sqrt{X} = \frac{xX\sqrt{X}}{4b} - \frac{a}{4b}\int \mathrm{d}x\sqrt{X}$$

$$\int x^3\mathrm{d}x\sqrt{X} = \left(\frac{x^2}{5b} - \frac{2a}{15b^2}\right)X\sqrt{X}$$

$$\int x^4\mathrm{d}x\sqrt{X} = \left(\frac{x^3}{6b} - \frac{ax}{8b^2}\right)X\sqrt{X} + \frac{a^2}{8b^2}\int \mathrm{d}x\sqrt{X}$$

$$\int x^5\mathrm{d}x\sqrt{X} = \left(\frac{x^4}{7b} - \frac{4ax^2}{35b^2} + \frac{8a^2}{105b^3}\right)X\sqrt{X}$$

$$\int x^6\mathrm{d}x\sqrt{X} = \left(\frac{x^5}{8b} - \frac{5ax^3}{48b^2} + \frac{5a^2x}{64b^3}\right)X\sqrt{X} - \frac{5a^3}{64b^3}\int \mathrm{d}x\sqrt{X}$$

$$\int x^7\mathrm{d}x\sqrt{X} = \left(\frac{x^6}{9b} - \frac{2ax^4}{21b^2} + \frac{8a^2x^2}{105b^3} - \frac{16a^3}{315b^4}\right)X\sqrt{X}$$

$$\int x^8\mathrm{d}x\sqrt{X} = \left(\frac{x^7}{10b} - \frac{7ax^5}{80b^2} + \frac{7a^2x^3}{96b^3} - \frac{7a^3x}{128b^4}\right)X\sqrt{X} + \frac{7a^4}{128b^4}\int \mathrm{d}x\sqrt{X}$$

$$\int x^9\mathrm{d}x\sqrt{X} = \left(\frac{x^8}{11b} - \frac{8ax^6}{99b^2} + \frac{16a^2x^4}{231b^3} - \frac{64a^3x^2}{1155b^4} + \frac{128a^4}{3465b^5}\right)X\sqrt{X}$$

$$\int x^{10}\mathrm{d}x\sqrt{X} = \left(\frac{x^9}{12b} - \frac{3ax^7}{40b^2} + \frac{21a^2x^5}{320b^3} - \frac{7a^3x^3}{128b^4} + \frac{21a^4x}{512b^5}\right)X\sqrt{X} - \frac{21a^5}{512b^5}\int \mathrm{d}x\sqrt{X}$$

$$\int x^{11}\mathrm{d}x\sqrt{X} = \left(\frac{x^{10}}{13b} - \frac{10ax^8}{143b^2} + \frac{80a^2x^6}{1287b^3} - \frac{160a^3x^4}{3003b^4} + \frac{128a^4x^2}{3003b^5} - \frac{256a^5}{9009b^6}\right)X\sqrt{X}$$

TAB. XXXV.

$$\int \frac{dx\sqrt{(a+bx^2)}}{x^m}$$

$$a + bx^2 = X$$

$$\int \frac{dx\sqrt{X}}{x} = \sqrt{X} + a\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx\sqrt{X}}{x^2} = -\frac{\sqrt{X}}{x} + b\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{dx\sqrt{X}}{x^3} = -\frac{\sqrt{X}}{2x^2} + \frac{b}{2}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx\sqrt{X}}{x^4} = -\frac{X\sqrt{X}}{3ax^3}$$

$$\int \frac{dx\sqrt{X}}{x^5} = -\frac{X\sqrt{X}}{4ax^4} + \frac{b\sqrt{X}}{8ax^2} - \frac{b^2}{8a}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx\sqrt{X}}{x^6} = \left(-\frac{1}{5ax^5} + \frac{2b}{15a^2x^3}\right) X\sqrt{X}$$

$$\int \frac{dx\sqrt{X}}{x^7} = \left(-\frac{1}{6ax^6} + \frac{b}{8a^2x^4}\right) X\sqrt{X} - \frac{b^2\sqrt{X}}{16a^2x^2} + \frac{b^3}{16a^2}\int \frac{dx}{x\sqrt{X}}.$$

$$\int \frac{dx\sqrt{X}}{x^8} = \left(-\frac{1}{7ax^7} + \frac{4b}{35a^2x^5} - \frac{8b^2}{105a^3x^3}\right) X\sqrt{X}$$

$$\int \frac{dx\sqrt{X}}{x^9} = \left(-\frac{1}{8ax^8} + \frac{5b}{48a^2x^6} - \frac{5b^2}{64a^3x^4}\right) X\sqrt{X} + \frac{5b^3\sqrt{X}}{128a^3x^2} - \frac{5b^4}{128a^3}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx\sqrt{X}}{x^{10}} = \left(-\frac{1}{9ax^9} + \frac{2b}{21a^2x^7} - \frac{8b^2}{105a^3x^5} + \frac{16b^3}{315a^4x^3}\right) X\sqrt{X}$$

$$\int \frac{dx\sqrt{X}}{x^{11}} = \left(-\frac{1}{10ax^{10}} + \frac{7b}{80a^2x^8} - \frac{7b^2}{96a^3x^6} + \frac{7b^3}{128a^4x^4}\right) X\sqrt{X} - \frac{7b^4\sqrt{X}}{256a^4x^2} + \frac{7b^5}{256a^4}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx\sqrt{X}}{x^{12}} = \left(-\frac{1}{11ax^{11}} + \frac{8b}{99a^2x^9} - \frac{16b^2}{231a^3x^7} + \frac{64b^3}{1155a^4x^5} - \frac{128b^4}{3465a^5x^3}\right) X\sqrt{X}$$

TAB. XXXVI.

$$\int x^m dx(a+bx^2)^{\frac{3}{2}}$$

$$a + bx^2 = X$$

$$\int dxX^{\frac{3}{2}} = \left(\frac{X}{4} + \frac{3a}{8}\right) x\sqrt{X} + \frac{3a^2}{8}\int\frac{dx}{\sqrt{X}}$$

$$\int xdxX^{\frac{3}{2}} = \frac{X^2\sqrt{X}}{5b}$$

$$\int x^2dxX^{\frac{3}{2}} = \frac{xX^2\sqrt{X}}{6b} - \frac{a}{6b}\int dxX^{\frac{3}{2}}$$

$$\int x^3dxX^{\frac{3}{2}} = \left(\frac{x^2}{7b} - \frac{2a}{35b^2}\right)X^2\sqrt{X}$$

$$\int x^4dxX^{\frac{3}{2}} = \left(\frac{x^3}{8b} - \frac{ax}{16b^2}\right)X^2\sqrt{X} + \frac{a^2}{16b^2}\int dxX^{\frac{3}{2}}$$

$$\int x^5dxX^{\frac{3}{2}} = \left(\frac{x^4}{9b} - \frac{4ax^2}{63b^2} + \frac{8a^2}{315b^3}\right)X^2\sqrt{X}$$

$$\int x^6dxX^{\frac{3}{2}} = \left(\frac{x^5}{10b} - \frac{ax^3}{16b^2} + \frac{a^2x}{32b^3}\right)X^2\sqrt{X} - \frac{a^3}{32b^3}\int dxX^{\frac{3}{2}}$$

$$\int x^7dxX^{\frac{3}{2}} = \left(\frac{x^6}{11b} - \frac{2ax^4}{33b^2} + \frac{8a^2x^2}{231b^3} - \frac{16a^3}{1155b^4}\right)X^2\sqrt{X}$$

$$\int x^8dxX^{\frac{3}{2}} = \left(\frac{x^7}{12b} - \frac{7ax^5}{120b^2} + \frac{7a^2x^3}{192b^3} - \frac{7a^3x}{384b^4}\right)X^2\sqrt{X} + \frac{7a^4}{384b^4}\int dxX^{\frac{3}{2}}$$

$$\int x^9dxX^{\frac{3}{2}} = \left(\frac{x^8}{13b} - \frac{8ax^6}{143b^2} + \frac{16a^2x^4}{429b^3} - \frac{64a^3x^2}{3003b^4} + \frac{128a^4}{15015b^5}\right)X^2\sqrt{X}$$

$$\int x^{10}dxX^{\frac{3}{2}} = \left(\frac{x^9}{14b} - \frac{3ax^7}{56b^2} + \frac{3a^2x^5}{80b^3} - \frac{3a^3x^3}{128b^4} + \frac{3a^4x}{256b^5}\right)X^2\sqrt{X} - \frac{3a^5}{256b^5}\int dxX^{\frac{3}{2}}$$

$$\int x^{11}dxX^{\frac{3}{2}} = \left(\frac{x^{10}}{15b} - \frac{2ax^8}{39b^2} + \frac{16a^2x^6}{429b^3} - \frac{32a^3x^4}{1287b^4} + \frac{128a^4x^2}{9009b^5} - \frac{256a^5}{45045b^6}\right)X^2\sqrt{X}$$

TAB. XXXVII.

$$\int \frac{dx(a+bx^2)^{\frac{3}{2}}}{x^m}$$

$$a+bx^2 = X$$

$$\int \frac{dx X^{\frac{3}{2}}}{x} = \left(\frac{X}{3}+a\right)\sqrt{X} + a^2 \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx X^{\frac{3}{2}}}{x^2} = -\frac{X^2\sqrt{X}}{ax} + \frac{4b}{a}\int dx X^{\frac{3}{2}}$$

$$\int \frac{dx X^{\frac{3}{2}}}{x^3} = -\frac{X^2\sqrt{X}}{2ax^2} + \frac{3b}{2a}\int \frac{dx X^{\frac{3}{2}}}{x}$$

$$\int \frac{dx X^{\frac{3}{2}}}{x^4} = \left(-\frac{1}{3ax^3} - \frac{2b}{3a^2x}\right) X^2\sqrt{X} + \frac{8b^2}{3a^2}\int dx X^{\frac{3}{2}}$$

$$\int \frac{dx X^{\frac{3}{2}}}{x^5} = \left(-\frac{1}{4ax^4} - \frac{b}{8a^2x^2}\right) X^2\sqrt{X} + \frac{3b^2}{8a^2}\int \frac{dx X^{\frac{3}{2}}}{x}$$

$$\int \frac{dx X^{\frac{3}{2}}}{x^6} = -\frac{X^2\sqrt{X}}{5ax^5}$$

$$\int \frac{dx X^{\frac{3}{2}}}{x^7} = \left(-\frac{1}{6ax^6} + \frac{b}{24a^2x^4} + \frac{b^2}{48a^3x^2}\right) X^2\sqrt{X} - \frac{b^3}{16a}\int \frac{dx X^{\frac{3}{2}}}{x}$$

$$\int \frac{dx X^{\frac{3}{2}}}{x^8} = \left(-\frac{1}{7ax^7} + \frac{2b}{35a^2x^5}\right) X^2\sqrt{X}$$

$$\int \frac{dx X^{\frac{3}{2}}}{x^9} = \left(-\frac{1}{8ax^8} + \frac{b}{16a^2x^6} - \frac{b^2}{64a^3x^4} - \frac{b^3}{128a^4x^2}\right) X^2\sqrt{X} + \frac{3b^4}{128a^4}\int \frac{dx X^{\frac{3}{2}}}{x}$$

$$\int \frac{dx X^{\frac{3}{2}}}{x^{10}} = \left(-\frac{1}{9ax^9} + \frac{4b}{63a^2x^7} - \frac{8b^2}{315a^3x^5}\right) X^2\sqrt{X}$$

$$\int \frac{dx X^{\frac{3}{2}}}{x^{11}} = \left(-\frac{1}{10ax^{10}} + \frac{b}{16a^2x^8} - \frac{b^2}{32a^3x^6} + \frac{b^3}{128a^4x^4} + \frac{b^4}{256a^5x^2}\right) X^2\sqrt{X} - \frac{3b^5}{256a^5}\int \frac{dx X^{\frac{3}{2}}}{x}$$

TAB. XXXVIII.

$$\int x^m dx(a+bx^2)^{\frac{5}{2}}$$

$$a+bx^2=X$$

$$\int dxX^{\frac{5}{2}}=\left(\frac{X^2}{6}+\frac{5aX}{24}+\frac{5a^2}{16}\right)x\sqrt{X}+\frac{5a^3}{16}\int\frac{dx}{\sqrt{X}}$$

$$\int xdxX^{\frac{5}{2}}=\frac{X^3\sqrt{X}}{7b}$$

$$\int x^2dxX^{\frac{5}{2}}=\frac{xX^3\sqrt{X}}{8b}-\frac{a}{8b}\int dxX^{\frac{5}{2}}$$

$$\int x^3dxX^{\frac{5}{2}}=\left(\frac{x^2}{9b}-\frac{2a}{63b^2}\right)X^3\sqrt{X}$$

$$\int x^4dxX^{\frac{5}{2}}=\left(\frac{x^3}{10b}-\frac{3ax}{80b^2}\right)X^3\sqrt{X}+\frac{3a^2}{80b^2}\int dxX^{\frac{5}{2}}$$

$$\int x^5dxX^{\frac{5}{2}}=\left(\frac{x^4}{11b}-\frac{4ax^2}{99b^2}+\frac{8a^2}{693b^3}\right)X^3\sqrt{X}$$

$$\int x^6dxX^{\frac{5}{2}}=\left(\frac{x^5}{12b}-\frac{ax^3}{24b^2}+\frac{a^2x}{64b^3}\right)X^3\sqrt{X}-\frac{a^3}{64b^3}\int dxX^{\frac{5}{2}}$$

$$\int x^7dxX^{\frac{5}{2}}=\left(\frac{x^6}{13b}-\frac{6ax^4}{143b^2}+\frac{8a^2x^2}{429b^3}-\frac{16a^3}{3003b^4}\right)X^3\sqrt{X}$$

$$\int x^8dxX^{\frac{5}{2}}=\left(\frac{x^7}{14b}-\frac{ax^5}{24b^2}+\frac{a^2x^3}{48b^3}-\frac{a^3x}{128b^4}\right)X^3\sqrt{X}+\frac{a^4}{128b^4}\int dxX^{\frac{5}{2}}$$

$$\int x^9dxX^{\frac{5}{2}}=\left(\frac{x^8}{15b}-\frac{8ax^6}{195b^2}+\frac{16a^2x^4}{715b^3}-\frac{64a^3x^2}{6435b^4}+\frac{128a^4}{45045b^5}\right)X^3\sqrt{X}$$

$$\int x^{10}dxX^{\frac{5}{2}}=\left(\frac{x^9}{16b}-\frac{9ax^7}{224b^2}+\frac{3a^2x^5}{128b^3}-\frac{3a^3x^3}{256b^4}+\frac{9a^4x}{2048b^5}\right)X^3\sqrt{X}$$
$$-\frac{9a^5}{2048b^5}\int dxX^{\frac{5}{2}}$$

$$\int x^{11}dxX^{\frac{5}{2}}=\left(\frac{x^{10}}{17b}-\frac{2ax^8}{51b^2}+\frac{16a^2x^6}{663b^3}-\frac{32a^3x^4}{2431b^4}+\frac{128a^4x^2}{21879b^5}\right.$$
$$\left.-\frac{256a^5}{153153b^6}\right)X^3\sqrt{X}$$

TAB. XXXIX.

$$\int \frac{dx(a+bx^2)^{\frac{5}{2}}}{x^m}$$

$$a + bx^2 = X$$

$$\int \frac{dx X^{\frac{5}{2}}}{x} = \left(\frac{X^2}{5} + \frac{aX}{3} + a^2\right)\sqrt{X} + a^3 \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^2} = -\frac{X^3\sqrt{X}}{ax} + \frac{6b}{a}\int dx X^{\frac{5}{2}}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^3} = -\frac{X^3\sqrt{X}}{2ax^2} + \frac{5b}{2a}\int \frac{dx X^{\frac{5}{2}}}{x}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^4} = \left(-\frac{1}{3ax^3} - \frac{4b}{3a^2x}\right) X^3\sqrt{X} + \frac{8b^2}{a^2}\int dx X^{\frac{5}{2}}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^5} = \left(-\frac{1}{4ax^4} - \frac{3b}{8a^2x^2}\right) X^3\sqrt{X} + \frac{15b^2}{5a^3}\int \frac{dx X^{\frac{5}{2}}}{x}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^6} = \left(-\frac{1}{5ax^5} - \frac{2b}{15a^2x^3} - \frac{8b^2}{15a^3x}\right) X^3\sqrt{X} + \frac{16b^3}{5a^3}\int dx X^{\frac{5}{2}}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^7} = \left(-\frac{1}{6ax^6} - \frac{b}{24a^2x^4} - \frac{b^2}{16a^3x^2}\right) X^3\sqrt{X} + \frac{5b^3}{16a^3}\int \frac{dx X^{\frac{5}{2}}}{x}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^8} = -\frac{X^3\sqrt{X}}{7ax^7}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^9} = \left(-\frac{1}{8ax^8} + \frac{b}{48a^2x^6} + \frac{b^2}{192a^3x^4} + \frac{b^3}{128a^4x^2}\right) X^3\sqrt{X} - \frac{5b^4}{128a^4}\int \frac{dx X^{\frac{5}{2}}}{x}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^{10}} = \left(-\frac{1}{9ax^9} + \frac{2b}{63a^2x^7}\right) X^3\sqrt{X}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^{11}} = \left(-\frac{1}{10ax^{10}} + \frac{3b}{80a^2x^8} - \frac{b^2}{160a^3x^6} - \frac{b^3}{640a^4x^4} - \frac{3b^4}{1280a^5x^2}\right) X^3\sqrt{X} + \frac{3b^5}{256a^5}\int \frac{dx X^{\frac{5}{2}}}{x}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^{12}} = \left(-\frac{1}{11ax^{11}} + \frac{4b}{99a^2x^9} - \frac{8b^2}{693a^3x^7}\right) X^3\sqrt{X}$$

TAB. XL.

$$\int x^m dx(a+bx^2)^{\frac{7}{2}}$$

$$a+bx^2=X$$

$$\int dxX^{\frac{7}{2}}=\left(\frac{X^3}{8}+\frac{7aX^2}{48}+\frac{35a^2X}{192}+\frac{35a^3}{128}\right)x\sqrt{X}+\frac{35a^4}{128}\int\frac{dx}{\sqrt{X}}$$

$$\int xdxX^{\frac{7}{2}}=\frac{X^4\sqrt{X}}{9b}$$

$$\int x^2dxX^{\frac{7}{2}}=\frac{xX^4\sqrt{X}}{10b}-\frac{a}{10b}\int dxX^{\frac{7}{2}}$$

$$\int x^3dxX^{\frac{7}{2}}=\left(\frac{x^2}{11b}-\frac{2a}{99b^2}\right)X^4\sqrt{X}$$

$$\int x^4dxX^{\frac{7}{2}}=\left(\frac{x^3}{12b}-\frac{ax}{40b^2}\right)X^4\sqrt{X}+\frac{a^2}{40b^2}\int dxX^{\frac{7}{2}}$$

$$\int x^5dxX^{\frac{7}{2}}=\left(\frac{x^4}{13b}-\frac{4ax^2}{143b^2}+\frac{8a^2}{1287b^3}\right)X^4\sqrt{X}$$

$$\int x^6dxX^{\frac{7}{2}}=\left(\frac{x^5}{14b}-\frac{5ax^3}{168b^2}+\frac{a^2x}{112b^3}\right)X^4\sqrt{X}-\frac{a^3}{112b^3}\int dxX^{\frac{7}{2}}$$

$$\int x^7dxX^{\frac{7}{2}}=\left(\frac{x^6}{15b}-\frac{2ax^4}{65b^2}+\frac{8a^2x^2}{715b^3}-\frac{16a^3}{6435b^4}\right)X^4\sqrt{X}$$

$$\int x^8dxX^{\frac{7}{2}}=\left(\frac{x^7}{16b}-\frac{ax^5}{32b^2}+\frac{5a^2x^3}{384b^3}-\frac{a^3x}{256b^4}\right)X^4\sqrt{X}+\frac{7a^4}{256b^4}\int dxX^{\frac{7}{2}}$$

$$\int x^9dxX^{\frac{7}{2}}=\left(\frac{x^8}{17b}-\frac{8ax^6}{255b^2}+\frac{16a^2x^4}{1105b^3}-\frac{64a^3x^2}{12155b^4}+\frac{128b^5}{109395b^5}\right)X^4\sqrt{X}$$

$$\int x^{10}dxX^{\frac{7}{2}}=\left(\frac{x^9}{18b}-\frac{ax^7}{32b^2}+\frac{a^2x^5}{64b^3}-\frac{5a^3x^3}{768b^4}+\frac{a^4x}{512b^5}\right)X^4\sqrt{X}$$
$$-\frac{7a^5}{512b^5}\int dxX^{\frac{7}{2}}$$

$$\int x^{11}dxX^{\frac{7}{2}}=\left(\frac{x^{10}}{19b}-\frac{10ax^8}{323b^2}+\frac{16a^2x^6}{969b^3}-\frac{32a^3x^4}{4199b^4}+\frac{128a^4x^2}{46189b^5}\right.$$
$$\left.-\frac{256a^5}{415701b^6}\right)X^4\sqrt{X}$$

TAB. XLI.

$$\int \frac{dx(a+bx^2)^{\frac{7}{2}}}{x^m}$$

$$a + bx^2 = X$$

$$\int \frac{dxX^{\frac{7}{2}}}{x} = \left(\frac{X^3}{7} + \frac{aX^2}{5} + \frac{a^2X}{3} + a^3\right)\sqrt{X} + a^4 \int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dxX^{\frac{7}{2}}}{x^2} = -\frac{X^4\sqrt{X}}{ax} + \frac{8b}{a} \int dxX^{\frac{7}{2}}$$

$$\int \frac{dxX^{\frac{7}{2}}}{x^3} = -\frac{X^4\sqrt{X}}{2ax^2} + \frac{7b}{2a} \int \frac{dxX^{\frac{7}{2}}}{x}$$

$$\int \frac{dxX^{\frac{7}{2}}}{x^4} = \left(-\frac{1}{3ax^3} - \frac{2b}{a^2x}\right) X^4\sqrt{X} + \frac{16b^2}{a^2} \int dxX^{\frac{7}{2}}$$

$$\int \frac{dxX^{\frac{7}{2}}}{x^5} = \left(-\frac{1}{4ax^4} - \frac{5b}{8a^2x^2}\right) X^4\sqrt{X} + \frac{35b^2}{8a^2} \int \frac{dxX^{\frac{7}{2}}}{x}$$

$$\int \frac{dxX^{\frac{7}{2}}}{x^6} = \left(-\frac{1}{5ax^5} - \frac{4b}{15a^2x^3} - \frac{8b^2}{5a^3x}\right) X^4\sqrt{X} + \frac{64b^3}{5a^3} \int dxX^{\frac{7}{2}}$$

$$\int \frac{dxX^{\frac{7}{2}}}{x^7} = \left(-\frac{1}{6ax^6} - \frac{b}{8a^2x^4} - \frac{5b^2}{16a^3x^2}\right) X^4\sqrt{X} + \frac{35b^3}{16a^3} \int \frac{dxX^{\frac{7}{2}}}{x}$$

$$\int \frac{dxX^{\frac{7}{2}}}{x^8} = \left(-\frac{1}{7ax^7} - \frac{2b}{35a^2x^5} - \frac{8b^2}{105a^3x^3} - \frac{16b^3}{35a^4x}\right) X^4\sqrt{X} + \frac{128b^4}{35a^4} \int dxX^{\frac{7}{2}}$$

$$\int \frac{dxX^{\frac{7}{2}}}{x^9} = \left(-\frac{1}{8ax^8} - \frac{b}{48a^2x^6} - \frac{b^2}{64a^3x^4} - \frac{5b^3}{128a^4x^2}\right) X^4\sqrt{X} + \frac{35b^4}{128a^4} \int \frac{dxX^{\frac{7}{2}}}{x}$$

$$\int \frac{dxX^{\frac{7}{2}}}{x^{10}} = -\frac{X^4\sqrt{X}}{9ax^9}$$

$$\int \frac{dxX^{\frac{7}{2}}}{x^{11}} = \left(-\frac{1}{10ax^{10}} + \frac{b}{80a^2x^8} + \frac{b^2}{480a^3x^6} + \frac{b^3}{640a^4x^4} + \frac{b^4}{256a^5x^2}\right) X^4\sqrt{X} - \frac{7b^5}{256a^5} \int \frac{dxX^{\frac{7}{2}}}{x}$$

$$\int \frac{dxX^{\frac{7}{2}}}{x^{12}} = \left(-\frac{1}{11ax^{11}} + \frac{2b}{99a^2x^9}\right) X^4\sqrt{X}$$

TAB. XLII.

$$\int x^m \mathrm{d}x(a+bx^2)^{\frac{9}{2}}, \quad \int \frac{\mathrm{d}x(a+bx^2)^{\frac{9}{2}}}{x^m}$$

$$a + bx^2 = X$$

$$\int \mathrm{d}xX^{\frac{9}{2}} = \left(\frac{X^4}{10}+\frac{9aX^3}{80}+\frac{21a^2X^2}{160}+\frac{21a^3X}{128}+\frac{63a^4}{256}\right)x\sqrt{X}+\frac{63a^5}{256}\int\frac{\mathrm{d}x}{\sqrt{X}}$$

$$\int x\mathrm{d}xX^{\frac{9}{2}} = \frac{X^5\sqrt{X}}{11b}$$

$$\int x^2\mathrm{d}xX^{\frac{9}{2}} = \frac{xX^5\sqrt{X}}{12b} - \frac{a}{12b}\int \mathrm{d}xX^{\frac{9}{2}}$$

$$\int x^3\mathrm{d}xX^{\frac{9}{2}} = \left(\frac{x^2}{13b} - \frac{2a}{143b^2}\right)X^5\sqrt{X}$$

$$\int x^4\mathrm{d}xX^{\frac{9}{2}} = \left(\frac{x^3}{14b} - \frac{ax}{56b^2}\right)X^5\sqrt{X} + \frac{a^2}{56b^2}\int \mathrm{d}xX^{\frac{9}{2}}$$

$$\int x^5\mathrm{d}xX^{\frac{9}{2}} = \left(\frac{x^4}{15b} - \frac{4ax^2}{195b^2} + \frac{8a^2}{2145b^3}\right)X^5\sqrt{X}$$

$$\int x^6\mathrm{d}xX^{\frac{9}{2}} = \left(\frac{x^5}{16b} - \frac{5ax^3}{224b^2} + \frac{5a^2x}{896b^3}\right)X^5\sqrt{X} - \frac{5a^3}{896b^3}\int \mathrm{d}xX^{\frac{9}{2}}$$

$$\int \frac{\mathrm{d}xX^{\frac{9}{2}}}{x} = \left(\frac{X^4}{9} + \frac{aX^3}{7} + \frac{a^2X^2}{5} + \frac{a^3X}{3} + a^4\right)\sqrt{X} + a^5\int\frac{\mathrm{d}x}{x\sqrt{X}}$$

$$\int \frac{\mathrm{d}xX^{\frac{9}{2}}}{x^2} = -\frac{X^5\sqrt{X}}{ax} + \frac{10b}{a}\int \mathrm{d}xX^{\frac{9}{2}}$$

$$\int \frac{\mathrm{d}xX^{\frac{9}{2}}}{x^3} = -\frac{X^5\sqrt{X}}{2ax^2} + \frac{9b}{2a}\int\frac{\mathrm{d}xX^{\frac{9}{2}}}{x}$$

$$\int \frac{\mathrm{d}xX^{\frac{9}{2}}}{x^4} = \left(-\frac{1}{3ax^3} - \frac{8b}{3a^2x}\right)X^5\sqrt{X} + \frac{80b^2}{3a^2}\int \mathrm{d}xX^{\frac{9}{2}}$$

$$\int \frac{\mathrm{d}xX^{\frac{9}{2}}}{x^5} = \left(-\frac{1}{4ax^4} - \frac{7b}{8a^2x^2}\right)X^5\sqrt{X} + \frac{63b^2}{8a^2}\int\frac{\mathrm{d}xX^{\frac{9}{2}}}{x}$$

$$\int \frac{\mathrm{d}xX^{\frac{9}{2}}}{x^6} = \left(-\frac{1}{5ax^5} - \frac{2b}{5a^2x^3} - \frac{16b^2}{5a^3x}\right)X^5\sqrt{X} + \frac{32b^3}{a^3}\int \mathrm{d}xX^{\frac{9}{2}}$$

TAB. XLIII.

$$\int \frac{dx}{(ax+bx^2)^{\frac{n}{2}}}$$

$$ax + bx^2 = X$$

$$\int \frac{dx}{X^{\frac{1}{2}}} = \int \frac{dx}{\sqrt{X}}$$ [see the following page.]

$$\int \frac{dx}{X^{\frac{3}{2}}} = -\frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{5}{2}}} = \left(-\frac{1}{3X} + \frac{8b}{3a^2}\right)\frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{7}{2}}} = \left(-\frac{1}{5X^2} + \frac{4^2 b}{15a^2X} - \frac{2\,.\,4^3b^2}{15a^4}\right)\frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{9}{2}}} = \left(-\frac{1}{7X^3} + \frac{6\,.\,4b}{35a^2X^2} - \frac{2\,.\,4^3b^2}{35a^4X} + \frac{4^5b^3}{35a^6}\right)\frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{11}{2}}} = \left(-\frac{1}{9X^4} + \frac{2\,.\,4^2b}{63a^2X^3} - \frac{4^4b^2}{105a^4X^2} + \frac{4^6b^3}{315a^6X} - \frac{2\,.\,4^7b^4}{315a^8}\right)\frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{13}{2}}} = \left(-\frac{1}{11X^5} + \frac{10\,.\,4b}{99a^2X^4} - \frac{5\,.\,4^4b^2}{693a^4X^3} + \frac{2\,.\,4^5b^3}{231a^6X^2} - \frac{2\,.\,4^7b^4}{693a^8X} + \frac{4^9b^5}{693a^{10}}\right)\frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{15}{2}}} = \left(-\frac{1}{13X^6} + \frac{3\,.\,4^2b}{143a^2X^5} - \frac{10\,.\,4^3b^2}{429a^4X^4} + \frac{5\,.\,4^6b^3}{3003a^6X^3} - \frac{2\,.\,4^7b^4}{1001a^8X^2} + \frac{2\,.\,4^9b^5}{3003a^{10}X} - \frac{4^{11}b^6}{3003a^{12}}\right)\frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{17}{2}}} = \left(-\frac{1}{15X^7} + \frac{14b}{195a^2X^6} - \frac{14\,.\,4^3b^2}{715a^4X^5} + \frac{7\,.\,4^5b^3}{1287a^6X^4} - \frac{2\,.\,4^7b^4}{1287a^8X^3} + \frac{4^9b^5}{2145a^{10}X^2} - \frac{4^{11}b^6}{6435a^{12}X} + \frac{2\,.\,4^{12}b^7}{6435a^{14}}\right)\frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{19}{2}}} = -\frac{2(2bx+a)}{17a^2X^{\frac{17}{2}}} - \frac{4^3b}{17a^2}\int \frac{dx}{X^{\frac{17}{2}}}$$

Note on the preceding Table.

In general

$$\int\frac{dx}{\sqrt{(ax+bx^2)}}=\frac{1}{\sqrt{b}}\log\frac{\sqrt{(ax+bx^2)}+x\sqrt{b}}{\sqrt{(ax+bx^2)}-x\sqrt{b}}$$

$$\text{or}\int\frac{dx}{\sqrt{(ax+bx^2)}}=\frac{2}{\sqrt{-b}}\text{arc tang}\frac{x\sqrt{-b}}{\sqrt{(ax+bx^2)}},$$

from which it follows, that in every case the first expression is real, when b is positive; the second, when b is negative. Hence we obtain

$$\text{I.}\int\frac{dx}{\sqrt{(ax+bx^2)}}=\pm\frac{1}{\sqrt{b}}\log\frac{\sqrt{(ax+bx^2)}\pm x\sqrt{b}}{\sqrt{(ax+bx^2)}\mp x\sqrt{b}}$$

$$=\pm\frac{1}{\sqrt{b}}\log\frac{\sqrt{(a+bx)}\pm\sqrt{bx}}{\sqrt{(a+bx)}\mp\sqrt{bx}}$$

$$=\pm\frac{1}{\sqrt{b}}\log\frac{2bx+a\pm2\sqrt{b}.\sqrt{(ax+bx^2)}}{a}$$

$$=\pm\frac{2}{\sqrt{b}}\log\frac{\sqrt{(a+bx)}\pm\sqrt{bx}}{\sqrt{a}}.$$

The upper signs are here taken together, as likewise are the lower.

$$\text{II.}\int\frac{dx}{\sqrt{(ax-bx^2)}}=\frac{2}{\sqrt{b}}\text{arc tang}\frac{x\sqrt{b}}{\sqrt{(ax-bx^2)}}=\frac{2}{\sqrt{b}}\text{arc tang}\sqrt{\frac{bx}{a-bx}}$$

$$=\frac{2}{\sqrt{b}}\text{arc cot}\sqrt{\frac{a-bx}{bx}}=\frac{2}{\sqrt{b}}\text{arc sec}\sqrt{\frac{a}{a-bx}}$$

$$=\frac{2}{\sqrt{b}}\text{arc cosec}\sqrt{\frac{a}{bx}}=\frac{2}{\sqrt{b}}\text{arc sin}\sqrt{\frac{bx}{a}}$$

$$=\frac{2}{\sqrt{b}}\text{arc cos}\sqrt{\frac{a-bx}{a}}=\frac{1}{\sqrt{b}}\text{arc cos}\frac{a-2bx}{a}$$

$$=\frac{1}{\sqrt{b}}\text{arc sin vers}\frac{2bx}{a}.$$

All the integrals in this page vanish when $x=0$.

Particular cases are

$$\int\frac{dx}{\sqrt{(x^2+x)}}=\pm\log[2x+1\pm2\sqrt{(x^2+x)}]$$

$$\int\frac{dx}{\sqrt{(x^2-x)}}=\pm\log[1-2x\mp2\sqrt{(x^2-x)}].$$

TAB. XLIV.

$$\int \frac{x^m dx}{\sqrt{(ax+bx^2)}}.$$

$$ax + bx^2 = X$$

$$\int \frac{dx}{\sqrt{X}} = \int \frac{dx}{\sqrt{X}} \text{ (see the preceding page)}$$

$$\int \frac{x\,dx}{\sqrt{X}} = \frac{\sqrt{X}}{b} - \frac{a}{2b}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^2 dx}{\sqrt{X}} = \left(\frac{x}{2b} - \frac{3a}{4b^2}\right)\sqrt{X} + \frac{3a^2}{8b^2}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^3 dx}{\sqrt{X}} = \left(\frac{x^2}{3b} - \frac{5ax}{12b^2} + \frac{5a^2}{8b^3}\right)\sqrt{X} - \frac{5a^3}{16b^3}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^4 dx}{\sqrt{X}} = \left(\frac{x^3}{4b} - \frac{7ax^2}{24b^2} + \frac{35a^2x}{96b^3} - \frac{35a^3}{64b^4}\right)\sqrt{X} + \frac{35a^4}{128b^4}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^5 dx}{\sqrt{X}} = \left(\frac{x^4}{5b} - \frac{9ax^3}{40b^2} + \frac{21a^2x^2}{80b^3} - \frac{21a^3x}{64b^4} + \frac{63a^4}{128b^5}\right)\sqrt{X} - \frac{63a^5}{256b^5}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^6 dx}{\sqrt{X}} = \left(\frac{x^5}{6b} - \frac{11ax^4}{60b^2} + \frac{33a^2x^3}{160b^3} - \frac{77a^3x^2}{320b^4} + \frac{77a^4x}{256b^5} - \frac{231a^5}{512b^6}\right)\sqrt{X}$$
$$+ \frac{231a^6}{1024b^6}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^7 dx}{\sqrt{X}} = \left(\frac{x^6}{7b} - \frac{13ax^5}{84b^2} + \frac{143a^2x^4}{840b^3} - \frac{429a^3x^3}{2240b^4} + \frac{143a^4x^2}{640b^5}\right.$$
$$\left. - \frac{143a^5x}{512b^6} + \frac{429a^6}{1024b^7}\right)\sqrt{X} - \frac{429a^7}{2048b^7}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^8 dx}{\sqrt{X}} = \left(\frac{x^7}{8b} - \frac{15ax^6}{112b^2} + \frac{65a^2x^5}{448b^3} - \frac{143a^3x^4}{896b^4} + \frac{1287a^4x^3}{7168b^5}\right.$$
$$\left. - \frac{429a^5x^2}{2048b^6} + \frac{2145a^6x}{8192b^7} - \frac{6435a^7}{16384b^8}\right)\sqrt{X} + \frac{6435a^8}{32768b^8}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^9 dx}{\sqrt{X}} = \frac{x^8\sqrt{X}}{9b} - \frac{17a}{18b}\int \frac{x^8 dx}{\sqrt{X}}$$

$$\int \frac{x^{10} dx}{\sqrt{X}} = \left(\frac{x^9}{10b} - \frac{19ax^8}{180b^2}\right)\sqrt{X} + \frac{323a^2}{360b^2}\int \frac{x^8 dx}{\sqrt{X}}$$

TAB. XLV.

$$\int \frac{dx}{x^m \sqrt{(ax+bx^2)}}$$

$$ax + bx^2 = X$$

$$\int \frac{dx}{x\sqrt{X}} = -\frac{2\sqrt{X}}{ax}$$

$$\int \frac{dx}{x^2\sqrt{X}} = \left(-\frac{1}{3ax^2} + \frac{2b}{3a^2x}\right)2\sqrt{X}$$

$$\int \frac{dx}{x^3\sqrt{X}} = \left(-\frac{1}{5ax^3} + \frac{4b}{15a^2x^2} - \frac{8b^2}{15a^3x}\right)2\sqrt{X}$$

$$\int \frac{dx}{x^4\sqrt{X}} = \left(-\frac{1}{7ax^4} + \frac{6b}{35a^2x^3} - \frac{8b^2}{35a^3x^2} + \frac{16b^3}{35a^4x}\right)2\sqrt{X}$$

$$\int \frac{dx}{x^5\sqrt{X}} = \left(-\frac{1}{9ax^5} + \frac{8b}{63a^2x^4} - \frac{16b^2}{105a^3x^3} + \frac{64b^3}{315a^4x^2} - \frac{128b^4}{315a^5x}\right)2\sqrt{X}$$

$$\int \frac{dx}{x^6\sqrt{X}} = \left(-\frac{1}{11ax^6} + \frac{10b}{99a^2x^5} - \frac{80b^2}{693a^3x^4} + \frac{32b^3}{231a^4x^3} - \frac{128b^4}{693a^5x^2} + \frac{256b^5}{693a^6x}\right)2\sqrt{X}$$

$$\int \frac{dx}{x^7\sqrt{X}} = \left(-\frac{1}{13ax^7} + \frac{12b}{143a^2x^6} - \frac{40b^2}{429a^3x^5} + \frac{320b^3}{3003a^4x^4} - \frac{128b^4}{1001a^5x^3} + \frac{512b^5}{3003a^6x^2} - \frac{1024b^6}{3003a^7x}\right)2\sqrt{X}$$

$$\int \frac{dx}{x^8\sqrt{X}} = \left(-\frac{1}{15ax^8} + \frac{14b}{195a^2x^7} - \frac{56b^2}{715a^3x^6} + \frac{112b^3}{1287a^4x^5} - \frac{128b^4}{1287a^5x^4} + \frac{256b^5}{2145a^6x^3} - \frac{1024b^6}{6435a^7x^2} + \frac{2048b^7}{6435a^8x}\right)2\sqrt{X}$$

$$\int \frac{dx}{x^9\sqrt{X}} = -\frac{2\sqrt{X}}{17ax^9} - \frac{16b}{17a}\int \frac{dx}{x^8\sqrt{X}}$$

$$\int \frac{dx}{x^{10}\sqrt{X}} = \left(-\frac{1}{19ax^{10}} + \frac{18b}{323a^2x^9}\right)2\sqrt{X} + \frac{288b^2}{323a^2}\int \frac{dx}{x^8\sqrt{X}}$$

TAB. XLVI.

$$\int \frac{x^m dx}{(ax+bx^2)^{\frac{3}{2}}}$$

$$ax+bx^2 = X$$

$$\int \frac{dx}{X^{\frac{3}{2}}} = -\frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{3}{2}}} = \frac{2x}{a\sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{3}{2}}} = \frac{2x}{b\sqrt{X}} + \frac{1}{b}\int\frac{dx}{\sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{3}{2}}} = \left(\frac{x^2}{b} + \frac{3ax}{b^2}\right)\frac{1}{\sqrt{X}} - \frac{3a}{2b^2}\int\frac{dx}{\sqrt{X}}$$

$$\int \frac{x^4 dx}{X^{\frac{3}{2}}} = \left(\frac{x^3}{2b} - \frac{5ax^2}{4b^2} - \frac{15a^2x}{4b^3}\right)\frac{1}{\sqrt{X}} + \frac{15a^2}{8b^3}\int\frac{dx}{\sqrt{X}}$$

$$\int \frac{x^5 dx}{X^{\frac{3}{2}}} = \left(\frac{x^4}{3b} - \frac{7ax^3}{12b^2} + \frac{35a^2x^2}{24b^3} + \frac{35a^3x}{8b^4}\right)\frac{1}{\sqrt{X}} - \frac{35a^3}{16b^4}\int\frac{dx}{\sqrt{X}}$$

$$\int \frac{x^6 dx}{X^{\frac{3}{2}}} = \left(\frac{x^5}{4b} - \frac{3ax^4}{8b^2} + \frac{21a^2x^3}{32b^3} - \frac{105a^3x^2}{64b^4} - \frac{315a^4x}{64b^5}\right)\frac{1}{\sqrt{X}} + \frac{315a^4}{128b^5}\int\frac{dx}{\sqrt{X}}$$

$$\int \frac{x^7 dx}{X^{\frac{3}{2}}} = \left(\frac{x^6}{5b} - \frac{11ax^5}{40b^2} + \frac{33a^2x^4}{80b^3} - \frac{231a^3x^3}{320b^4} + \frac{231a^4x^2}{128b^5} + \frac{693a^5x}{128b^6}\right)\frac{1}{\sqrt{X}} - \frac{693a^5}{256b^6}\int\frac{dx}{\sqrt{X}}$$

$$\int \frac{x^8 dx}{X^{\frac{3}{2}}} = \left(\frac{x^7}{6b} - \frac{13ax^6}{60b^2} + \frac{143a^2x^5}{480b^3} - \frac{143a^3x^4}{320b^4} + \frac{1001a^4x^3}{1280b^5} - \frac{1001a^5x^2}{512b^6} - \frac{3003a^6x}{512b^7}\right)\frac{1}{\sqrt{X}} + \frac{3003a^6}{1024b^7}\int\frac{dx}{\sqrt{X}}$$

$$\int \frac{x^9 dx}{X^{\frac{3}{2}}} = \frac{x^8}{7b\sqrt{X}} - \frac{15a}{14b}\int\frac{x^8 dx}{X^{\frac{3}{2}}}$$

$$\int \frac{x^{10} dx}{X^{\frac{3}{2}}} = \left(\frac{x^9}{8b} - \frac{17a}{112b^2}\right)\frac{1}{\sqrt{X}} - \frac{255a^2}{224b^2}\int\frac{x^8 dx}{X^{\frac{3}{2}}}$$

TAB. XLVII.

$$\int \frac{dx}{x^m(ax+bx^2)^{\frac{3}{2}}}$$

$$ax + bx^2 = X$$

$$\int \frac{dx}{xX^{\frac{3}{2}}} = -\frac{2}{3ax\sqrt{X}} - \frac{4b}{3a}\int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^2X^{\frac{3}{2}}} = \left(-\frac{1}{5ax^2} + \frac{2b}{5a^2x}\right)\frac{2}{\sqrt{X}} + \frac{8b^2}{5a^2}\int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{3}{2}}} = \left(-\frac{1}{7ax^3} + \frac{8b}{35a^2x^2} - \frac{16b^2}{35a^3x}\right)\frac{2}{\sqrt{X}} - \frac{64b^3}{35a^3}\int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{3}{2}}} = \left(-\frac{1}{9ax^4} + \frac{10b}{63a^2x^3} - \frac{16b^2}{63a^3x^2} + \frac{32b^3}{63a^4x}\right)\frac{2}{\sqrt{X}} + \frac{128b^4}{63a^4}\int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{3}{2}}} = \left(-\frac{1}{11ax^5} + \frac{4b}{33a^2x^4} - \frac{40b^2}{231a^3x^3} + \frac{64b^3}{231a^4x^2} - \frac{128b^4}{231a^5x}\right)\frac{2}{\sqrt{X}} - \frac{512b^5}{231a^5}\int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^6X^{\frac{3}{2}}} = \left(-\frac{1}{13ax^6} + \frac{14b}{143a^2x^5} - \frac{56b^2}{429a^3x^4} + \frac{80b^3}{429a^4x^3} - \frac{128b^4}{429a^5x^2} + \frac{256b^5}{429a^6x}\right)\frac{2}{\sqrt{X}} + \frac{1024b^6}{429a^6}\int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^7X^{\frac{3}{2}}} = \left(-\frac{1}{15ax^7} + \frac{16b}{195a^2x^6} - \frac{224b^2}{2145a^3x^5} + \frac{896b^3}{6435a^4x^4} - \frac{256b^4}{1287a^5x^3} + \frac{2048b^5}{6435a^6x^2} - \frac{4096b^6}{6435a^7x}\right)\frac{2}{\sqrt{X}} - \frac{16384b^7}{6435a^7}\int \frac{dx}{X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^8X^{\frac{3}{2}}} = -\frac{2}{17ax^8\sqrt{X}} - \frac{18b}{17a}\int \frac{dx}{x^7X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^9X^{\frac{3}{2}}} = \left(-\frac{1}{19ax^9} + \frac{20b}{323a^2x^8}\right)\frac{2}{\sqrt{X}} + \frac{360b^2}{323a^2}\int \frac{dx}{x^7X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^{10}X^{\frac{3}{2}}} = \left(-\frac{1}{21ax^{10}} + \frac{22b}{399a^2x^9} - \frac{440b^2}{6783a^3x^8}\right)\frac{2}{\sqrt{X}} - \frac{2640b^3}{2261a^3}\int \frac{dx}{x^7X^{\frac{3}{2}}}$$

TAB. XLVIII.

$$\int \frac{x^m dx}{(ax + bx^2)^{\frac{5}{2}}}$$

$$ax + bx^2 = X$$

$$\int \frac{dx}{X^{\frac{5}{2}}} = \left(-\frac{2}{3X} + \frac{16b}{3a^2}\right)\frac{2bx+a}{a^2\sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{5}{2}}} = \frac{2x}{3aX\sqrt{X}} - \frac{8(2bx+a)}{3a^3\sqrt{X}} = \left(\frac{1}{a+bx} - \frac{4(2bx+a)}{a^2}\right)\frac{2}{3a\sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{5}{2}}} = \left(\frac{2x^2}{3aX} + \frac{4x}{3a^2}\right)\frac{1}{\sqrt{X}} = \left(\frac{x}{a+bx} + \frac{2x}{a}\right)\frac{2}{3a\sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{5}{2}}} = \frac{2x^3}{3aX\sqrt{X}} = \frac{2x^2}{3a(a+bx)\sqrt{X}}$$

$$\int \frac{x^4 dx}{X^{\frac{5}{2}}} = \left(-\frac{8x^3}{3b} - \frac{2ax^2}{b^2}\right)\frac{1}{X\sqrt{X}} + \frac{1}{b^2}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^5 dx}{X^{\frac{5}{2}}} = \left(\frac{x^4}{b} + \frac{20ax^3}{3b^2} + \frac{5a^2x^2}{b^3}\right)\frac{1}{X\sqrt{X}} - \frac{5a}{2b^3}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^6 dx}{X^{\frac{5}{2}}} = \left(\frac{x^5}{2b} - \frac{7ax^4}{4b^2} - \frac{35a^2x^3}{3b^3} - \frac{35a^3x^2}{4b^4}\right)\frac{1}{X\sqrt{X}} + \frac{35a^2}{8b^4}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^7 dx}{X^{\frac{5}{2}}} = \left(\frac{x^6}{3b} - \frac{3ax^5}{4b^2} + \frac{21a^2x^4}{8b^3} + \frac{35a^3x^3}{2b^4} + \frac{105a^4x^2}{8b^5}\right)\frac{1}{X\sqrt{X}}$$
$$- \frac{105a^3}{16b^5}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^8 dx}{X^{\frac{5}{2}}} = \left(\frac{x^7}{4b} - \frac{11ax^6}{24b^2} + \frac{33a^2x^5}{32b^3} - \frac{231a^3x^4}{64b^4} - \frac{385a^4x^3}{16b^5}\right.$$
$$\left. - \frac{1155a^5x^2}{64b^6}\right)\frac{1}{X\sqrt{X}} + \frac{1155a^4}{128b^6}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^9 dx}{X^{\frac{5}{2}}} = \left(\frac{x^8}{5b} - \frac{13ax^7}{40b^2} + \frac{143a^2x^6}{240b^3} - \frac{429a^3x^5}{320b^4} + \frac{3003a^4x^4}{640b^5}\right.$$
$$\left. + \frac{1001a^5x^3}{32b^6} + \frac{3003a^6x^2}{128b^7}\right)\frac{1}{X\sqrt{X}} - \frac{3003a^5}{256b^7}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^{10} dx}{X^{\frac{5}{2}}} = \frac{x^9}{6bX\sqrt{X}} - \frac{5a}{4b}\int \frac{x^9 dx}{X^{\frac{5}{2}}}$$

TAB. XLIX.

$$\int \frac{dx}{x^m(ax+bx^2)^{\frac{5}{2}}}$$

$$ax + bx^2 = X$$

$$\int \frac{dx}{xX^{\frac{5}{2}}} = -\frac{2}{3axX\sqrt{X}} - \frac{8b}{5a}\int \frac{dx}{X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^2X^{\frac{5}{2}}} = \left(-\frac{1}{7ax^2} + \frac{2b}{7a^2x}\right)\frac{2}{X\sqrt{X}} + \frac{16b^2}{7a^2}\int \frac{dx}{X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{5}{2}}} = \left(-\frac{1}{9ax^3} + \frac{4b}{21a^2x^2} - \frac{8b^2}{21a^3x}\right)\frac{2}{X\sqrt{X}} - \frac{64b^3}{21a^3}\int \frac{dx}{X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{5}{2}}} = \left(-\frac{1}{11ax^4} + \frac{14b}{99a^2x^3} - \frac{8b^2}{33a^3x^2} + \frac{16b^3}{33a^4x}\right)\frac{2}{X\sqrt{X}} - \frac{128b^4}{33a^4}\int \frac{dx}{X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{5}{2}}} = \left(-\frac{1}{13ax^5} + \frac{16b}{143a^2x^4} - \frac{224b^2}{1287a^3x^3} + \frac{128b^3}{429a^4x^2} - \frac{256b^4}{429a^5x}\right)\frac{2}{X\sqrt{X}} - \frac{2048b^5}{429a^5}\int \frac{dx}{X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^6X^{\frac{5}{2}}} = \left(-\frac{1}{15ax^6} + \frac{6b}{65a^2x^5} - \frac{96b^2}{715a^3x^4} + \frac{448b^3}{2145a^4x^3} - \frac{256b^4}{715a^5x^2} + \frac{512b^5}{715a^6x}\right)\frac{2}{X\sqrt{X}} + \frac{4096b^6}{715a^6}\int \frac{dx}{X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^7X^{\frac{5}{2}}} = \left(-\frac{1}{17ax^7} + \frac{4b}{51a^2x^6} - \frac{24b^2}{221a^3x^5} + \frac{384b^3}{2431a^4x^4} - \frac{1792b^4}{7293a^5x^3} + \frac{1024b^5}{2431a^6x^2} - \frac{2048b^6}{2431a^7x}\right)\frac{2}{X\sqrt{X}} - \frac{16384b^7}{2431a^7}\int \frac{dx}{X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^8X^{\frac{5}{2}}} = -\frac{2}{19ax^8X\sqrt{X}} - \frac{22b}{19a}\int \frac{dx}{x^7X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^9X^{\frac{5}{2}}} = \left(-\frac{1}{21ax^9} + \frac{8b}{133a^2x^8}\right)\frac{2}{X\sqrt{X}} + \frac{176b^2}{133a^2}\int \frac{dx}{x^7X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^{10}X^{\frac{5}{2}}} = \left(-\frac{1}{23ax^{10}} + \frac{26b}{483a^2x^9} - \frac{208b^2}{3059a^3x^8}\right)\frac{2}{X\sqrt{X}} - \frac{4576b^3}{3059a^3}\int \frac{dx}{x^7X^{\frac{5}{2}}}$$

TAB. L.

$$\int \frac{x^m\,dx}{(ax+bx^2)^{\frac{7}{2}}}$$

$$ax + bx^2 = X$$

$$\int \frac{dx}{X^{\frac{7}{2}}} = \left(-\frac{1}{X^3} + \frac{16b}{3a^2X} - \frac{128b^2}{3a^4}\right)\frac{2(2bx+a)}{5a^2\sqrt{X}}$$

$$\int \frac{x\,dx}{X^{\frac{7}{2}}} = \frac{2x}{5aX^2\sqrt{X}} - \left(\frac{1}{X} - \frac{8b}{a^2}\right)\frac{16(2bx+a)}{15a^3\sqrt{X}}$$

$$\int \frac{x^2\,dx}{X^{\frac{7}{2}}} = \left(\frac{x^2}{X^2} + \frac{2x}{aX}\right)\frac{2}{5a\sqrt{X}} - \frac{16(2bx+a)}{5a^4\sqrt{X}}$$

$$\int \frac{x^3\,dx}{X^{\frac{7}{2}}} = \left(\frac{x^3}{X^2} + \frac{4x^2}{3aX} + \frac{8x}{3a^2}\right)\frac{2}{5a\sqrt{X}}$$

$$\int \frac{x^4\,dx}{X^{\frac{7}{2}}} = \left(\frac{x^4}{X^2} + \frac{2x^3}{3aX}\right)\frac{2}{5a\sqrt{X}}$$

$$\int \frac{x^5\,dx}{X^{\frac{7}{2}}} = \frac{2x^5}{5aX^2\sqrt{X}}$$

$$\int \frac{x^6\,dx}{X^{\frac{7}{2}}} = \frac{2x^6}{5aX^2\sqrt{X}} - \frac{2}{5a}\int \frac{x^5\,dx}{X^{\frac{5}{2}}}$$

$$\int \frac{x^7\,dx}{X^{\frac{7}{2}}} = -\frac{2x^6}{5bX^2\sqrt{X}} + \frac{7}{5b}\int \frac{x^5\,dx}{X^{\frac{5}{2}}}$$

$$\int \frac{x^8\,dx}{X^{\frac{7}{2}}} = \left(\frac{x^7}{2b} + \frac{9ax^6}{10b^2}\right)\frac{1}{X^2\sqrt{X}} - \frac{63a}{20b^2}\int \frac{x^5\,dx}{X^{\frac{5}{2}}}$$

$$\int \frac{x^9\,dx}{X^{\frac{7}{2}}} = \left(\frac{x^8}{3b} - \frac{11ax^7}{12b^2} - \frac{33a^2x^6}{20b^3}\right)\frac{1}{X^2\sqrt{X}} + \frac{231a^2}{40b^3}\int \frac{x^5\,dx}{X^{\frac{5}{2}}}$$

$$\int \frac{x^{10}\,dx}{X^{\frac{7}{2}}} = \left(\frac{x^9}{4b} - \frac{13ax^8}{24b^2} + \frac{143a^2x^7}{96b^3} + \frac{429a^3x^6}{160b^4}\right)\frac{1}{X^2\sqrt{X}} - \frac{3003a^3}{320b^4}\int \frac{x^5\,dx}{X^{\frac{5}{2}}}$$

TAB. LI.

$$\int \frac{dx}{x^m (ax+bx^2)^{\frac{7}{2}}}$$

$$ax + bx^2 = X$$

$$\int \frac{dx}{xX^{\frac{7}{2}}} = -\frac{2}{7axX^2\sqrt{X}} - \frac{12b}{7a}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^2X^{\frac{7}{2}}} = \left(-\frac{1}{9ax^2} + \frac{2b}{9a^2x}\right)\frac{2}{X^3\sqrt{X}} + \frac{8b^2}{3a^2}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{7}{2}}} = \left(-\frac{1}{11ax^3} + \frac{16b}{99a^2x^2} - \frac{32b^2}{99a^3x}\right)\frac{2}{X^2\sqrt{X}} - \frac{128b^3}{33a^3}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{7}{2}}} = \left(-\frac{1}{13ax^4} + \frac{18b}{143a^2x^3} - \frac{32b^2}{143a^3x^2} + \frac{64b^3}{143a^4x}\right)\frac{2}{X^2\sqrt{X}} + \frac{768b^4}{143a^4}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{7}{2}}} = \left(-\frac{1}{15ax^5} + \frac{4b}{39a^2x^4} - \frac{24b^2}{143a^3x^3} + \frac{128b^3}{429a^4x^2} - \frac{256b^4}{429a^5x}\right)\frac{2}{X^2\sqrt{X}} - \frac{1024b^5}{143a^5}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^6X^{\frac{7}{2}}} = \left(-\frac{1}{17ax^6} + \frac{22b}{255a^2x^5} - \frac{88b^2}{663a^3x^4} + \frac{48b^3}{221a^4x^3} - \frac{256b^4}{663a^5x^2} + \frac{512b^5}{663a^6x}\right)\frac{2}{X^2\sqrt{X}} + \frac{2048b^6}{221a^6}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^7X^{\frac{7}{2}}} = \left(-\frac{1}{19ax^7} + \frac{24b}{323a^2x^6} - \frac{176b^2}{1615a^3x^5} + \frac{704b^3}{4199a^4x^4} - \frac{1152b^4}{4199a^5x^3} + \frac{2048b^5}{4199a^6x^2} - \frac{4096b^6}{4199a^7x}\right)\frac{2}{X^2\sqrt{X}} - \frac{49152b^7}{4199a^7}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^8X^{\frac{7}{2}}} = -\frac{2}{21ax^8X^2\sqrt{X}} - \frac{26b}{21a}\int \frac{dx}{x^7X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^9X^{\frac{7}{2}}} = \left(-\frac{1}{23ax^9} + \frac{4b}{69a^2x^8}\right)\frac{2}{X^3\sqrt{X}} + \frac{104b^2}{69a^2}\int \frac{dx}{x^7X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^{10}X^{\frac{7}{2}}} = \left(-\frac{1}{25ax^{10}} + \frac{6b}{115a^2x^9} - \frac{8b^2}{115a^3x^8}\right)\frac{2}{X^3\sqrt{X}} - \frac{208b^3}{115a^3}\int \frac{dx}{x^7X^{\frac{7}{2}}}$$

TAB. LII.

$$\int\frac{x^m dx}{(ax+bx^2)^{\frac{7}{2}}},\quad \int\frac{dx}{x^m(ax+bx^2)^{\frac{7}{2}}}$$

$$ax + bx^2 = X$$

$$\int\frac{dx}{X^{\frac{7}{2}}} = \left(-\frac{1}{X^3} + \frac{24b}{5a^2X^2} - \frac{128b^2}{5a^4X} + \frac{1024b^3}{5a^6}\right)\frac{2(2bx+a)}{7a^2\sqrt{X}}$$

$$\int\frac{x\,dx}{X^{\frac{7}{2}}} = \frac{2x}{7aX^3\sqrt{X}} - \left(\frac{1}{X^2} - \frac{16b}{3a^2X} + \frac{128b^2}{3a^4}\right)\frac{24(2bx+a)}{35a^3\sqrt{X}}$$

$$\int\frac{x^2dx}{X^{\frac{7}{2}}} = \left(\frac{x^2}{X^3} + \frac{2x}{aX^2}\right)\frac{2}{7a\sqrt{X}} - \left(\frac{1}{X} - \frac{8b}{a^2}\right)\frac{32(2bx+a)}{21a^4\sqrt{X}}$$

$$\int\frac{x^3dx}{X^{\frac{7}{2}}} = \left(\frac{x^3}{X^3} + \frac{8x^2}{5aX^2} + \frac{16x}{5a^2X}\right)\frac{2}{7a\sqrt{X}} - \frac{128(2bx+a)}{35a^5\sqrt{X}}$$

$$\int\frac{x^4dx}{X^{\frac{7}{2}}} = \left(\frac{x^4}{X^3} + \frac{6x^3}{5aX^2} + \frac{8x^2}{5a^2X} + \frac{16x}{5a^3}\right)\frac{2}{7a\sqrt{X}}$$

$$\int\frac{x^5dx}{X^{\frac{7}{2}}} = \left(\frac{x^5}{X^3} + \frac{4x^4}{5aX^2} + \frac{8x^3}{15a^2X}\right)\frac{2}{7a\sqrt{X}}$$

$$\int\frac{x^6dx}{X^{\frac{7}{2}}} = \left(\frac{x^6}{X^3} + \frac{2x^5}{5aX^2}\right)\frac{2}{7a\sqrt{X}} = \left(\frac{1}{(a+bx)^3} + \frac{2}{5a(a+bx)^2}\right)\frac{2x^3}{7a\sqrt{X}}$$

$$\int\frac{x^7dx}{X^{\frac{7}{2}}} = \frac{2x^7}{7aX^3\sqrt{X}} = \frac{2x^4}{7a(a+bx)^3\sqrt{X}}$$

$$\int\frac{dx}{xX^{\frac{7}{2}}} = -\frac{2}{9axX^3\sqrt{X}} - \frac{16b}{9a}\int\frac{dx}{X^{\frac{7}{2}}}$$

$$\int\frac{dx}{x^2X^{\frac{7}{2}}} = \left(-\frac{1}{11ax^2} + \frac{2b}{11a^2x}\right)\frac{2}{X^3\sqrt{X}} + \frac{32b^2}{11a^2}\int\frac{dx}{X^{\frac{7}{2}}}$$

$$\int\frac{dx}{x^3X^{\frac{7}{2}}} = \left(-\frac{1}{13ax^3} + \frac{20b}{143a^2x^2} - \frac{40b^2}{143a^3x}\right)\frac{2}{X^3\sqrt{X}} - \frac{640b^3}{143a^3}\int\frac{dx}{X^{\frac{7}{2}}}$$

$$\int\frac{dx}{x^4X^{\frac{7}{2}}} = \left(-\frac{1}{15ax^4} + \frac{22b}{195a^2x^3} - \frac{8b^2}{39a^3x^2} + \frac{16b^3}{39a^4x}\right)\frac{2}{X^3\sqrt{X}} + \frac{256b^4}{39a^4}\int\frac{dx}{X^{\frac{7}{2}}}$$

$$\int\frac{dx}{x^5X^{\frac{7}{2}}} = \left(-\frac{1}{17ax^5} + \frac{8b}{85a^2x^4} - \frac{176b^2}{1105a^3x^3} + \frac{64b^3}{221a^4x^2} - \frac{128b^4}{221a^5x}\right)\frac{2}{X^3\sqrt{X}} - \frac{2048b^5}{221a^5}\int\frac{dx}{X^{\frac{7}{2}}}$$

TAB. LIII.

$$\int x^m dx\sqrt{(ax+bx^2)}$$

$$ax + bx^2 = X$$

$$\int dx\sqrt{X} = \left(\frac{x}{2} + \frac{a}{4b}\right)\sqrt{X} - \frac{a^2}{8b}\int\frac{dx}{\sqrt{X}}$$

$$\int x dx\sqrt{X} = \frac{X\sqrt{X}}{3b} - \frac{a}{2b}\int dx\sqrt{X}$$

$$\int x^2 dx\sqrt{X} = \left(\frac{x}{4b} - \frac{5a}{24b^2}\right)X\sqrt{X} - \frac{5a^2}{16b^2}\int dx\sqrt{X}$$

$$\int x^3 dx\sqrt{X} = \left(\frac{x^2}{5b} - \frac{7ax}{40b^2} + \frac{7a^2}{48b^3}\right)X\sqrt{X} - \frac{7a^3}{32b^3}\int dx\sqrt{X}$$

$$\int x^4 dx\sqrt{X} = \left(\frac{x^3}{6b} - \frac{3ax^2}{20b^2} + \frac{21a^2x}{160b^3} - \frac{7a^3}{64b^4}\right)X\sqrt{X} + \frac{21a^4}{128b^4}\int dx\sqrt{X}$$

$$\int x^5 dx\sqrt{X} = \left(\frac{x^4}{7b} - \frac{11ax^3}{84b^2} + \frac{33a^2x^2}{280b^3} - \frac{33a^3x}{320b^4} + \frac{11a^4}{128b^5}\right)X\sqrt{X}$$
$$- \frac{33a^5}{256b^5}\int dx\sqrt{X}$$

$$\int x^6 dx\sqrt{X} = \left(\frac{x^5}{8b} - \frac{13ax^4}{112b^2} + \frac{143a^2x^3}{1344b^3} - \frac{429a^3x^2}{4480b^4} + \frac{429a^4x}{5120b^5}\right.$$
$$\left. - \frac{143a^5}{2048b^6}\right)X\sqrt{X} + \frac{429a^6}{4096b^6}\int dx\sqrt{X}$$

$$\int x^7 dx\sqrt{X} = \left(\frac{x^6}{9b} - \frac{5ax^5}{48b^2} + \frac{65a^2x^4}{672b^3} - \frac{715a^3x^3}{8064b^4} + \frac{715a^4x^2}{8960b^4}\right.$$
$$\left. - \frac{143a^5x}{2048b^6} - \frac{715a^6}{12288b^6}\right)X\sqrt{X} - \frac{715a^7}{8192b^7}\int dx\sqrt{X}$$

$$\int x^8 dx\sqrt{X} = \frac{x^7X\sqrt{X}}{10b} - \frac{17a}{20b}\int x^7 dx\sqrt{X}$$

$$\int x^9 dx\sqrt{X} = \left(\frac{x^8}{11b} - \frac{19ax^7}{220b^2}\right)X\sqrt{X} + \frac{323a^2}{440b^2}\int x^7 dx\sqrt{X}$$

$$\int x^{10} dx\sqrt{X} = \left(\frac{x^9}{12b} - \frac{7ax^8}{88b^2} + \frac{133a^2x^7}{1760b^3}\right)X\sqrt{X} - \frac{2261a^3}{3520b^3}\int x^7 dx\sqrt{X}$$

TAB. LIV.

$$\int \frac{dx\sqrt{(ax+bx^2)}}{x^m}$$

$$ax+bx^2=X$$

$$\int\frac{dx\sqrt{X}}{x}=\sqrt{X}+\frac{a}{2}\int\frac{dx}{\sqrt{X}}$$

$$\int\frac{dx\sqrt{X}}{x^2}=-\frac{2\sqrt{X}}{x}+b\int\frac{dx}{\sqrt{X}}$$

$$\int\frac{dx\sqrt{X}}{x^3}=-\frac{2X\sqrt{X}}{3ax^3}=-\frac{2(a+bx)\sqrt{X}}{3ax^2}$$

$$\int\frac{dx\sqrt{X}}{x^4}=\left(-\frac{1}{5ax^4}+\frac{2b}{15a^2x^3}\right)2X\sqrt{X}$$

$$\int\frac{dx\sqrt{X}}{x^5}=\left(-\frac{1}{7ax^5}+\frac{4b}{35a^2x^4}-\frac{8b^2}{105a^3x^3}\right)2X\sqrt{X}$$

$$\int\frac{dx\sqrt{X}}{x^6}=\left(-\frac{1}{9ax^6}+\frac{2b}{21a^2x^5}-\frac{8b^2}{105a^3x^4}+\frac{16b^3}{315a^4x^3}\right)2X\sqrt{X}$$

$$\int\frac{dx\sqrt{X}}{x^7}=\left(-\frac{1}{11ax^7}+\frac{8b}{99a^2x^6}-\frac{16b^2}{231a^3x^5}+\frac{64b^3}{1155a^4x^4}-\frac{128b^4}{3465a^5x^3}\right)2X\sqrt{X}$$

$$\int\frac{dx\sqrt{X}}{x^8}=\left(-\frac{1}{13ax^8}+\frac{10b}{143a^2x^7}-\frac{80b^2}{1287a^3x^6}+\frac{160b^3}{3003a^4x^5}-\frac{128b^4}{3003a^5x^4}+\frac{256b^5}{9009a^6x^3}\right)2X\sqrt{X}$$

$$\int\frac{dx\sqrt{X}}{x^9}=\left(-\frac{1}{15ax^9}+\frac{4b}{65a^2x^8}-\frac{8b^2}{143a^3x^7}+\frac{64b^3}{1287a^4x^6}-\frac{128b^4}{3003a^5x^5}+\frac{512b^5}{15015a^6x^4}-\frac{1024b^6}{45045a^7x^3}\right)2X\sqrt{X}$$

$$\int\frac{dx\sqrt{X}}{x^{10}}=-\frac{2X\sqrt{X}}{17ax^{10}}-\frac{14b}{17a}\int\frac{dx\sqrt{X}}{x^9}$$

$$\int\frac{dx\sqrt{X}}{x^{11}}=\left(-\frac{1}{19ax^{11}}+\frac{16b}{323a^2x^{10}}\right)2X\sqrt{X}+\frac{224b^2}{323a^2}\int\frac{dx\sqrt{X}}{x^9}$$

TAB. LV.

$$\int x^m dx (ax+bx^2)^{\frac{3}{2}}$$

$$ax + bx^2 = X$$

$$\int dx X^{\frac{3}{2}} = \left(\frac{X}{b} - \frac{3a^2}{8b^2}\right)\frac{2bx+a}{8}\sqrt{X} + \frac{3a^4}{128b^2}\int\frac{dx}{\sqrt{X}}$$

$$\int x dx X^{\frac{3}{2}} = \frac{X^2\sqrt{X}}{5b} - \frac{a}{2b}\int dx X^{\frac{3}{2}}$$

$$\int x^2 dx X^{\frac{3}{2}} = \left(\frac{x}{6b} - \frac{7a}{60b^2}\right)X^2\sqrt{X} + \frac{7a^2}{24b^2}\int dx X^{\frac{3}{2}}$$

$$\int x^3 dx X^{\frac{3}{2}} = \left(\frac{x^2}{7b} - \frac{3ax}{28b^2} + \frac{3a^2}{40b^3}\right)X^2\sqrt{X} - \frac{3a^3}{16b^3}\int dx X^{\frac{3}{2}}$$

$$\int x^4 dx X^{\frac{3}{2}} = \left(\frac{x^3}{8b} - \frac{11ax^2}{112b^2} + \frac{33a^2x}{448b^3} - \frac{33a^3}{640b^4}\right)X^2\sqrt{X} + \frac{33a^4}{256b^4}\int dx X^{\frac{3}{2}}$$

$$\int x^5 dx X^{\frac{3}{2}} = \left(\frac{x^4}{9b} - \frac{13ax^3}{144b^2} + \frac{143a^2x^2}{2016b^3} - \frac{143a^3x}{2688b^4} + \frac{143a^4}{3840b^5}\right)X^2\sqrt{X}$$
$$- \frac{143a^5}{1536b^5}\int dx X^{\frac{3}{2}}$$

$$\int x^6 dx X^{\frac{3}{2}} = \left(\frac{x^5}{10b} - \frac{ax^4}{12b^2} + \frac{13a^2x^3}{192b^3} - \frac{143a^3x^2}{2688b^4} + \frac{143a^4x}{3584b^5}\right.$$
$$\left. - \frac{143a^5}{5120b^6}\right)X^2\sqrt{X} + \frac{143a^6}{2048b^6}\int dx X^{\frac{3}{2}}$$

$$\int x^7 dx X^{\frac{3}{2}} = \left(\frac{x^6}{11b} - \frac{17ax^5}{220b^2} + \frac{17a^2x^4}{264b^3} - \frac{221a^3x^3}{4224b^4} + \frac{221a^4x^2}{5376b^5}\right.$$
$$\left. - \frac{221a^5x}{7168b^6} + \frac{221a^6}{10240b^7}\right)X^2\sqrt{X} - \frac{221a^7}{4096b^7}\int dx X^{\frac{3}{2}}$$

$$\int x^8 dx X^{\frac{3}{2}} = \frac{x^7X^2\sqrt{X}}{12b} - \frac{19a}{24b}\int x^7 dx X^{\frac{3}{2}}$$

$$\int x^9 dx X^{\frac{3}{2}} = \left(\frac{x^8}{13b} - \frac{7ax^7}{104b^2}\right)X^2\sqrt{X} + \frac{133a^2}{208b^2}\int x^7 dx X^{\frac{3}{2}}$$

$$\int x^{10} dx X^{\frac{3}{2}} = \left(\frac{x^9}{14b} - \frac{23ax^8}{364b^2} + \frac{23a^2x^7}{416b^3}\right)X^2\sqrt{X} - \frac{437a^3}{832b^3}\int x^7 dx X^{\frac{3}{2}}$$

TAB. LVI.

$$\int\frac{dx(ax+bx^2)^{\frac{3}{2}}}{x^m}$$

$$ax+bx^2=X$$

$$\int\frac{dxX^{\frac{3}{2}}}{x}=\frac{X\sqrt{X}}{3}+\frac{a}{2}\int dx\sqrt{X}$$

$$\int\frac{dxX^{\frac{3}{2}}}{x^2}=\frac{X\sqrt{X}}{2x}+\frac{3a}{4}\sqrt{X}+\frac{3a^2}{8}\int\frac{dx}{\sqrt{X}}$$

$$=\left(\frac{5a}{4}+\frac{bx}{2}\right)\sqrt{X}+\frac{3a^2}{8}\int\frac{dx}{\sqrt{X}}$$

$$\int\frac{dxX^{\frac{3}{2}}}{x^3}=\frac{X\sqrt{X}}{x^2}-\frac{3a\sqrt{X}}{x}+\frac{3ab}{2}\int\frac{dx}{\sqrt{X}}$$

$$=\left(b-\frac{2a}{x}\right)\sqrt{X}+\frac{3ab}{2}\int\frac{dx}{\sqrt{X}}$$

$$\int\frac{dxX^{\frac{3}{2}}}{x^4}=-\frac{2X^2\sqrt{X}}{3ax^4}+\frac{2b}{3a}\int\frac{dxX^{\frac{3}{2}}}{x^3}$$

$$=-\left(\frac{2a}{3x^2}+\frac{8b}{3x}\right)\sqrt{X}+b^2\int\frac{dx}{\sqrt{X}}$$

$$\int\frac{dxX^{\frac{3}{2}}}{x^5}=-\frac{2X^2\sqrt{X}}{5ax^5}=-\frac{2(a+bx)^2\sqrt{X}}{5ax^3}$$

$$\int\frac{dxX^{\frac{3}{2}}}{x^6}=\left(-\frac{1}{7ax^6}+\frac{2b}{35a^2x^5}\right)2X^2\sqrt{X}$$

$$\int\frac{dxX^{\frac{3}{2}}}{x^7}=\left(-\frac{1}{9ax^7}+\frac{4b}{63a^2x^6}-\frac{8b^2}{315a^3x^5}\right)2X^2\sqrt{X}$$

$$\int\frac{dxX^{\frac{3}{2}}}{x^8}=\left(-\frac{1}{11ax^8}+\frac{2b}{33a^2x^7}-\frac{8b^2}{231a^3x^6}+\frac{16b^3}{1155a^4x^5}\right)2X^2\sqrt{X}$$

$$\int\frac{dxX^{\frac{3}{2}}}{x^9}=\left(-\frac{1}{13ax^9}+\frac{8b}{143a^2x^8}-\frac{16b^2}{429a^3x^7}+\frac{64b^3}{3003a^4x^6}\right.$$
$$\left.-\frac{128b^4}{15015a^5x^5}\right)2X^2\sqrt{X}$$

$$\int\frac{dxX^{\frac{3}{2}}}{x^{10}}=\left(-\frac{1}{15ax^{10}}+\frac{2b}{39a^2x^9}-\frac{16b^2}{429a^3x^8}+\frac{32b^3}{1287a^4x^7}\right.$$
$$\left.-\frac{128b^4}{9009a^5x^6}+\frac{256b^5}{45045a^6x^5}\right)2X^2\sqrt{X}$$

TAB. LVII.

$$\int x^m dx (ax+bx^2)^{\frac{5}{2}}$$

$$ax + bx^2 = X$$

$$\int dx X^{\frac{5}{2}} = \left(\frac{X^2}{b} - \frac{5a^2X}{16b^2} + \frac{15a^4}{124b^3}\right)\frac{2bx+a}{12}\, X - \frac{5a^6}{1024b^3}\int\frac{dx}{\sqrt{X}}$$

$$\int x dx X^{\frac{5}{2}} = \frac{X^3\sqrt{X}}{7b} - \frac{a}{2b}\int dx X^{\frac{5}{2}}$$

$$\int x^2 dx X^{\frac{5}{2}} = \left(\frac{x}{8b} - \frac{9a}{112b^2}\right)X^3\sqrt{X} + \frac{9a^2}{32b^2}\int dx X^{\frac{5}{2}}$$

$$\int x^3 dx X^{\frac{5}{2}} = \left(\frac{x^2}{9b} - \frac{11ax}{144b^2} + \frac{11a^2}{224b^3}\right)X^3\sqrt{X} - \frac{11a^3}{64b^3}\int dx X^{\frac{5}{2}}$$

$$\int x^4 dx X^{\frac{5}{2}} = \left(\frac{x^3}{10b} - \frac{13ax^2}{180b^2} + \frac{143ax^2}{2880b^3} - \frac{143a^3}{4480b^4}\right)X^3\sqrt{X}$$
$$+ \frac{143a^4}{1280b^4}\int dx X^{\frac{5}{2}}$$

$$\int x^5 dx X^{\frac{5}{2}} = \left(\frac{x^4}{14b} - \frac{3ax^3}{44b^2} + \frac{39a^2x^2}{792b^3} - \frac{39a^3x}{1152b^4} + \frac{39a^4}{1792b^5}\right)X^3\sqrt{X}$$
$$+ \frac{39a^5}{512b^5}\int dx X^{\frac{5}{2}}$$

$$\int x^6 dx X^{\frac{5}{2}} = \left(\frac{x^5}{12b} - \frac{17ax^4}{264b^2} + \frac{17a^2x^3}{352b^3} - \frac{221a^3x^2}{6336b^4} + \frac{221a^4x}{9216b^5}\right.$$
$$\left. - \frac{221a^5}{14336b^6}\right)X^3\sqrt{X} + \frac{221a^6}{4096b^6}\int dx X^{\frac{5}{2}}$$

$$\int x^7 dx X^{\frac{5}{2}} = \frac{x^6X^3\sqrt{X}}{13b} - \frac{19a}{26b}\int x^6 dx X^{\frac{5}{2}}$$

$$\int x^8 dx X^{\frac{5}{2}} = \left(\frac{x^7}{14b} - \frac{3ax^6}{52b^2}\right)X^3\sqrt{X} + \frac{57a^2}{104b^2}\int x^6 dx X^{\frac{5}{2}}$$

$$\int x^9 dx X^{\frac{5}{2}} = \left(\frac{x^8}{15b} - \frac{23ax^7}{420b^2} + \frac{23a^2x^6}{520b^3}\right)X^3\sqrt{X} - \frac{437a^3}{1040b^3}\int x^6 dx X^{\frac{5}{2}}$$

$$\int x^{10} dx X^{\frac{5}{2}} = \left(\frac{x^9}{16b} - \frac{5ax^8}{96b^2} + \frac{115a^2x^7}{2688b^3} - \frac{115a^3x^6}{3328b^4}\right)X^3\sqrt{X}$$
$$+ \frac{2185a^4}{6656b^4}\int x^6 dx X^{\frac{5}{2}}$$

TAB. LVIII.

$$\int \frac{\mathrm{d}x(ax+bx^2)^{\frac{5}{2}}}{x^m}$$

$$ax+bx^2 = X$$

$$\int \frac{\mathrm{d}x X^{\frac{5}{2}}}{x} = \frac{X^2\sqrt{X}}{5} + \frac{a}{2}\int \mathrm{d}x X^{\frac{3}{2}}$$

$$\int \frac{\mathrm{d}x X^{\frac{5}{2}}}{x^2} = \left(\frac{X^2}{4x} + \frac{5aX}{24}\right)\sqrt{X} + \frac{5a^2}{16}\int \mathrm{d}x\sqrt{X}$$

$$\int \frac{\mathrm{d}x X^{\frac{5}{2}}}{x^3} = \left(\frac{X^2}{3x^2} + \frac{5aX}{12x} + \frac{5a^2}{8}\right)\sqrt{X} + \frac{5a^3}{16}\int \frac{\mathrm{d}x}{\sqrt{X}}$$

$$= \left(\frac{11a^2}{8} + \frac{13abx}{12} + \frac{b^2x^2}{3}\right)\sqrt{X} + \frac{5a^3}{16}\int \frac{\mathrm{d}x}{\sqrt{X}}$$

$$\int \frac{\mathrm{d}x X^{\frac{5}{2}}}{x^4} = \left(\frac{X^2}{2x^3} + \frac{5aX}{4x^2} - \frac{15a^2}{4x}\right)\sqrt{X} + \frac{15a^2b}{8}\int \frac{\mathrm{d}x}{\sqrt{X}}$$

$$= \left(-\frac{2a^2}{x} + \frac{9ab}{4} + \frac{b^2x}{2}\right)\sqrt{X} + \frac{15a^2b}{8}\int \frac{\mathrm{d}x}{\sqrt{X}}$$

$$\int \frac{\mathrm{d}x X^{\frac{5}{2}}}{x^5} = -\frac{2X^3\sqrt{X}}{3ax^5} + \frac{4b}{3a}\int \frac{\mathrm{d}x X^{\frac{5}{2}}}{x^4}$$

$$\int \frac{\mathrm{d}x X^{\frac{5}{2}}}{x^6} = \left(-\frac{1}{5ax^6} - \frac{2b}{15a^2x^5}\right)2X^3\sqrt{X} + \frac{8b^2}{15a^2}\int \frac{\mathrm{d}x X^{\frac{5}{2}}}{x^4}$$

$$\int \frac{\mathrm{d}x X^{\frac{5}{2}}}{x^7} = -\frac{2X^3\sqrt{X}}{7ax^7} = -\frac{2(a+bx)^3\sqrt{X}}{7ax^4}$$

$$\int \frac{\mathrm{d}x X^{\frac{5}{2}}}{x^8} = \left(-\frac{1}{9ax^8} + \frac{2b}{63a^2x^7}\right)2X^3\sqrt{X}$$

$$\int \frac{\mathrm{d}x X^{\frac{5}{2}}}{x^9} = \left(-\frac{1}{11ax^9} + \frac{4b}{99a^2x^8} - \frac{8b^2}{693a^3x^7}\right)2X^3\sqrt{X}$$

$$\int \frac{\mathrm{d}x X^{\frac{5}{2}}}{x^{10}} = \left(-\frac{1}{13ax^{10}} + \frac{6b}{143a^2x^9} - \frac{8b^2}{429a^3x^8} + \frac{16b^3}{3003a^4x^7}\right)2X^3\sqrt{X}$$

$$\int \frac{\mathrm{d}x X^{\frac{5}{2}}}{x^{11}} = \left(-\frac{1}{15ax^{11}} + \frac{8b}{195a^2x^{10}} - \frac{16b^2}{715a^3x^9} + \frac{64b^3}{6435a^4x^8}\right.$$

$$\left. - \frac{128b^4}{45045a^5x^7}\right)2X^3\sqrt{X}$$

TAB. LIX.

$$\int x^m dx (ax+bx^2)^{\frac{7}{2}}$$

$$ax + bx^2 = X$$

$$\int dx X^{\frac{7}{2}} = \left(\frac{X^3}{b} - \frac{7a^2X^2}{24b^2} + \frac{35a^4X}{384b^3} - \frac{35a^6}{1024b^4}\right)\frac{2bx+a}{16}\sqrt{X} + \frac{35a^8}{32768b^4}\int\frac{dx}{\sqrt{X}}$$

$$\int x dx X^{\frac{7}{2}} = \frac{X^4\sqrt{X}}{9b} - \frac{a}{2b}\int dx X^{\frac{7}{2}}$$

$$\int x^2 dx X^{\frac{7}{2}} = \left(\frac{x}{10b} - \frac{11a}{180b^2}\right)X^4\sqrt{X} + \frac{11a^2}{40b^2}\int dx X^{\frac{7}{2}}$$

$$\int x^3 dx X^{\frac{7}{2}} = \left(\frac{x^2}{11b} - \frac{13ax}{220b^2} + \frac{13a^2}{360b^3}\right)X^4\sqrt{X} - \frac{13a^3}{80b^3}\int dx X^{\frac{7}{2}}$$

$$\int x^4 dx X^{\frac{7}{2}} = \left(\frac{x^3}{12b} - \frac{5ax^2}{88b^2} + \frac{13a^2x}{352b^3} - \frac{13a^3}{576b^4}\right)X^4\sqrt{X} + \frac{13a^4}{128b^4}\int dx X^{\frac{7}{2}}$$

$$\int x^5 dx X^{\frac{7}{2}} = \left(\frac{x^4}{13b} - \frac{17ax^3}{312b^2} + \frac{85a^2x^2}{2288b^3} - \frac{17a^3x}{704b^4} + \frac{17a^4}{1152b^5}\right)X^4\sqrt{X} + \frac{17a^5}{256b^5}\int dx X^{\frac{7}{2}}$$

$$\int x^6 dx X^{\frac{7}{2}} = \left(\frac{x^5}{14b} - \frac{19ax^4}{364b^2} + \frac{323a^2x^3}{8736b^3} - \frac{1615a^3x^2}{64064b^4} + \frac{323a^4x}{19712b^5} - \frac{323a^5}{32256b^6}\right)X^4\sqrt{X} + \frac{323a^6}{7168b^6}\int dx X^{\frac{7}{2}}$$

$$\int x^7 dx X^{\frac{7}{2}} = \frac{x^6X^4\sqrt{X}}{15b} - \frac{7a}{10b}\int x^6 dx X^{\frac{7}{2}}$$

$$\int x^8 dx X^{\frac{7}{2}} = \left(\frac{x^7}{16b} - \frac{23ax^6}{480b^2}\right)X^4\sqrt{X} + \frac{161a^2}{320b^2}\int x^6 dx X^{\frac{7}{2}}$$

$$\int x^9 dx X^{\frac{7}{2}} = \left(\frac{x^8}{17b} - \frac{25ax^7}{544b^2} + \frac{115a^2x^6}{3264b^3}\right)X^4\sqrt{X} - \frac{805a^3}{2176b^3}\int x^6 dx X^{\frac{7}{2}}$$

$$\int x^{10} dx X^{\frac{7}{2}} = \left(\frac{x^9}{18b} - \frac{3ax^8}{68b^2} + \frac{75a^2x^7}{2176b^3} - \frac{345a^3x^6}{13056b^4}\right)X^4\sqrt{X} + \frac{2415a^4}{8704b^4}\int x^6 dx X^{\frac{7}{2}}$$

TAB. LX.

$$\int \frac{dx(ax+bx^2)^{\frac{7}{2}}}{x^m}$$

$$ax + bx^2 = X$$

$$\int \frac{dx X^{\frac{7}{2}}}{x} = \frac{X^3\sqrt{X}}{7} + \frac{a}{2}\int dx X^{\frac{5}{2}}$$

$$\int \frac{dx X^{\frac{7}{2}}}{x^2} = \left(\frac{X^3}{6x} + \frac{7aX^2}{60}\right)\sqrt{X} + \frac{7a^2}{24}\int dx X^{\frac{3}{2}}$$

$$\int \frac{dx X^{\frac{7}{2}}}{x^3} = \left(\frac{X^3}{5x^2} + \frac{7aX^2}{40x} + \frac{7a^2X}{48}\right)\sqrt{X} + \frac{7a^3}{32}\int dx\sqrt{X}$$

$$\int \frac{dx X^{\frac{7}{2}}}{x^4} = \left(\frac{X^3}{4x^3} + \frac{7aX^2}{24x^2} + \frac{35a^2X}{96x} + \frac{35a^3}{64}\right)\sqrt{X} + \frac{35a^4}{128}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{dx X^{\frac{7}{2}}}{x^5} = \left(\frac{X^3}{3x^4} + \frac{7aX^2}{12x^3} + \frac{35a^2X}{24x^2} - \frac{35a^3}{8x}\right)\sqrt{X} + \frac{35a^3b}{16}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{dx X^{\frac{7}{2}}}{x^6} = -\frac{2X^3\sqrt{X}}{3x^5} + \frac{7b}{3}\int \frac{dx X^{\frac{5}{2}}}{x^4}$$

$$\int \frac{dx X^{\frac{7}{2}}}{x^7} = \left(-\frac{1}{5x^6} - \frac{7b}{15ax^5}\right)2X^3\sqrt{X} + \frac{28b^2}{15a}\int \frac{dx X^{\frac{5}{2}}}{x^4}$$

$$\int \frac{dx X^{\frac{7}{2}}}{x^8} = \left(-\frac{1}{7x^7} - \frac{b}{5ax^6} - \frac{2b^2}{15a^2x^5}\right)2X^3\sqrt{X} + \frac{8b^3}{15a^2}\int \frac{dx X^{\frac{5}{2}}}{x^4}$$

$$\int \frac{dx X^{\frac{7}{2}}}{x^9} = -\frac{2X^4\sqrt{X}}{9ax^9} = -\frac{2(a+bx)^4\sqrt{X}}{9ax^5}$$

$$\int \frac{dx X^{\frac{7}{2}}}{x^{10}} = \left(-\frac{1}{11ax^{10}} + \frac{2b}{99a^2x^9}\right)2X^4\sqrt{X}$$

$$\int \frac{dx X^{\frac{7}{2}}}{x^{11}} = \left(-\frac{1}{13ax^{11}} + \frac{4b}{143a^2x^{10}} - \frac{8b^2}{1287a^3x^9}\right)2X^4\sqrt{X}$$

TAB. LXI.

$$\int x^m dx (ax+bx^2)^{\frac{9}{2}}, \quad \int \frac{dx(ax+bx^2)^{\frac{9}{2}}}{x^m}$$

$$ax + bx^2 = X$$

$$\int dx X^{\frac{9}{2}} = \left(\frac{X^4}{b} - \frac{9a^2X^3}{32b^2} + \frac{21a^4X^2}{256b^3} - \frac{105a^6X}{4096b^4} + \frac{315a^8}{32768b^5}\right)\frac{2bx+a}{20}\sqrt{X} - \frac{63a^{10}}{262144b^5}\int\frac{dx}{\sqrt{X}}$$

$$\int x dx X^{\frac{9}{2}} = \frac{X^5\sqrt{X}}{11b} - \frac{a}{2b}\int dx X^{\frac{9}{2}}$$

$$\int x^2 dx X^{\frac{9}{2}} = \left(\frac{x}{12b} - \frac{13a}{264b^2}\right)X^5\sqrt{X} + \frac{13a^2}{48b^2}\int dx X^{\frac{9}{2}}$$

$$\int x^3 dx X^{\frac{9}{2}} = \left(\frac{x^2}{13b} - \frac{5ax}{104b^2} + \frac{5a^2}{176b^3}\right)X^5\sqrt{X} - \frac{5a^3}{32b^3}\int dx X^{\frac{9}{2}}$$

$$\int x^4 dx X^{\frac{9}{2}} = \frac{x^3X^5\sqrt{X}}{14b} - \frac{17a}{28b}\int x^3 dx X^{\frac{9}{2}}$$

$$\int x^5 dx X^{\frac{9}{2}} = \left(\frac{x^4}{15b} - \frac{19ax^3}{420b^2}\right)X^5\sqrt{X} + \frac{323a^2}{840b^2}\int x^3 dx X^{\frac{9}{2}}$$

$$\int x^6 dx X^{\frac{9}{2}} = \left(\frac{x^5}{16b} - \frac{7ax^4}{160b^2} + \frac{19a^2x^3}{640b^3}\right)X^5\sqrt{X} - \frac{323a^3}{1280b^3}\int x^3 dx X^{\frac{9}{2}}$$

$$\int\frac{dx X^{\frac{9}{2}}}{x} = \frac{X^4\sqrt{X}}{9} + \frac{a}{2}\int dx X^{\frac{7}{2}}$$

$$\int\frac{dx X^{\frac{9}{2}}}{x^2} = \left(\frac{X^4}{8x} + \frac{9aX^3}{112}\right)\sqrt{X} + \frac{9a^2}{32}\int dx X^{\frac{5}{2}}$$

$$\int\frac{dx X^{\frac{9}{2}}}{x^3} = \left(\frac{X^4}{7x^2} + \frac{3aX^3}{28x} + \frac{3a^2X^2}{40}\right)\sqrt{X} + \frac{3a^3}{16}\int dx X^{\frac{3}{2}}$$

$$\int\frac{dx X^{\frac{9}{2}}}{x^4} = \left(\frac{X^4}{6x^3} + \frac{3aX^3}{20x^2} + \frac{21a^2X^2}{160x} + \frac{7a^3X}{64}\right)\sqrt{X} + \frac{21a^4}{128}\int dx\sqrt{X}$$

$$\int\frac{dx X^{\frac{9}{2}}}{x^5} = \left(\frac{X^4}{5x^4} + \frac{9aX^3}{40x^3} + \frac{21a^2X^2}{80x^2} + \frac{21a^3X}{64x} + \frac{63a^4}{128}\right)\sqrt{X} + \frac{63a^5}{256}\int\frac{dx}{\sqrt{X}}$$

TAB. LXII.

$$\int \frac{dx}{(a+bx+cx^2)^{\frac{n}{2}}}$$

$$a + bx + cx = X,\quad 4ac - b^2 = k$$

$$\int \frac{dx}{X^{\frac{1}{2}}} = \int \frac{dx}{\sqrt{X}}$$ [see the following page.]

$$\int \frac{dx}{X^{\frac{3}{2}}} = \frac{2(2cx+b)}{k\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{5}{2}}} = \left(\frac{1}{3kX} + \frac{8c}{3k^2}\right)\frac{2(2cx+b)}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{7}{2}}} = \left(\frac{1}{5kX^2} + \frac{4^2c}{15k^2X} + \frac{2\cdot4^3c^2}{15k^3}\right)\frac{2(2cx+b)}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{9}{2}}} = \left(\frac{1}{7kX^3} + \frac{6\cdot4c}{35k^2X^2} + \frac{2\cdot4^3c^2}{35k^3X} + \frac{4^5c^3}{35k^4}\right)\frac{2(2cx+b)}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{11}{2}}} = \left(\frac{1}{9kX^4} + \frac{2\cdot4^2c}{63k^2X^3} + \frac{4^4c^2}{105k^3X^2} + \frac{4^6c^3}{315k^4X} + \frac{2\cdot4^7c^4}{315k^5}\right)\frac{2(2cx+b)}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{13}{2}}} = \left(\frac{1}{11kX^5} + \frac{10\cdot4c}{99k^2X^4} + \frac{5\cdot4^4c^2}{693k^3X^3} + \frac{2\cdot4^5c^3}{231k^4X^2} + \frac{2\cdot4^7c^4}{693k^5X} + \frac{4^9c^5}{693k^6}\right)\frac{2(2cx+b)}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{15}{2}}} = \left(\frac{1}{13kX^6} + \frac{3\cdot4^2c}{143k^2X^5} + \frac{10\cdot4^3c^2}{429k^3X^4} + \frac{5\cdot4^6c^3}{3003k^4X^3} + \frac{2\cdot4^7c^4}{1001k^5X^2} + \frac{2\cdot4^9c^5}{3003k^6X} + \frac{4^{11}c^6}{3003k^7}\right)\frac{2(2cx+b)}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{17}{2}}} = \left(\frac{1}{15kX^7} + \frac{14\cdot4c}{195k^2X^6} + \frac{14\cdot4^3c^2}{715k^3X^5} + \frac{7\cdot4^5c^3}{1287k^4X^4} + \frac{2\cdot4^7c^4}{1287k^5X^3} + \frac{4^9c^5}{2145k^6X^2} + \frac{4^{11}c^6}{6435k^7X} + \frac{2\cdot4^{12}c^7}{6435k^8}\right)\frac{2(2cx+b)}{\sqrt{X}}$$

$$\int \frac{dx}{X^{\frac{19}{2}}} = \frac{2(2cx+b)}{17kX^8\sqrt{X}} - \frac{64c}{17k}\int \frac{dx}{X^{\frac{17}{2}}}$$

Note on the preceding Table.

In general

$$\int \frac{dx}{\sqrt{(a+bx+cx^2)}} = \frac{1}{\sqrt{c}} \log\left[2cx+b+2\sqrt{c}.\sqrt{(a+bx+cx^2)}\right]+\text{const.}$$

or

$$\int \frac{dx}{\sqrt{(a+bx+cx^2)}} = \frac{-1}{\sqrt{-c}} \text{ arc sin } \frac{2cx+b}{\sqrt{(b^2-4ac)}} + \text{const.}$$

The first form is real, when c is positive; the second, when c is negative. Hence it follows that:

$$\text{I. } \int \frac{dx}{\sqrt{X}} = \int \frac{dx}{\sqrt{(a+bx+cx^2)}} = \pm \frac{1}{\sqrt{c}} \log\left(2cx+b \pm 2\sqrt{c}.\sqrt{X}\right)$$

and when this integral vanishes for $x=0$,

$$\int \frac{dx}{\sqrt{X}} = \pm \frac{1}{\sqrt{c}} \log \frac{2cx+b \pm 2\sqrt{c}\,.\sqrt{X}}{b \pm 2\sqrt{ac}}$$

The upper signs must here be taken together; so likewise the lower signs.

$$\text{II. } \int \frac{dx}{\sqrt{X}} = \int \frac{dx}{\sqrt{(a+bx-cx^2)}} = \frac{1}{\sqrt{c}} \text{ arc sin } \frac{2cx-b}{\sqrt{(b^2+4ac)}}$$

$$= \frac{1}{\sqrt{c}} \text{ arc cos } \frac{2\sqrt{cX}}{\sqrt{(b^2+4ac)}} = \frac{1}{\sqrt{c}} \text{ arc tang } \frac{2cx-b}{2\sqrt{cX}}$$

$$= \frac{1}{\sqrt{c}} \text{ arc cot } \frac{2\sqrt{cX}}{2cx-b} = \frac{1}{\sqrt{c}} \text{ arc sec } \frac{\sqrt{(b^2+4ac)}}{2\sqrt{cX}}$$

$$= \frac{1}{\sqrt{c}} \text{ arc cosec } \frac{\sqrt{(b^2+4ac)}}{2cx-b} = \frac{1}{2\sqrt{c}} \text{ arc sin vers } \frac{2(2cx-b)^2}{b^2+4ac}$$

and these circular arcs all vanish when $x = \frac{b}{2c}$. If they vanish when $x=0$, we have

$$\int \frac{dx}{\sqrt{X}} = \int \frac{dx}{\sqrt{(a+bx-cx^2)}} = \frac{1}{\sqrt{c}} \text{ arc sin } \frac{2(2cx-b)\sqrt{ac}+2b\sqrt{cX}}{b^2+4ac}$$

$$= \frac{1}{\sqrt{c}} \text{ arc cos } \frac{4c\sqrt{aX}-b(2cx-b)}{b^2+4ac} = \text{\&c.}$$

In practice it may be better not to connect the constants with the arcs.

TAB. LXIII.

$$\int dx\,(a + bx + cx^2)^{\frac{n}{2}}$$

$$a + bx + cx^2 = X,\quad 4ac - b^2 = k$$

$$\int dx X^{\frac{1}{2}} = \frac{(2cx+b)\sqrt{X}}{4c} + \frac{k}{8c}\int\frac{dx}{\sqrt{X}}$$

$$\int dx X^{\frac{3}{2}} = \left(\frac{X}{8c} + \frac{3k}{64c^2}\right)(2cx+b)\sqrt{X} + \frac{3k^2}{128c^2}\int\frac{dx}{\sqrt{X}}$$

$$\int dx X^{\frac{5}{2}} = \left(\frac{X^2}{12c} + \frac{5kX}{192c^2} + \frac{5k^2}{512c^3}\right)(2cx+b)\sqrt{X} + \frac{5k^3}{1024c^3}\int\frac{dx}{\sqrt{X}}$$

$$\int dx X^{\frac{7}{2}} = \left(\frac{X^3}{16c} + \frac{7kX^2}{6\,.\,4^3c^2} + \frac{35k^2X}{6\,.\,4^5c^3} + \frac{35k^3}{4^7c^4}\right)(2cx+b)\sqrt{X} + \frac{35k^4}{2\,.\,4^7c^4}\int\frac{dx}{\sqrt{X}}$$

$$\int dx X^{\frac{9}{2}} = \left(\frac{X^4}{20c} + \frac{9kX^3}{10\,.\,4^3c^2} + \frac{21k^2X^2}{5\,.\,4^5c^3} + \frac{21k^3X}{4^7c^4} + \frac{63k^4}{2\,.\,4^8c^5}\right)\times (2cx+b)\sqrt{X} + \frac{63k^5}{4^9c^5}\int\frac{dx}{\sqrt{X}}$$

$$\int dx X^{\frac{11}{2}} = \left(\frac{X^5}{24c} + \frac{11kX^4}{15\,.\,4^3c^2} + \frac{33k^2X^3}{10\,.\,4^5c^3} + \frac{77k^3X^2}{5\,.\,4^7c^4} + \frac{77k^4X}{4^9c^5} + \frac{231k^5}{2\,.\,4^{10}c^6}\right)(2cx+b)\sqrt{X} + \frac{231k^6}{4^{11}c^6}\int\frac{dx}{\sqrt{X}}$$

$$\int dx X^{\frac{13}{2}} = \left(\frac{X^6}{28c} + \frac{13kX^5}{21\,.\,4^3c^2} + \frac{143k^2X^4}{210\,.\,4^4c^3} + \frac{429k^3X^3}{35\,.\,4^7c^4} + \frac{143k^4X^2}{10\,.\,4^8c^5} + \frac{143k^5X}{2\,.\,4^{10}c^6} + \frac{429k^6}{4^{12}c^7}\right)(2cx+b)\sqrt{X} + \frac{429k^7}{2\,.\,4^{12}c^7}\int\frac{dx}{\sqrt{X}}$$

$$\int dx X^{\frac{15}{2}} = \left(\frac{X^7}{32c} + \frac{15kX^6}{7\,.\,4^4c^2} + \frac{65k^2X^5}{7\,.\,4^6c^3} + \frac{143k^3X^4}{14\,.\,4^7c^4} + \frac{1287k^4X^3}{7\,.\,4^{10}c^5} + \frac{429k^5X^2}{2\,.\,4^{11}c^6} + \frac{2145k^6X}{2\,.\,4^{13}c^7} + \frac{6435k^7}{4^{15}c^8}\right)(2cx+b)\sqrt{X} + \frac{6435k^8}{2\,.\,4^{15}c^8}\int\frac{dx}{\sqrt{X}}$$

TAB. LXIV.

$$\int \frac{x^m dx}{\sqrt{(a+bx+cx^2)}}$$

$$a + bx + cx^2 = X$$

$$\int \frac{dx}{\sqrt{X}} = \int \frac{dx}{\sqrt{X}} \text{ (page 160)}$$

$$\int \frac{x dx}{\sqrt{X}} = \frac{\sqrt{X}}{c} - \frac{b}{2c}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^2 dx}{\sqrt{X}} = \left(\frac{x}{2c} - \frac{3b}{4c^2}\right)\sqrt{X} + \left(\frac{3b^2}{8c^2} - \frac{a}{2c}\right)\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^3 dx}{\sqrt{X}} = \left(\frac{x^2}{3c} - \frac{5bx}{12c^2} + \frac{5b^2}{3c^3} - \frac{2a}{3c^2}\right)\sqrt{X} - \left(\frac{5b^3}{16c^3} - \frac{3ab}{4c^2}\right)\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^4 dx}{\sqrt{X}} = \left[\frac{x^3}{4c} - \frac{7bx^2}{24c^2} + \left(\frac{35b^2}{96c^3} - \frac{3a}{8c^2}\right)x - \frac{35b^3}{64c^4} + \frac{55ab}{48c^3}\right]\sqrt{X} + \left(\frac{35b^4}{128c^4} - \frac{15ab^2}{16c^3} + \frac{3a^2}{8c^2}\right)\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^5 dx}{\sqrt{X}} = \frac{x^4\sqrt{X}}{5c} - \frac{4a}{5c}\int \frac{x^3 dx}{\sqrt{X}} - \frac{9b}{10c}\int \frac{x^4 dx}{\sqrt{X}}$$

$$\int \frac{x^6 dx}{\sqrt{X}} = \left(\frac{x^5}{6c} - \frac{11bx^4}{60c^2}\right)\sqrt{X} + \frac{11ab}{15c^2}\int \frac{x^3 dx}{\sqrt{X}} + \left(\frac{33b^2}{40c^2} - \frac{5a}{6c}\right)\int \frac{x^4 dx}{\sqrt{X}}$$

$$\int \frac{x^7 dx}{\sqrt{X}} = \left[\frac{x^6}{7c} - \frac{13bx^5}{84c^2} + \left(\frac{143b^2}{850c^3} - \frac{6a}{35c^2}\right)x^4\right]\sqrt{X} - \left(\frac{143ab^2}{210c^3} - \frac{24a^2}{35c^2}\right)\int \frac{x^3 dx}{\sqrt{X}} - \left(\frac{429b^3}{56c^3} - \frac{649ab}{420c^2}\right)\int \frac{x^4 dx}{\sqrt{X}}$$

$$\int \frac{x^8 dx}{\sqrt{X}} = \left[\frac{x^7}{8c} - \frac{15bx^6}{112c^2} + \left(\frac{65b^2}{448c^3} - \frac{7a}{48c^2}\right)x^5 - \left(\frac{143b^3}{896c^4} - \frac{1079ab}{3360c^3}\right)x^4\right]\sqrt{X} + \left(\frac{143ab^3}{224c^4} - \frac{1079a^2b}{840c^3}\right)\int \frac{x^3 dx}{\sqrt{X}} + \left(\frac{6435b^4}{896c^4} - \frac{2431ab^2}{1120c^3} + \frac{35a^2}{48c^2}\right)\int \frac{x^4 dx}{\sqrt{X}}$$

$$\int \frac{x^9 dx}{\sqrt{X}} = \frac{x^8\sqrt{X}}{9c} - \frac{8a}{9c}\int \frac{x^7 dx}{\sqrt{X}} - \frac{17b}{18c}\int \frac{x^8 dx}{\sqrt{X}}$$

TAB. LXV.

$$\int \frac{dx}{x^m \sqrt{(a+bx+cx^2)}}$$

$$a + bx + cx^2 = X$$

$$\int \frac{dx}{x\sqrt{X}} = \int \frac{dx}{x\sqrt{X}} \text{ (see the following page)}$$

$$\int \frac{dx}{x^2\sqrt{X}} = -\frac{\sqrt{X}}{ax} - \frac{b}{2a}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^3\sqrt{X}} = \left(-\frac{1}{2ax^2} + \frac{3b}{4a^2x}\right)\sqrt{X} + \left(\frac{3b^2}{8a^2} - \frac{c}{2a}\right)\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^4\sqrt{X}} = \left[-\frac{1}{3ax^3} + \frac{5b}{12a^2x^2} - \left(\frac{5b^2}{8a^3} - \frac{2c}{3a^2}\right)\frac{1}{x}\right]\sqrt{X}$$
$$- \left(\frac{5b^3}{16a^3} - \frac{3bc}{4a^2}\right)\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^5\sqrt{X}} = \left[-\frac{1}{4ax^4} + \frac{7b}{24a^2x^3} - \left(\frac{35b^2}{96a^3} - \frac{3c}{8a^2}\right)\frac{1}{x^2} + \left(\frac{35b^3}{64a^4} - \frac{55bc}{48a^3}\right)\frac{1}{x}\right]\sqrt{X}$$
$$+ \left(\frac{35b^4}{128a^4} - \frac{15b^2c}{16a^3} + \frac{3c^2}{8a^2}\right)\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^6\sqrt{X}} = -\frac{\sqrt{X}}{5ax^5} - \frac{9b}{10a}\int \frac{dx}{x^5\sqrt{X}} - \frac{4c}{5a}\int \frac{dx}{x^4\sqrt{X}}$$

$$\int \frac{dx}{x^7\sqrt{X}} = \left(-\frac{1}{6ax^6} + \frac{11b}{60a^2x^5}\right)\sqrt{X} + \left(\frac{33b^2}{40a^2} - \frac{5c}{6a}\right)\int \frac{dx}{x^5\sqrt{X}}$$
$$+ \frac{11bc}{15a^2}\int \frac{dx}{x^4\sqrt{X}}$$

$$\int \frac{dx}{x^8\sqrt{X}} = \left[-\frac{1}{7ax^7} + \frac{13b}{84a^2x^6} - \left(\frac{143b^2}{840a^3} - \frac{6c}{35a^2}\right)\frac{1}{x^5}\right]\sqrt{X}$$
$$-\left(\frac{429b^3}{560a^3} - \frac{649bc}{420a^2}\right)\int \frac{dx}{x^5\sqrt{X}} - \left(\frac{143b^2c}{210a^3} - \frac{24c^2}{35a^2}\right)\int \frac{dx}{x^4\sqrt{X}}$$

$$\int \frac{dx}{x^9\sqrt{X}} = \left[-\frac{1}{8ax^8} + \frac{15b}{112a^2x^7} - \left(\frac{65b^2}{448a^3} - \frac{7c}{48a^2}\right)\frac{1}{x^6} + \left(\frac{143b^3}{896a^4}\right.\right.$$
$$\left.\left. - \frac{1079bc}{3360a^3}\right)\frac{1}{x^5}\right]\sqrt{X} + \left(\frac{1287b^4}{1792a^4} - \frac{2431b^2c}{1120a^3} + \frac{35c^2}{48a^2}\right)\int \frac{dx}{x^5\sqrt{X}}$$
$$+ \left(\frac{143b^3c}{224a^4} - \frac{1079bc^2}{840a^3}\right)\int \frac{dx}{x^4\sqrt{X}}$$

$$\int \frac{dx}{x^{10}\sqrt{X}} = \frac{\sqrt{X}}{9ax^9} - \frac{17b}{18a}\int \frac{dx}{x^9\sqrt{X}} - \frac{8c}{9a}\int \frac{dx}{x^8\sqrt{X}}$$

Note on the preceding Table.

In general

$$\int\frac{dx}{x\sqrt{X}}=\frac{1}{\sqrt{a}}\log\frac{2a+bx-2\sqrt{a}\,.\sqrt{X}}{x}+\text{const.}$$

$$\text{or}\int\frac{dx}{x\sqrt{X}}=\frac{1}{\sqrt{-a}}\text{ arc tang }\frac{2a+bx}{2\sqrt{-a}\,.\sqrt{X}}+\text{const.}$$

The first form is real, when a is positive; the second when a is negative. Hence it follows that:

$$\text{I. }\int\frac{dx}{x\sqrt{X}}=\int\frac{dx}{x\sqrt{(a+bx+cx^2)}}$$

$$=\pm\frac{1}{\sqrt{a}}\log\frac{2a+bx\mp2\sqrt{aX}}{x}+\text{const.}$$

$$=\pm\frac{1}{\sqrt{a}}\log\frac{2a+bx\mp2\sqrt{aX}}{kx}$$

The k in the latter expression denotes an arbitrary constant. The upper signs are taken together, as also the lower signs. When $x=0$, the integral does not vanish.

$$\text{II. }\int\frac{dx}{x\sqrt{X}}=\int\frac{dx}{x\sqrt{(-a+bx+cx^2)}}=\frac{1}{\sqrt{a}}\text{ arc tang }\frac{bx-2a}{2\sqrt{aX}}$$

$$=\frac{1}{\sqrt{a}}\text{ arc cot }\frac{2\sqrt{aX}}{bx-2a}=\frac{1}{\sqrt{a}}\text{ arc sec }\frac{x\sqrt{(b^2+4ac)}}{2\sqrt{aX}}$$

$$=\frac{1}{\sqrt{a}}\text{ arc cosec }\frac{x\sqrt{(b^2+4ac)}}{bx-2a}=\frac{1}{\sqrt{a}}\text{ arc sin }\frac{bx-2a}{x\sqrt{(b^2+4ac)}}$$

$$=\frac{1}{\sqrt{a}}\text{ arc cos }\frac{2\sqrt{aX}}{x\sqrt{(b^2+4ac)}}=\frac{1}{2\sqrt{a}}\text{ arc sin vers }\frac{2\,(bx-2a)^2}{(b^2+4ac)x^2}.$$

These circular arcs all vanish when $x=\frac{2a}{b}$. When $x=0$, they do not vanish.

TAB. LXVI.

$$\int \frac{x^m dx}{(a+bx+cx^2)^{\frac{3}{2}}}$$

$$a + bx + cx^2 = X,\ 4ac - b^2 = k$$

$$\int \frac{dx}{X^{\frac{3}{2}}} = \frac{2(2cx+b)}{k\sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{3}{2}}} = -\frac{2(2a+bx)}{k\sqrt{X}}$$

$$\int \frac{x^2 dx}{X^{\frac{3}{2}}} = -\frac{(4ac-2b^2)x-2ab}{ck\sqrt{X}} + \frac{1}{c}\int \frac{dx}{\sqrt{X}}$$

$$\int \frac{x^3 dx}{X^{\frac{3}{2}}} = \frac{x^2}{c\sqrt{X}} - \frac{2a}{c}\int \frac{x dx}{X^{\frac{3}{2}}} - \frac{3b}{2c}\int \frac{x^2 dx}{X^{\frac{3}{2}}}$$

$$\int \frac{x^4 dx}{X^{\frac{3}{2}}} = \left(\frac{x^3}{2c} - \frac{5bx^2}{4c^2}\right)\frac{1}{\sqrt{X}} + \frac{5ab}{2c^2}\int \frac{x dx}{X^{\frac{3}{2}}} + \left(\frac{15b^2}{8c^2} - \frac{3a}{2c}\right)\int \frac{x^2 dx}{X^{\frac{3}{2}}}$$

$$\int \frac{x^5 dx}{X^{\frac{3}{2}}} = \left[\frac{x^4}{3c} - \frac{7bx^3}{12c^2} + \left(\frac{35b^2}{24c^3} - \frac{4a}{3c^2}\right)x^2\right]\frac{1}{\sqrt{X}} - \left(\frac{35ab^2}{12c^3} - \frac{8a^2}{3c^2}\right)\int \frac{x dx}{X^{\frac{3}{2}}}$$
$$- \left(\frac{35b^3}{16c^3} - \frac{15ab}{4c^2}\right)\int \frac{x^2 dx}{X^{\frac{3}{2}}}$$

$$\int \frac{x^6 dx}{X^{\frac{3}{2}}} = \left[\frac{x^5}{4c} - \frac{3bx^4}{8c^2} + \left(\frac{21b^2}{32c^3} - \frac{5a}{8c^2}\right)x^3 - \left(\frac{105b^3}{64c^4} - \frac{49ab}{16c^3}\right)x^2\right]\frac{1}{\sqrt{X}}$$
$$+ \left(\frac{105ab^3}{32c^4} - \frac{49a^2b}{8c^3}\right)\int \frac{x dx}{X^{\frac{3}{2}}} + \left(\frac{315b^4}{128c^4} - \frac{105ab^2}{16c^3} + \frac{15a^2}{8c^2}\right)\int \frac{x^2 dx}{X^{\frac{3}{2}}}$$

$$\int \frac{x^7 dx}{X^{\frac{3}{2}}} = \frac{x^6}{5c\sqrt{X}} - \frac{6a}{5c}\int \frac{x^5 dx}{X^{\frac{3}{2}}} - \frac{11b}{10c}\int \frac{x^6 dx}{X^{\frac{3}{2}}}$$

$$\int \frac{x^8 dx}{X^{\frac{3}{2}}} = \left(\frac{x^7}{6c} - \frac{13bx^6}{60c^2}\right)\frac{1}{\sqrt{X}} + \frac{13ab}{10c^2}\int \frac{x^5 dx}{X^{\frac{3}{2}}} + \left(\frac{143b^2}{120c^2} - \frac{7a}{6c}\right)\int \frac{x^6 dx}{X^{\frac{3}{2}}}$$

$$\int \frac{x^9 dx}{X^{\frac{3}{2}}} = \left[\frac{x^8}{7c} - \frac{5bx^7}{28c^2} + \left(\frac{13b^2}{56c^3} - \frac{8a}{35c^2}\right)x^6\right]\frac{1}{\sqrt{X}} - \left(\frac{39ab^2}{28c^3} - \frac{48a^2}{35c^2}\right)\int \frac{x^5 dx}{X^{\frac{3}{2}}}$$
$$- \left(\frac{143b^3}{112c^3} - \frac{351ab}{140c^2}\right)\int \frac{x^6 dx}{X^{\frac{3}{2}}}$$

TAB. LXVII.

$$\int \frac{dx}{x^m(a+bx+cx^2)^{\frac{3}{2}}}$$

$$a + bx + cx^2 = X$$

$$\int \frac{dx}{xX^{\frac{3}{2}}} = \frac{1}{a\sqrt{X}} - \frac{b}{2a}\int\frac{dx}{X^{\frac{3}{2}}} + \frac{1}{a}\int\frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{3}{2}}} = \left(-\frac{1}{ax} - \frac{3b}{2a^2}\right)\frac{1}{\sqrt{X}} + \left(\frac{3b^2}{4a^2} - \frac{2c}{a}\right)\int\frac{dx}{X^{\frac{3}{2}}} - \frac{3b}{2a^2}\int\frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^3X^{\frac{3}{2}}} = \left(-\frac{1}{2ax^2} + \frac{5b}{4a^2x} + \frac{15b^2}{8a^3} - \frac{3c}{2a^2}\right)\frac{1}{\sqrt{X}} - \left(\frac{15b^3}{16a^3} - \frac{13bc}{4a^2}\right)\int\frac{dx}{X^{\frac{3}{2}}}$$
$$+ \left(\frac{15b^2}{8a^3} - \frac{3c}{2a^2}\right)\int\frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^4X^{\frac{3}{2}}} = \left[-\frac{1}{3ax^3} + \frac{7b}{12a^2x^2} - \left(\frac{35b^2}{24a^3} - \frac{4c}{3a^2}\right)\frac{1}{x} - \left(\frac{35b^3}{16a^4} - \frac{15bc}{4a^3}\right)\right]\frac{1}{\sqrt{X}}$$
$$+ \left(\frac{35b^4}{32a^4} - \frac{115b^2c}{24a^3} + \frac{8c^2}{3a^2}\right)\int\frac{dx}{X^{\frac{3}{2}}} - \left(\frac{35b^3}{16a^4} - \frac{15bc}{4a^3}\right)\int\frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^5X^{\frac{3}{2}}} = -\frac{1}{4ax^4\sqrt{X}} - \frac{9b}{8a}\int\frac{dx}{x^4X^{\frac{3}{2}}} - \frac{5c}{4a}\int\frac{dx}{x^3X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^6X^{\frac{3}{2}}} = \left(-\frac{1}{5ax^5} + \frac{11b}{40a^2x^4}\right)\frac{1}{\sqrt{X}} + \left(\frac{99b^2}{80a^2} - \frac{6c}{5a}\right)\int\frac{dx}{x^4X^{\frac{3}{2}}}$$
$$+ \frac{11bc}{8a^2}\int\frac{dx}{x^3X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^7X^{\frac{3}{2}}} = \left[-\frac{1}{6ax^6} + \frac{13b}{60a^2x^5} - \left(\frac{143b^2}{480a^3} - \frac{7c}{24a^2}\right)\frac{1}{x^4}\right]\frac{1}{\sqrt{X}}$$
$$- \left(\frac{429b^3}{320a^3} - \frac{209bc}{80a^2}\right)\int\frac{dx}{x^4X^{\frac{3}{2}}} - \left(\frac{143b^2c}{96a^3} - \frac{35c^2}{24a^2}\right)\int\frac{dx}{x^4X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^8X^{\frac{3}{2}}} = -\frac{1}{7ax^7\sqrt{X}} - \frac{15b}{14a}\int\frac{dx}{x^7X^{\frac{3}{2}}} - \frac{8c}{7a}\int\frac{dx}{x^6X^{\frac{3}{2}}}$$

$$\int \frac{dx}{x^9X^{\frac{3}{2}}} = \left(-\frac{1}{8ax^8} + \frac{17b}{112a^2x^7}\right)\frac{1}{\sqrt{X}} + \left(\frac{255b^2}{224a^2} - \frac{9c}{8a}\right)\int\frac{dx}{x^7X^{\frac{3}{2}}}$$
$$+ \frac{17bc}{14a^2}\int\frac{dx}{x^6X^{\frac{3}{2}}}$$

TAB. LXVIII.

$$\int \frac{x^m dx}{(a+bx+cx^2)^{\frac{5}{2}}}$$

$$a + bx + cx^2 = X, \quad 4ac - b^2 = k$$

$$\int \frac{dx}{X^{\frac{5}{2}}} = \left(\frac{1}{3kX} + \frac{8c}{3k^2}\right)\frac{2(2cx+b)}{\sqrt{X}}$$

$$\int \frac{x\,dx}{X^{\frac{5}{2}}} = -\frac{1}{3cX\sqrt{X}} - \frac{b}{2c}\int \frac{dx}{X^{\frac{5}{2}}}$$

$$\int \frac{x^2 dx}{X^{\frac{5}{2}}} = \left(-\frac{x}{2c} + \frac{b}{12c^2}\right)\frac{1}{X\sqrt{X}} + \left(\frac{b^2}{8c^2} + \frac{a}{2c}\right)\int \frac{dx}{X^{\frac{5}{2}}}$$

$$\int \frac{x^3 dx}{X^{\frac{5}{2}}} = \left(-\frac{x^2}{c} - \frac{bx}{4c^2} + \frac{b^2}{24c^3} - \frac{2a}{3c^2}\right)\frac{1}{X\sqrt{X}} + \left(\frac{b^3}{16c^3} - \frac{3ab}{4c^2}\right)\int \frac{dx}{X^{\frac{5}{2}}}$$

$$\int \frac{x^4 dx}{X^{\frac{5}{2}}} = \frac{1}{c}\int \frac{x^2 dx}{X^{\frac{3}{2}}} - \frac{a}{c}\int \frac{x^2 dx}{X^{\frac{5}{2}}} - \frac{b}{c}\int \frac{x^3 dx}{X^{\frac{5}{2}}}$$

$$\int \frac{x^5 dx}{X^{\frac{5}{2}}} = \frac{x^4}{cX\sqrt{X}} - \frac{4a}{c}\int \frac{x^3 dx}{X^{\frac{5}{2}}} - \frac{5b}{2c}\int \frac{x^4 dx}{X^{\frac{5}{2}}}$$

$$\int \frac{x^6 dx}{X^{\frac{5}{2}}} = \left(\frac{x^5}{2c} - \frac{7bx^4}{4c^2}\right)\frac{1}{X\sqrt{X}} + \frac{7ab}{c^2}\int \frac{x^3 dx}{X^{\frac{5}{2}}} + \left(\frac{35b^2}{8c^2} - \frac{5a}{2c}\right)\int \frac{x^4 dx}{X^{\frac{5}{2}}}$$

$$\int \frac{x^7 dx}{X^{\frac{5}{2}}} = \left[\frac{x^6}{3c} - \frac{3bx^5}{4c^2} + \left(\frac{21b^2}{8c^3} - \frac{2a}{c^2}\right)x^4\right]\frac{1}{X\sqrt{X}} - \left(\frac{21ab^2}{2c^3} - \frac{8a^2}{c^2}\right)\int \frac{x^3 dx}{X^{\frac{5}{2}}}$$
$$- \left(\frac{105b^3}{16c^3} - \frac{35ab}{4c^2}\right)\int \frac{x^4 dx}{X^{\frac{5}{2}}}$$

$$\int \frac{x^8 dx}{X^{\frac{5}{2}}} = \frac{x^7}{4cX\sqrt{X}} - \frac{7a}{4c}\int \frac{x^6 dx}{X^{\frac{5}{2}}} - \frac{11b}{8c}\int \frac{x^7 dx}{X^{\frac{5}{2}}}$$

$$\int \frac{x^9 dx}{X^{\frac{5}{2}}} = \left(\frac{x^8}{5c} - \frac{13bx^7}{40c^2}\right)\frac{1}{X\sqrt{X}} + \frac{91ab}{40c^2}\int \frac{x^6 dx}{X^{\frac{5}{2}}} + \left(\frac{143b^2}{80c^2} - \frac{8a}{5c}\right)\int \frac{x^7 dx}{X^{\frac{5}{2}}}$$

TAB. LXIX.

$$\int \frac{dx}{x^m(a+bx+cx^2)^{\frac{5}{2}}}$$

$$a + bx + cx^2 = X$$

$$\int \frac{dx}{xX^{\frac{5}{2}}} = \left(\frac{1}{3aX} + \frac{1}{a^2}\right)\frac{1}{\sqrt{X}} - \frac{b}{2a}\int \frac{dx}{X^{\frac{5}{2}}} - \frac{b}{2a^2}\int \frac{dx}{X^{\frac{3}{2}}} + \frac{1}{a^2}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{5}{2}}} = -\frac{1}{axX\sqrt{X}} - \frac{5b}{2a}\int \frac{dx}{xX^{\frac{5}{2}}} - \frac{4c}{a}\int \frac{dx}{X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{5}{2}}} = \left(-\frac{1}{2ax^2} + \frac{7b}{4a^2x}\right)\frac{1}{X\sqrt{X}} + \left(\frac{35b^2}{8a^2} - \frac{5c}{2a}\right)\int \frac{dx}{xX^{\frac{5}{2}}} + \frac{7bc}{a^2}\int \frac{dx}{X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^4X^{\frac{5}{2}}} = \left[-\frac{1}{3ax^3} + \frac{3b}{4a^2x^2} - \left(\frac{21b^2}{8a^3} - \frac{2c}{a^2}\right)\frac{1}{x}\right]\frac{1}{X\sqrt{X}} - \left(\frac{105b^3}{16a^3} - \frac{35bc}{4a^2}\right)\int \frac{dx}{xX^{\frac{5}{2}}} - \left(\frac{21b^2c}{2a^3} - \frac{8c^2}{a^2}\right)\int \frac{dx}{X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{5}{2}}} = -\frac{1}{4ax^4X\sqrt{X}} - \frac{11b}{8a}\int \frac{dx}{x^4X^{\frac{5}{2}}} - \frac{7c}{4a}\int \frac{dx}{x^3X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^6X^{\frac{5}{2}}} = \left(-\frac{1}{5ax^5} + \frac{13b}{40a^2x^4}\right)\frac{1}{X\sqrt{X}} + \left(\frac{143b^2}{80a^2} - \frac{8c}{5a}\right)\int \frac{dx}{x^4X^{\frac{5}{2}}} + \frac{91bc}{40a^2}\int \frac{dx}{x^3X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^7X^{\frac{5}{2}}} = \left[-\frac{1}{6ax^6} + \frac{b}{4a^2x^5} - \left(\frac{13b^2}{32a^3} - \frac{3c}{8a^4}\right)\frac{1}{x^4}\right]\frac{1}{X\sqrt{X}} - \left(\frac{143b^3}{64a^3} - \frac{65bc}{16a^2}\right)\int \frac{dx}{x^4X^{\frac{5}{2}}} - \left(\frac{91b^2c}{64a^3} - \frac{21c^2}{8a^2}\right)\int \frac{dx}{x^3X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^8X^{\frac{5}{2}}} = -\frac{1}{7ax^7X\sqrt{X}} - \frac{17b}{14a}\int \frac{dx}{x^7X^{\frac{5}{2}}} - \frac{10c}{7a}\int \frac{dx}{x^6X^{\frac{5}{2}}}$$

$$\int \frac{dx}{x^9X^{\frac{5}{2}}} = \left(-\frac{1}{8ax^8} + \frac{19b}{112a^2x^7}\right)\frac{1}{X\sqrt{X}} + \left(\frac{323b^2}{224a^2} - \frac{11c}{8a}\right)\int \frac{dx}{x^7X^{\frac{5}{2}}} + \frac{95bc}{56a^2}\int \frac{dx}{x^6X^{\frac{5}{2}}}$$

TAB. LXX.

$$\int \frac{x^m dx}{(a+bx+cx^2)^{\frac{7}{2}}}$$

$$a + bx + cx^2 = X,\ 4ac - b^2 = k$$

$$\int \frac{dx}{X^{\frac{7}{2}}} = \left(\frac{1}{5kX^2} + \frac{16c}{15k^2X} + \frac{128c^2}{15k^3}\right)\frac{2(2cx+b)}{\sqrt{X}}$$

$$\int \frac{x dx}{X^{\frac{7}{2}}} = -\frac{1}{5cX^2\sqrt{X}} - \frac{b}{2c}\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{x^2 dx}{X^{\frac{7}{2}}} = \left(-\frac{x}{4c} + \frac{3b}{40c^2}\right)\frac{1}{X^2\sqrt{X}} + \left(\frac{3b^2}{16c^2} + \frac{a}{4c}\right)\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{x^3 dx}{X^{\frac{7}{2}}} = \left(-\frac{x^2}{3c} + \frac{bx}{24c^2} - \frac{b^2}{80c^3} - \frac{2a}{15c^2}\right)\frac{1}{X^2\sqrt{X}} - \left(\frac{b^3}{32c^3} + \frac{3ab}{8c^2}\right)\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{x^4 dx}{X^{\frac{7}{2}}} = \left[-\frac{x^3}{2c} - \frac{bx^2}{12c^2} + \left(\frac{b^2}{96c^3} - \frac{3a}{8c^2}\right)x - \frac{b^3}{320c^4} + \frac{19ab}{240c^3}\right]\frac{1}{X^2\sqrt{X}}$$
$$- \left(\frac{b^4}{128c^4} - \frac{3ab^2}{16c^3} - \frac{3a^2}{8c^2}\right)\int \frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{x^5 dx}{X^{\frac{7}{2}}} = \left[-\frac{x^4}{c} - \frac{3bx^3}{4c^2} - \left(\frac{b^2}{8c^3} + \frac{4a}{3c^2}\right)x^2 + \left(\frac{b^3}{64c^4} - \frac{19ab}{48c^3}\right)x\right.$$
$$\left. - \frac{3b^4}{640c^5} + \frac{11ab^2}{160c^4} - \frac{8a^2}{15c^3}\right]\frac{1}{X^2\sqrt{X}} - \left(\frac{3b^5}{256c^5} - \frac{5ab^3}{32c^4} + \frac{15a^2b}{16c^3}\right)\frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{x^6 dx}{X^{\frac{7}{2}}} = \frac{1}{c}\int \frac{x^4 dx}{X^{\frac{5}{2}}} - \frac{a}{c}\int \frac{x^4 dx}{X^{\frac{7}{2}}} - \frac{b}{c}\int \frac{x^5 dx}{X^{\frac{7}{2}}}$$

$$\int \frac{x^7 dx}{X^{\frac{7}{2}}} = \frac{x^6}{cX^2\sqrt{X}} - \frac{6a}{c}\int \frac{x^5 dx}{X^{\frac{7}{2}}} - \frac{7b}{2c}\int \frac{x^6 dx}{X^{\frac{7}{2}}}$$

$$\int \frac{x^8 dx}{X^{\frac{7}{2}}} = \left(\frac{x^7}{2c} - \frac{9bx^6}{4c^2}\right)\frac{1}{X^2\sqrt{X}} + \frac{27ab}{2c^2}\int \frac{x^5 dx}{X^{\frac{7}{2}}} + \left(\frac{63b^2}{8c^2} - \frac{7a}{2c}\right)\int \frac{x^6 dx}{X^{\frac{7}{2}}}$$

$$\int \frac{x^9 dx}{X^{\frac{7}{2}}} = \left[\frac{x^8}{3c} - \frac{11bx^7}{12c^2} + \left(\frac{33b^2}{8c^3} - \frac{8a}{3c^2}\right)x^6\right]\frac{1}{X^2\sqrt{X}}$$
$$- \left(\frac{231b^3}{16c^3} - \frac{63ab}{4c^2}\right)\int \frac{x^6 dx}{X^{\frac{7}{2}}}$$

TAB. LXXI.

$$\int \frac{dx}{x^m (a+bx+cx^2)^{\frac{7}{2}}}$$

$$+ bx + cx^2 = X$$

$$\int \frac{dx}{xX^{\frac{7}{2}}} = \left(\frac{1}{5aX^2} + \frac{1}{3a^2X} + \frac{1}{a^3}\right)\frac{1}{\sqrt{X}} - \frac{b}{2a}\int\frac{dx}{X^{\frac{7}{2}}} - \frac{b}{2a^2}\int\frac{dx}{X^{\frac{5}{2}}} - \frac{b}{2a^3}\int\frac{dx}{X^{\frac{3}{2}}} + \frac{1}{a^3}\int\frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{x^2X^{\frac{7}{2}}} = -\frac{1}{axX^2\sqrt{X}} - \frac{7b}{2a}\int\frac{dx}{xX^{\frac{7}{2}}} - \frac{6c}{a}\int\frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^3X^{\frac{7}{2}}} = \left(-\frac{1}{2ax^2} + \frac{9b}{4a^2x}\right)\frac{1}{X^2\sqrt{X}} + \left(\frac{63b^2}{8a^2} - \frac{7c}{2a}\right)\left(\int\frac{dx}{xX^{\frac{7}{2}}} + \frac{27bc}{2a^2}\int\frac{dx}{X^{\frac{7}{2}}}\right.$$

$$\int \frac{dx}{x^4X^{\frac{7}{2}}} = \left[-\frac{1}{3ax^3} + \frac{11b}{12a^2x^2} - \left(\frac{33b^2}{8a^3} - \frac{8c}{3a^2}\right)\frac{1}{x}\right]\frac{1}{X^2\sqrt{X}} - \left(\frac{231b^3}{16a^3} - \frac{63bc}{4a^2}\right)\int\frac{dx}{xX^{\frac{7}{2}}} - \left(\frac{99b^2c}{4a^3} - \frac{16c^2}{a^2}\right)\int\frac{dx}{X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^5X^{\frac{7}{2}}} = -\frac{1}{4ax^4X^2\sqrt{X}} - \frac{13b}{8a}\int\frac{dx}{x^4X^{\frac{7}{2}}} - \frac{9c}{4a}\int\frac{dx}{x^3X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^6X^{\frac{7}{2}}} = \left(-\frac{1}{5ax^5} + \frac{3b}{8a^2x^4}\right)\frac{1}{X^2\sqrt{X}} + \left(\frac{39b^2}{16a^2} - \frac{2c}{a}\right)\int\frac{dx}{x^4X^{\frac{7}{2}}} + \frac{27bc}{8a^2}\int\frac{dx}{x^3X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^7X^{\frac{7}{2}}} = \left[-\frac{1}{6ax^6} + \frac{17b}{60a^2x^5} - \left(\frac{17b^2}{32a^3} - \frac{11c}{24a^2}\right)\frac{1}{x^4}\right]\frac{1}{X^2\sqrt{X}} - \left(\frac{221b^3}{64a^3} - \frac{93bc}{16a^2}\right)\int\frac{dx}{x^4X^{\frac{7}{2}}} - \left(\frac{153b^2c}{32a^3} - \frac{33c^2}{8a^2}\right)\int\frac{dx}{x^3X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^8X^{\frac{7}{2}}} = -\frac{1}{7ax^7X^2\sqrt{X}} - \frac{19b}{14a}\int\frac{dx}{x^7X^{\frac{7}{2}}} - \frac{12c}{7a}\int\frac{dx}{x^6X^{\frac{7}{2}}}$$

$$\int \frac{dx}{x^9X^{\frac{7}{2}}} = \left(-\frac{1}{8ax^8} + \frac{3b}{16a^2x^7}\right)\frac{1}{X^2\sqrt{X}} + \left(\frac{57b^2}{32a^4} - \frac{13c}{8a}\right)\int\frac{dx}{x^7X^{\frac{7}{2}}} + \frac{9bc}{4a^2}\int\frac{dx}{x^6X^{\frac{7}{2}}}$$

TAB. LXXII.

$$\int\frac{x^m dx}{(a+bx+cx^2)^{\frac{9}{2}}};\ \int\frac{dx}{x^m(a+bx+cx^2)^{\frac{9}{2}}}$$

$$a+bx+cx^2=X,\ 4ac-b^2=k$$

$$\int\frac{dx}{X^{\frac{9}{2}}}=\left(\frac{1}{7kX^3}+\frac{24c}{35k^2X^2}+\frac{128c^2}{35k^3X}+\frac{1024c^3}{35k^4}\right)\frac{2(2cx+b)}{\sqrt{X}}$$

$$\int\frac{xdx}{X^{\frac{9}{2}}}=-\frac{1}{7cX^3\sqrt{X}}-\frac{b}{2c}\int\frac{dx}{X^{\frac{9}{2}}}$$

$$\int\frac{x^2dx}{X^{\frac{9}{2}}}=\left(-\frac{x}{6c}+\frac{5b}{84c^2}\right)\frac{1}{X^3\sqrt{X}}+\left(\frac{5b^2}{24c^2}+\frac{a}{6c}\right)\int\frac{dx}{X^{\frac{9}{2}}}$$

$$\int\frac{x^3dx}{X^{\frac{9}{2}}}=\left(-\frac{x^2}{5c}+\frac{bx}{20c^2}-\frac{b^2}{56c^3}-\frac{2a}{35c^2}\right)\frac{1}{X^3\sqrt{X}}-\left(\frac{b^3}{16c^3}+\frac{ab}{4c^2}\right)\int\frac{dx}{X^{\frac{9}{2}}}$$

$$\int\frac{x^4dx}{X^{\frac{9}{2}}}=\left[-\frac{x^3}{4c}+\frac{bx^2}{40c^2}-\left(\frac{b^2}{160c^3}+\frac{a}{8c^2}\right)x+\frac{b^3}{448c^4}+\frac{29ab}{560c^3}\right]\frac{1}{X^3\sqrt{X}}$$
$$+\left(\frac{b^4}{128c^4}+\frac{3ab^2}{16c^3}+\frac{a^2}{8c^2}\right)\int\frac{dx}{X^{\frac{9}{2}}}$$

$$\int\frac{x^7dx}{X^{\frac{9}{2}}}=-\frac{x^4}{3cX^3\sqrt{X}}+\frac{4a}{3c}\int\frac{x^3dx}{X^{\frac{9}{2}}}+\frac{b}{6c}\int\frac{x^4dx}{X^{\frac{9}{2}}}$$

$$\int\frac{dx}{xX^{\frac{9}{2}}}=\left(\frac{1}{7aX^3}+\frac{1}{5a^2X^2}+\frac{1}{3a^3X}+\frac{1}{a^4}\right)\frac{1}{\sqrt{X}}-\frac{b}{2a}\int\frac{dx}{X^{\frac{9}{2}}}$$
$$-\frac{b}{2a^2}\int\frac{dx}{X^{\frac{7}{2}}}-\frac{b}{2a^3}\int\frac{dx}{X^{\frac{5}{2}}}-\frac{b}{2a^4}\int\frac{dx}{X^{\frac{3}{2}}}+\frac{1}{a^4}\int\frac{dx}{x\sqrt{X}}$$

$$\int\frac{dx}{x^2X^{\frac{9}{2}}}=-\frac{1}{axX^3\sqrt{X}}-\frac{9b}{2a}\int\frac{dx}{xX^{\frac{9}{2}}}-\frac{8c}{a}\int\frac{dx}{X^{\frac{9}{2}}}$$

$$\int\frac{dx}{x^3X^{\frac{9}{2}}}=\left(-\frac{1}{2ax^2}+\frac{11b}{4a^2x}\right)\frac{1}{X^3\sqrt{X}}+\left(\frac{99b^2}{8a^2}-\frac{9c}{2a}\right)\int\frac{dx}{xX^{\frac{9}{2}}}$$
$$+\frac{22bc}{a^2}\int\frac{dx}{X^{\frac{9}{2}}}$$

$$\int\frac{dx}{x^4X^{\frac{9}{2}}}=\left[-\frac{1}{3ax^3}+\frac{13b}{12a^2x^2}-\left(\frac{143b^2}{24a^3}-\frac{10c}{3a^2}\right)\frac{1}{x}\right]\frac{1}{X^3\sqrt{X}}$$
$$-\left(\frac{429b^3}{16a^3}-\frac{99bc}{4a^2}\right)\int\frac{dx}{xX^{\frac{9}{2}}}-\left(\frac{143b^2c}{3a^3}-\frac{80c^2}{3a^2}\right)\int\frac{dx}{X^{\frac{9}{2}}}$$

TAB. LXXIII.

$$\int x^m dx\sqrt{(a+bx+cx^2)}$$

$$a+bx+cx^2=X$$

$$\int dx\sqrt{X}=\frac{(2cx+b)\sqrt{X}}{4c}+\frac{4ac-b^2}{8c}\int\frac{dx}{\sqrt{X}}$$

$$\int x\,dx\sqrt{X}=\frac{X\sqrt{X}}{3c}-\frac{b}{2c}\int dx\sqrt{X}$$

$$\int x^2dx\sqrt{X}=\left(\frac{x}{4c}-\frac{5b}{24c^2}\right)X\sqrt{X}+\left(\frac{5b^2}{16c^2}-\frac{a}{4c}\right)\int dx\sqrt{X}$$

$$\int x^3dx\sqrt{X}=\left(\frac{x^2}{5c}-\frac{7bx}{40c^2}+\frac{7b^2}{48c^3}-\frac{2a}{15c^2}\right)X\sqrt{X}-\left(\frac{7b^3}{32c^3}-\frac{3ab}{8c^2}\right)\int dx\sqrt{X}$$

$$\int x^4dx\sqrt{X}=\left[\frac{x^3}{6c}-\frac{3bx^2}{20c^2}+\left(\frac{21b^2}{160c^3}-\frac{a}{8c^2}\right)x-\frac{7b^3}{64c^4}+\frac{49ab}{240c^3}\right]X\sqrt{X}$$
$$+\left(\frac{21b^4}{128c^4}-\frac{7ab^2}{16c^3}+\frac{a^2}{8c^2}\right)\int dx\sqrt{X}$$

$$\int x^5dx\sqrt{X}=\frac{x^4X\sqrt{X}}{7c}-\frac{4a}{7c}\int x^3dx\sqrt{X}-\frac{11b}{14c}\int x^4dx\sqrt{X}$$

$$\int x^6dx\sqrt{X}=\left(\frac{x^5}{8c}-\frac{13bx^4}{112c^2}\right)X\sqrt{X}+\frac{13ab}{28c^2}\int x^3dx\sqrt{X}$$
$$+\left(\frac{143b^2}{224c^2}-\frac{5a}{8c}\right)\int x^4dx\sqrt{X}$$

$$\int x^7dx\sqrt{X}=\left[\frac{x^6}{9c}-\frac{5bx^5}{48c^2}+\left(\frac{65b^2}{672c^3}-\frac{2a}{21c^2}\right)x^4\right]X\sqrt{X}$$
$$-\left(\frac{65ab^2}{168c^3}-\frac{8a^2}{21c^2}\right)\int x^3dx\sqrt{X}-\left(\frac{715b^3}{1344c^3}-\frac{117ab}{112c^2}\right)\int x^4dx\sqrt{X}$$

$$\int x^8dx\sqrt{X}=\frac{x^7X\sqrt{X}}{10c}-\frac{7a}{10c}\int x^6dx\sqrt{X}-\frac{17b}{20c}\int x^7dx\sqrt{X}$$

$$\int x^9dx\sqrt{X}=\left(\frac{x^8}{11c}-\frac{19bx^7}{220c^2}\right)X\sqrt{X}+\frac{133ab}{220c^2}\int x^6dx\sqrt{X}$$
$$+\left(\frac{323b^2}{440c^2}-\frac{8a}{11c}\right)\int x^7dx\sqrt{X}$$

TAB. LXXIV.

$$\int \frac{dx\sqrt{(a+bx+cx^2)}}{x^m}$$

$$a + bx + cx^2 = X$$

$$\int\frac{dx\sqrt{X}}{x} = \sqrt{X} + a\int\frac{dx}{x\sqrt{X}} + \frac{b}{2}\int\frac{dx}{\sqrt{X}}$$

$$\int\frac{dx\sqrt{X}}{x^2} = -\frac{\sqrt{X}}{x} + \frac{b}{2}\int\frac{dx}{x\sqrt{X}} + c\int\frac{dx}{\sqrt{X}}$$

$$\int\frac{dx\sqrt{X}}{x^3} = -\left(\frac{1}{2x^2} + \frac{b}{4ax}\right)\sqrt{X} - \left(\frac{b^2}{8a} - \frac{c}{2}\right)\int\frac{dx}{x\sqrt{X}}$$

$$\int\frac{dx\sqrt{X}}{x^4} = -\frac{X\sqrt{X}}{3ax^3} + \left(\frac{b}{4ax^2} + \frac{b^2}{8a^2x}\right)\sqrt{X} + \left(\frac{b^3}{16a^2} - \frac{bc}{4a}\right)\int\frac{dx}{x\sqrt{X}}$$

$$\int\frac{dx\sqrt{X}}{x^5} = \left(-\frac{1}{4ax^4} + \frac{5b}{24a^2x^3}\right)X\sqrt{X} - \left[\left(\frac{5b^2}{32a^2} - \frac{c}{8a}\right)\frac{1}{x}\right.$$
$$\left.+\left(\frac{5b^3}{64a^3} - \frac{bc}{16a^2}\right)\frac{1}{x}\right]\sqrt{X} - \left(\frac{5b^4}{128a^3} - \frac{3b^2c}{16a^2} + \frac{c^2}{8a}\right)\int\frac{dx}{x\sqrt{X}}$$

$$\int\frac{dx\sqrt{X}}{x^6} = -\frac{X\sqrt{X}}{5ax^5} - \frac{7b}{10a}\int\frac{dx\sqrt{X}}{x^5} - \frac{2c}{5a}\int\frac{dx\sqrt{X}}{x^4}$$

$$\int\frac{dx\sqrt{X}}{x^7} = \left(-\frac{1}{6ax^6} + \frac{3b}{20a^2x^5}\right)X\sqrt{X} + \left(\frac{21b^2}{40a^2} - \frac{c}{2a}\right)\int\frac{dx\sqrt{X}}{x^5}$$
$$+\frac{3bc}{10a^2}\int\frac{dx\sqrt{X}}{x^4}$$

$$\int\frac{dx\sqrt{X}}{x^8} = \left[-\frac{1}{7ax^7} + \frac{11b}{84a^2x^6} - \left(\frac{33b^2}{280a^3} - \frac{4c}{35a^2}\right)\frac{1}{x^5}\right]X\sqrt{X}$$
$$-\left(\frac{33b^3}{80a^3} - \frac{111bc}{140a^2}\right)\int\frac{dx\sqrt{X}}{x^5} - \left(\frac{33b^2c}{140a^3} - \frac{8c^2}{35a^2}\right)\int\frac{dx\sqrt{X}}{x^4}$$

$$\int\frac{dx\sqrt{X}}{x^9} = -\frac{X\sqrt{X}}{8ax^8} - \frac{13b}{16a}\int\frac{dx\sqrt{X}}{x^8} - \frac{5c}{8a}\int\frac{dx}{x^7\sqrt{X}}$$

$$\int\frac{dx\sqrt{X}}{x^{10}} = \left(-\frac{1}{9ax^9} + \frac{5b}{48a^2x^8}\right)X\sqrt{X} + \left(\frac{65b^2}{96a^2} - \frac{2c}{3a}\right)\int\frac{dx\sqrt{X}}{x^8}$$
$$+\frac{25bc}{48a^2}\int\frac{dx}{x^7\sqrt{X}}$$

TAB. LXXV.

$$\int x^m \mathrm{d}x(a+bx+cx^2)^{\frac{3}{2}}$$

$$a+bx+cx^2 = X,\ 4ac - b^2 = k$$

$$\int \mathrm{d}xX^{\frac{3}{2}} = \left(\frac{X}{8c}+\frac{3k}{64c^2}\right)(2cx+b)\sqrt{X} + \frac{3k^2}{128c^2}\int\frac{\mathrm{d}x}{\sqrt{X}}$$

$$\int x\mathrm{d}xX^{\frac{3}{2}} = \frac{X^2\sqrt{X}}{5c} - \frac{b}{2c}\int \mathrm{d}xX^{\frac{3}{2}}$$

$$\int x^2\mathrm{d}xX^{\frac{3}{2}} = \left(\frac{x}{6c}-\frac{7b}{60c^2}\right)X^2\sqrt{X} + \left(\frac{7b^2}{24c^2}-\frac{a}{6c}\right)\int \mathrm{d}xX^{\frac{3}{2}}$$

$$\int x^3\mathrm{d}xX^{\frac{3}{2}} = \left(\frac{x^2}{7c} - \frac{3bx}{28c^2} + \frac{3b^2}{40c^3} - \frac{2a}{35c^2}\right)X^2\sqrt{X} - \left(\frac{3b^3}{16c^3}-\frac{ab}{4c^2}\right)\int \mathrm{d}xX^{\frac{3}{2}}$$

$$\int x^4\mathrm{d}xX^{\frac{3}{2}} = \left[\frac{x^3}{8c} - \frac{11bx^2}{112c^2} + \left(\frac{33b^2}{448c^3} - \frac{a}{16c^2}\right)x - \frac{33b^3}{640c^4} + \frac{93ab}{1120c^3}\right]X^2\sqrt{X} + \left(\frac{33b^4}{256c^4} - \frac{9ab^2}{32c^3} + \frac{a^2}{16c^2}\right)\int \mathrm{d}xX^{\frac{3}{2}}$$

$$\int x^5\mathrm{d}xX^{\frac{3}{2}} = \frac{x^4X^2\sqrt{X}}{9c} - \frac{4a}{9c}\int x^3\mathrm{d}xX^{\frac{3}{2}} - \frac{13b}{18c}\int x^4\mathrm{d}xX^{\frac{3}{2}}$$

$$\int x^6\mathrm{d}xX^{\frac{3}{2}} = \left(\frac{x^5}{10c} - \frac{bx^4}{12c^2}\right)X^2\sqrt{X} + \frac{ab}{3c^2}\int x^3\mathrm{d}xX^{\frac{3}{2}} + \left(\frac{13b^2}{24c^2}-\frac{a}{2c}\right)\int x^4\mathrm{d}xX^{\frac{3}{2}}$$

$$\int x^7\mathrm{d}xX^{\frac{3}{2}} = \left[\frac{x^6}{11c} - \frac{17bx^5}{220c^2} + \left(\frac{17b^2}{264c^3} - \frac{2a}{33c^2}\right)x^4\right]X^2\sqrt{X} - \left(\frac{17ab^2}{66c^3}-\frac{8a^2}{33c^2}\right)\int x^3\mathrm{d}xX^{\frac{3}{2}} - \left(\frac{221b^3}{264c^3}-\frac{103ab}{113c^2}\right)\int x^4\mathrm{d}xX^{\frac{3}{2}}$$

$$\int x^8\mathrm{d}xX^{\frac{3}{2}} = \frac{x^7X^2\sqrt{X}}{12c} - \frac{7a}{12c}\int x^6\mathrm{d}xX^{\frac{3}{2}} - \frac{19b}{24c}\int x^7\mathrm{d}xX^{\frac{3}{2}}$$

$$\int x^9\mathrm{d}xX^{\frac{3}{2}} = \left(\frac{x^8}{13c} - \frac{7bx^7}{104c^2}\right)X^2\sqrt{X} + \frac{49ab}{104c^2}\int x^6\mathrm{d}xX^{\frac{3}{2}} + \left(\frac{133b^2}{208c^2}-\frac{8a}{13c}\right)\int x^7\mathrm{d}xX^{\frac{3}{2}}$$

TAB. LXXVI.

$$\int \frac{dx(a+bx+cx^2)^{\frac{3}{2}}}{x^m}$$

$$a + bx + cx^2 = X$$

$$\int\frac{dxX^{\frac{3}{2}}}{x} = \left(\frac{X}{3}+a\right)\sqrt{X} + a^2\int\frac{dx}{x\sqrt{X}} + \frac{ab}{2}\int\frac{dx}{\sqrt{X}} + \frac{b}{2}\int dx\sqrt{X}$$

$$\int\frac{dxX^{\frac{3}{2}}}{x^2} = -\frac{X^2\sqrt{X}}{ax} + \frac{3b}{2a}\int\frac{dxX^{\frac{3}{2}}}{x} + \frac{4c}{a}\int dxX^{\frac{3}{2}}$$

$$\int\frac{dxX^{\frac{3}{2}}}{x^3} = \left(-\frac{1}{2ax^2} - \frac{b}{4a^2x}\right)X^2\sqrt{X} + \left(\frac{3b^2}{8a^2} + \frac{3c}{2a}\right)\int\frac{dxX^{\frac{3}{2}}}{x} + \frac{bc}{a^2}\int dxX^{\frac{3}{2}}$$

$$\int\frac{dxX^{\frac{3}{2}}}{x^4} = \left[-\frac{1}{3ax^3} + \frac{b}{12a^2x^2} + \left(\frac{b^2}{24a^3} - \frac{2c}{3a^2}\right)\frac{1}{x}\right]X^2\sqrt{X} - \left(\frac{b^3}{16a^3} - \frac{3bc}{4a^2}\right)\int\frac{dxX^{\frac{3}{2}}}{x} - \left(\frac{b^2c}{6a^3} - \frac{8c^2}{3a^2}\right)\int dxX^{\frac{3}{2}}$$

$$\int\frac{dxX^{\frac{3}{2}}}{x^5} = \left[-\frac{1}{4ax^4} + \frac{b}{8a^2x^3} - \left(\frac{b^2}{32a^3} + \frac{c}{8a^2}\right)\frac{1}{x^2} - \left(\frac{b^3}{64a^4} - \frac{3bc}{16a^3}\right)\frac{1}{x}\right]X^2\sqrt{X} + \left(\frac{3b^4}{128a^4} - \frac{3b^2c}{16a^3} + \frac{3c^2}{8a^2}\right)\int\frac{dxX^{\frac{3}{2}}}{x} + \left(\frac{b^3c}{16a^4} - \frac{3bc^2}{4a^3}\right)\int dxX^{\frac{3}{2}}$$

$$\int\frac{dxX^{\frac{3}{2}}}{x^6} = -\frac{X^2\sqrt{X}}{5ax^5} - \frac{b}{2a}\int\frac{dxX^{\frac{3}{2}}}{x^5}$$

$$\int\frac{dxX^{\frac{3}{2}}}{x^7} = \left(-\frac{1}{6ax^6} + \frac{7b}{60a^2x^5}\right)X^2\sqrt{X} + \left(\frac{7b^2}{24a^2} - \frac{c}{6a}\right)\int\frac{dxX^{\frac{3}{2}}}{x^5}$$

$$\int\frac{dxX^{\frac{3}{2}}}{x^8} = \left[-\frac{1}{7ax^7} + \frac{3b}{28a^2x^6} - \left(\frac{3b^2}{40a^3} - \frac{2c}{35a^2}\right)\frac{1}{x^5}\right]X^2\sqrt{X} - \left(\frac{3b^3}{16a^3} - \frac{bc}{4a^2}\right)\int\frac{dxX^{\frac{3}{2}}}{x^5}$$

$$\int\frac{dxX^{\frac{3}{2}}}{x^9} = \left[-\frac{1}{8ax^8} + \frac{11b}{112a^2x^7} - \left(\frac{33b^2}{448a^3} - \frac{c}{16a^2}\right)\frac{1}{x^6} + \left(\frac{33b^3}{640a^4} - \frac{93bc}{1120a^3}\right)\frac{1}{x^5}\right]X^2\sqrt{X} + \left(\frac{33b^4}{256a^4} - \frac{9b^2c}{32a^3} + \frac{c^2}{16a^2}\right)\int\frac{dxX^{\frac{3}{2}}}{x^5}$$

TAB. LXXVII.

$$\int x^m dx (a+bx+cx^2)^{\frac{5}{2}}$$

$$a + bx + cx^2 = X,\ 4ac - b^2 = k$$

$$\int dx X^{\frac{5}{2}} = \left(\frac{X^2}{12c} + \frac{5kX}{192c^2} + \frac{5k^2}{512c^3}\right)(2cx+b)\sqrt{X} + \frac{5k^3}{1024c^3}\int\frac{dx}{\sqrt{X}}$$

$$\int x dx X^{\frac{5}{2}} = \frac{X^3\sqrt{X}}{7c} - \frac{b}{2c}\int dx X^{\frac{5}{2}}$$

$$\int x^2 dx X^{\frac{5}{2}} = \left(\frac{x}{8c} - \frac{9b}{112c^2}\right)X^3\sqrt{X} + \left(\frac{9b^2}{32c^2} - \frac{a}{8c}\right)\int dx X^{\frac{5}{2}}$$

$$\int x^3 dx X^{\frac{5}{2}} = \left(\frac{x^2}{9c} - \frac{11bx}{144c^2} + \frac{11b^2}{224c^3} - \frac{2a}{63c^2}\right)X^3\sqrt{X}$$
$$- \left(\frac{11b^3}{64c^3} - \frac{3ab}{16c^2}\right)\int dx X^{\frac{5}{2}}$$

$$\int x^4 dx X^{\frac{5}{2}} = \left[\frac{x^3}{10c} - \frac{13bx^2}{180c^2} + \left(\frac{143b^2}{2880c^3} - \frac{3a}{80c^2}\right)x - \frac{143b^3}{4480c^4}\right.$$
$$\left. + \frac{451ab}{10080c^3}\right]X^3\sqrt{X} + \left(\frac{143b^4}{1280c^4} - \frac{33ab^2}{160c^3} + \frac{3a^2}{80c^2}\right)\int dx X^{\frac{5}{2}}$$

$$\int x^5 dx X^{\frac{5}{2}} = \frac{x^4X^3\sqrt{X}}{11c} - \frac{4a}{11c}\int x^3 dx X^{\frac{5}{2}} - \frac{15b}{22c}\int x^4 dx X^{\frac{5}{2}}$$

$$\int x^6 dx X^{\frac{5}{2}} = \left(\frac{x^5}{12c} - \frac{17bx^4}{264c^2}\right)X^3\sqrt{X} + \frac{17ab}{66c^2}\int x^3 dx X^{\frac{5}{2}}$$
$$+ \left(\frac{85b^2}{176c^2} - \frac{5a}{12c}\right)\int x^4 dx X^{\frac{5}{2}}$$

$$\int x^7 dx X^{\frac{5}{2}} = \left[\frac{x^6}{13c} - \frac{19bx^5}{312c^2} + \left(\frac{323b^2}{6864c^3} - \frac{6a}{143c^2}\right)x^4\right]X^3\sqrt{X}$$
$$- \left(\frac{323ab^2}{1716c^3} - \frac{24a^2}{143c^2}\right)\int x^3 dx X^{\frac{5}{2}} - \left(\frac{1615b^3}{4576c^3} - \frac{2125ab}{3432c^2}\right)\int x^4 dx X^{\frac{5}{2}}$$

$$\int x^8 dx X^{\frac{5}{2}} = \frac{x^7X^3\sqrt{X}}{14c} - \frac{a}{2c}\int x^6 dx X^{\frac{5}{2}} - \frac{3b}{4c}\int x^7 dx X^{\frac{5}{2}}$$

$$\int x^9 dx X^{\frac{5}{2}} = \left(\frac{x^8}{15c} - \frac{23bx^7}{420c^2}\right)X^3\sqrt{X} + \frac{23ab}{60c^2}\int x^6 dx X^{\frac{5}{2}}$$
$$+ \left(\frac{23b^2}{40c^2} - \frac{8a}{15c}\right)\int x^7 dx X^{\frac{5}{2}}$$

TAB. LXXVIII.

$$\int \frac{dx(ax+bx+cx^2)^{\frac{5}{2}}}{x^m}$$

$$a+bx+cx^2 = X$$

$$\int \frac{dx X^{\frac{5}{2}}}{x} = \left(\frac{X^2}{5} + \frac{aX}{3} + a^2\right)\sqrt{X} + a^3 \int \frac{dx}{x\sqrt{X}} + \frac{a^2 b}{2}\int \frac{dx}{\sqrt{X}}$$
$$+ \frac{ab}{2}\int dx\sqrt{X} + \frac{b}{2}\int dx X^{\frac{3}{2}}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^2} = -\frac{X^2\sqrt{X}}{ax} + \frac{5b}{2a}\int \frac{dx X^{\frac{5}{2}}}{x} + \frac{6c}{a}\int dx X^{\frac{5}{2}}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^3} = \left(-\frac{1}{2ax^2} - \frac{3b}{4a^2x}\right) X^3\sqrt{X} + \left(\frac{15b^2}{8a^2} + \frac{5c}{2a}\right)\int \frac{dx X^{\frac{5}{2}}}{x}$$
$$+ \frac{9bc}{2a^2}\int dx X^{\frac{5}{2}}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^4} = \left[-\frac{1}{3ax^3} - \frac{b}{12a^2x^2} - \left(\frac{b^2}{8a^3} + \frac{4c}{3a^2}\right)\frac{1}{x}\right] X^3\sqrt{X}$$
$$+ \left(\frac{5b^3}{16a^3} + \frac{15bc}{4a^2}\right)\int \frac{dx X^{\frac{5}{2}}}{x} + \left(\frac{3b^2c}{4a^3} + \frac{8c^2}{a^2}\right)\int dx X^{\frac{5}{2}}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^5} = -\frac{X^3\sqrt{X}}{4ax^4} - \frac{b}{8a}\int \frac{dx X^{\frac{5}{2}}}{x^4} + \frac{3c}{4a}\int \frac{dx X^{\frac{5}{2}}}{x^3}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^6} = \left(-\frac{1}{5ax^5} + \frac{3b}{40a^2x^4}\right) X^3\sqrt{X} + \left(\frac{3b^2}{80a^2} + \frac{2c}{5a}\right)\int \frac{dx X^{\frac{5}{2}}}{x^4}$$
$$- \frac{9bc}{40a^2}\int \frac{dx X^{\frac{5}{2}}}{x^3}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^7} = \left[-\frac{1}{6ax^6} + \frac{b}{12a^2x^5} - \left(\frac{b^2}{32a^3} + \frac{c}{24a^2}\right)\frac{1}{x^4}\right] X^3\sqrt{X}$$
$$- \left(\frac{b^3}{64a^3} + \frac{3bc}{16a^2}\right)\int \frac{dx X^{\frac{5}{2}}}{x^4} + \left(\frac{3b^2c}{32a^3} + \frac{c^2}{8a^2}\right)\int \frac{dx X^{\frac{5}{2}}}{x^3}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^8} = -\frac{X^3\sqrt{X}}{7ax^7} - \frac{b}{2a}\int \frac{dx X^{\frac{5}{2}}}{x^7}$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^9} = \left(-\frac{1}{8ax^8} + \frac{9b}{112a^2x^7}\right) X^3\sqrt{X} + \left(\frac{9b^2}{32a^2} - \frac{c}{8a}\right)\int \frac{dx X^{\frac{5}{2}}}{x^7}$$

TAB. LXXIX.

$$\int x^m dx (a+bx+cx^2)^{\frac{7}{2}}$$

$$a + bx + cx^2 = X,\ 4ac - b^2 = k$$

$$\int dx X^{\frac{7}{2}} = \left(\frac{X^3}{16c} + \frac{7kX^2}{384c^2} + \frac{35k^2X}{6144c^3} + \frac{35k^3}{16384c^4}\right)(2cx+b)\sqrt{X} + \frac{35k^4}{32768c^4}\int\frac{dx}{\sqrt{X}}$$

$$\int x dx X^{\frac{7}{2}} = \frac{X^4\sqrt{X}}{9c} - \frac{b}{2c}\int dx X^{\frac{7}{2}}$$

$$\int x^2 dx X^{\frac{7}{2}} = \left(\frac{x}{10c} - \frac{11b}{180c^2}\right)X^4\sqrt{X} + \left(\frac{11b^2}{40c^2} - \frac{a}{10c}\right)\int dx X^{\frac{7}{2}}$$

$$\int x^3 dx X^{\frac{7}{2}} = \left(\frac{x^2}{11c} - \frac{13bx}{220c^2} + \frac{13b^2}{360c^3} - \frac{2a}{99c^2}\right)X^4\sqrt{X} - \left(\frac{13b^3}{80c^3} - \frac{3ab}{20c^2}\right)\int dx X^{\frac{7}{2}}$$

$$\int x^4 dx X^{\frac{7}{2}} = \left[\frac{x^3}{12c} - \frac{5bx^2}{88c^2} + \left(\frac{13b^2}{352c^3} - \frac{a}{40c^2}\right)x - \frac{13b^3}{576c^4} + \frac{221ab}{7920c^3}\right]X^4\sqrt{X} + \left(\frac{13b^4}{128c^4} - \frac{13ab^2}{80c^3} + \frac{a^2}{40c^2}\right)\int dx X^{\frac{7}{2}}$$

$$\int x^5 dx X^{\frac{7}{2}} = \frac{x^4X^4\sqrt{X}}{13c} - \frac{4a}{13c}\int x^3 dx X^{\frac{7}{2}} - \frac{17b}{26c}\int x^4 dx X^{\frac{7}{2}}$$

$$\int x^6 dx X^{\frac{7}{2}} = \left(\frac{x^5}{14c} - \frac{19bx^4}{364c^2}\right)X^4\sqrt{X} + \frac{19ab}{91c^2}\int x^3 dx X^{\frac{7}{2}} + \left(\frac{323b^2}{728c^2} - \frac{5a}{14c}\right)\int x^4 dx X^{\frac{7}{2}}$$

$$\int x^7 dx X^{\frac{7}{2}} = \left[\frac{x^6}{15c} - \frac{bx^5}{20c^2} + \left(\frac{19b^2}{520c^3} - \frac{2a}{65c^2}\right)x^4\right]X^4\sqrt{X} - \left(\frac{19ab^2}{130c^3} - \frac{8a^2}{65c^2}\right)\int x^3 dx X^{\frac{7}{2}} - \left(\frac{323b^3}{1040c^3} - \frac{133ab}{260c^2}\right)\int x^4 dx X^{\frac{7}{2}}$$

$$\int x^8 dx X^{\frac{7}{2}} = \frac{x^7X^4\sqrt{X}}{16c} - \frac{7a}{16c}\int x^6 dx X^{\frac{7}{2}} - \frac{23b}{32c}\int x^7 dx X^{\frac{7}{2}}$$

$$\int x^9 dx X^{\frac{7}{2}} = \left(\frac{x^8}{17c} - \frac{25bx^7}{544c^2}\right)X^4\sqrt{X} + \frac{175ab}{544c^2}\int x^6 dx X^{\frac{7}{2}} + \left(\frac{575b^2}{1088c^2} - \frac{8a}{17c}\right)\int x^7 dx X^{\frac{7}{2}}$$

TAB. LXXX.

$$\int \frac{\mathrm{d}x(a+bx+cx^2)^{\frac{7}{2}}}{x^m}$$

$$a + bx + cx^2 = X$$

$$\int \frac{\mathrm{d}x X^{\frac{7}{2}}}{x} = \left(\frac{X^3}{7} + \frac{aX^2}{5} + \frac{a^2X}{3} + a^3\right)\sqrt{X} + a^4 \int \frac{\mathrm{d}x}{x\sqrt{X}} + \frac{a^3b}{2}\int \frac{\mathrm{d}x}{\sqrt{X}}$$
$$+ \frac{a^2b}{2}\int \mathrm{d}x\sqrt{X} + \frac{ab}{2}\int \mathrm{d}x X^{\frac{3}{2}} + \frac{b}{2}\int \mathrm{d}x X^{\frac{5}{2}}$$

$$\int \frac{\mathrm{d}x X^{\frac{7}{2}}}{x^2} = -\frac{X^4\sqrt{X}}{ax} + \frac{7b}{2a}\int \frac{\mathrm{d}x X^{\frac{7}{2}}}{x} + \frac{8c}{a}\int \mathrm{d}x X^{\frac{7}{2}}$$

$$\int \frac{\mathrm{d}x X^{\frac{7}{2}}}{x^3} = \left(-\frac{1}{2ax^2} + \frac{5b}{4a^2x}\right) X^4\sqrt{X} + \left(\frac{35b^2}{8a^2} + \frac{7c}{2a}\right)\int \frac{\mathrm{d}x X^{\frac{7}{2}}}{x}$$
$$+ \frac{10bc}{a^2}\int \mathrm{d}x X^{\frac{7}{2}}$$

$$\int \frac{\mathrm{d}x X^{\frac{7}{2}}}{x^4} = \left(-\frac{1}{3ax^3} - \frac{b}{4a^2x^2} - \left(\frac{5b^2}{8a^3} + \frac{2c}{a^2}\right)\frac{1}{x}\right] X^4\sqrt{X}$$
$$+ \left(\frac{35b^3}{16a^5} + \frac{35bc}{4a^2}\right)\int \frac{\mathrm{d}x X^{\frac{7}{2}}}{x} + \left(\frac{5b^2c}{a^3} + \frac{16c^2}{a^2}\right)\int \mathrm{d}x X^{\frac{7}{2}}$$

$$\int \frac{\mathrm{d}x X^{\frac{7}{2}}}{x^5} = \left[-\frac{1}{4ax^4} - \frac{b}{24a^2x^3} - \left(\frac{b^2}{32a^3} + \frac{5c}{8a^2}\right)\frac{1}{x^2} - \left(\frac{5b^3}{64a^4} + \frac{29bc}{16a^3}\right)\frac{1}{x}\right] X^4\sqrt{X}$$
$$+ \left(\frac{35b^4}{128a^4} + \frac{105b^2c}{16a^3} + \frac{35c^2}{8a^2}\right)\int \frac{\mathrm{d}x X^{\frac{7}{2}}}{x} + \left(\frac{5b^3c}{8a^4} + \frac{29bc^2}{2a^3}\right)\int \mathrm{d}x X^{\frac{7}{2}}$$

$$\int \frac{\mathrm{d}x X^{\frac{7}{2}}}{x^6} = -\frac{X^4\sqrt{X}}{5ax^5} - \frac{b}{10a}\int \frac{\mathrm{d}x X^{\frac{7}{2}}}{x^5} + \frac{4c}{5a}\int \frac{\mathrm{d}x X^{\frac{7}{2}}}{x^4}$$

$$\int \frac{\mathrm{d}x X^{\frac{7}{2}}}{x^7} = \left(-\frac{1}{6ax^6} + \frac{b}{20a^2x^5}\right) X^4\sqrt{X} + \left(\frac{b^2}{40a^2} + \frac{c}{2a}\right)\int \frac{\mathrm{d}x X^{\frac{7}{2}}}{x^5}$$
$$+ \frac{bc}{5a^2}\int \frac{\mathrm{d}x X^{\frac{7}{2}}}{x^4}$$

$$\int \frac{\mathrm{d}x X^{\frac{7}{2}}}{x^8} = \left[-\frac{1}{7ax^7} + \frac{5b}{84a^2x^6} - \left(\frac{b^2}{56a^3} + \frac{2c}{35a^2}\right)\frac{1}{x^5}\right] X^4\sqrt{X}$$
$$-\left(\frac{b^3}{112a^3} + \frac{29bc}{140a^2}\right)\int \frac{\mathrm{d}x X^{\frac{7}{2}}}{x^5} + \left(\frac{b^2c}{14a^3} + \frac{8c^2}{35a^2}\right)\int \frac{\mathrm{d}x X^{\frac{7}{2}}}{x^4}$$

TAB. LXXXI.

$$\int x^m dx (a+bx+cx^2)^{\frac{9}{2}}, \quad \int \frac{dx\,(a+bx+cx^2)^{\frac{9}{2}}}{x^m}$$

$$a + bx + cx^2 = X, \; 4ac - b^2 = k$$

$$\int dx X^{\frac{9}{2}} = \int dx X^{\frac{9}{2}} \text{ (page 161)}$$

$$\int x dx X^{\frac{9}{2}} = \frac{X^5\sqrt{X}}{11c} - \frac{b}{2c}\int dx X^{\frac{9}{2}}$$

$$\int x^2 dx X^{\frac{9}{2}} = \left(\frac{x}{12c} - \frac{13b}{264c^2}\right) X^5\sqrt{X} + \left(\frac{13b^2}{48c^2} - \frac{a}{12c}\right)\int dx X^{\frac{9}{2}}$$

$$\int x^3 dx X^{\frac{9}{2}} = \left(\frac{x^2}{13c} - \frac{5bx}{104c^2} + \frac{5b^2}{176c^3} - \frac{2a}{143c^2}\right) X^5\sqrt{X} - \left(\frac{5b^3}{32c^3} - \frac{ab}{8c^2}\right)\int dx X^{\frac{9}{2}}$$

$$\int x^4 dx X^{\frac{9}{2}} = \frac{x^3 X^5\sqrt{X}}{14c} - \frac{3a}{14c}\int x^2 dx X^{\frac{9}{2}} - \frac{17b}{28c}\int x^3 dx X^{\frac{9}{2}}$$

$$\int x^5 dx X^{\frac{9}{2}} = \left(\frac{x^4}{15c} - \frac{19bx^3}{420c^2}\right) X^5\sqrt{X} + \frac{19ab}{140c^2}\int x^2 dx X^{\frac{9}{2}} + \left(\frac{323b^2}{840c^2} - \frac{4a}{15c}\right)\int x^3 dx X^{\frac{9}{2}}$$

$$\int \frac{dx X^{\frac{9}{2}}}{x} = \left(\frac{X^4}{9} + \frac{aX^3}{7} + \frac{a^2X^2}{5} + \frac{a^3X}{3} + a^4\right)\sqrt{X} + a^5\int\frac{dx}{x\sqrt{X}} + \frac{a^4b}{2}\int\frac{dx}{\sqrt{X}} + \frac{a^3b}{2}\int dx\sqrt{X} + \frac{a^2b}{2}\int dx X^{\frac{3}{2}} + \frac{ab}{2}\int dx X^{\frac{5}{2}} + \frac{b}{2}\int dx X^{\frac{7}{2}}$$

$$\int \frac{dx X^{\frac{9}{2}}}{x^2} = -\frac{X^5\sqrt{X}}{ax} + \frac{9b}{2a}\int\frac{dx X^{\frac{9}{2}}}{x} + \frac{10c}{a}\int dx X^{\frac{7}{2}}$$

$$\int \frac{dx X^{\frac{9}{2}}}{x^3} = \left[-\frac{1}{2ax^2} - \frac{7b}{4a^2x}\right) X^5\sqrt{X} + \left(\frac{63b^2}{8a^2} + \frac{9c}{2a}\right)\int\frac{dx X^{\frac{9}{2}}}{x} + \frac{70bc}{4a^2}\int dx X^{\frac{9}{2}}$$

$$\int \frac{dx X^{\frac{9}{2}}}{x^4} = \left[-\frac{1}{3ax^3} - \frac{5b}{12a^2x^2} - \left(\frac{35b^2}{24a^3} + \frac{8c}{3a^2}\right)\frac{1}{x}\right] X^5\sqrt{X} + \left(\frac{105b^3}{16a^3} + \frac{63bc}{4a^2}\right)\int\frac{dx X^{\frac{9}{2}}}{x} + \left(\frac{175b^2c}{12a^3} + \frac{80c^2}{3a^2}\right)\int dx X^{\frac{9}{2}}$$

TAB. LXXXII.

$$\int \frac{dx}{(a+bx)^n \sqrt{x}}$$

$$a + bx = X$$

$$\int \frac{dx}{X\sqrt{x}} = \left\{ \begin{array}{c} \pm \dfrac{2}{\sqrt{ab}} \text{ arc tang } \sqrt{\dfrac{bx}{a}} \\ \text{or} \\ \dfrac{1}{\sqrt{-ab}} \log \dfrac{a - bx + 2\sqrt{x}\,.\,\sqrt{-ab}}{X} \end{array} \right\}^{*} + \text{const.}$$

$$\int \frac{dx}{X^2\sqrt{x}} = \frac{\sqrt{x}}{aX} + \frac{1}{2a}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{dx}{X^3\sqrt{x}} = \left(\frac{1}{2aX^2} + \frac{3}{4a^2X}\right)\sqrt{x} + \frac{3}{8a^2}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{dx}{X^4\sqrt{x}} = \left(\frac{1}{3aX^3} + \frac{5}{12a^2X^2} + \frac{5}{8a^3X}\right)\sqrt{x} + \frac{5}{16a^3}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{dx}{X^5\sqrt{x}} = \left(\frac{1}{4aX^4} + \frac{7}{24a^2X^3} + \frac{35}{96a^3X^2} + \frac{35}{64a^4X}\right)\sqrt{x} + \frac{35}{128a^4}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{dx}{X^6\sqrt{x}} = \left(\frac{1}{5aX^5} + \frac{9}{40a^2X^4} + \frac{21}{80a^3X^3} + \frac{21}{64a^4X^2} + \frac{63}{128a^5X}\right)\sqrt{x} + \frac{63}{256a^5}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{dx}{X^7\sqrt{x}} = \left(\frac{1}{6aX^6} + \frac{11}{60a^2X^5} + \frac{33}{160a^3X^4} + \frac{77}{320a^4X^3} + \frac{77}{256a^5X^2} + \frac{231}{512a^6X}\right)\sqrt{x} + \frac{231}{1024a^6}\int \frac{dx}{X\sqrt{x}}$$

* The first expression is taken when a and b have the same signs, and then the upper sign makes a positive, and the lower a negative; the second expression, on the contrary, is taken when a and b have different signs. Both forms vanish when $x = 0$. Moreover, we have, arc tang $\sqrt{\frac{bx}{a}}$ = arc cot $\sqrt{\frac{a}{bx}}$ = arc sec $\sqrt{\frac{a+bx}{a}}$ = arc cosec $\sqrt{\frac{a+bx}{bx}}$ = arc cos $\sqrt{\frac{a}{a+bx}}$ = $\frac{1}{2}$ arc cos $\frac{a-bx}{a+bx}$ = arc sin $\sqrt{\frac{bx}{a+bx}}$ = $\frac{1}{2}$ arc sin $\frac{2\sqrt{abx}}{a+bx}$ = $\frac{1}{2}$ arc sin vers $\frac{2bx}{a+bx}$.

TAB. LXXXIII.

$$\int \frac{x^m dx\sqrt{x}}{a+bx}, \quad \int \frac{x^m dx\sqrt{x}}{(a+bx)^2}$$

$$a + bx = X$$

$$\int \frac{dx\sqrt{x}}{X} = \frac{2\sqrt{x}}{b} - \frac{a}{b}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{xdx\sqrt{x}}{X} = \left(\frac{x}{3b} - \frac{a}{b^2}\right)2\sqrt{x} + \frac{a^2}{b^2}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{x^2dx\sqrt{x}}{X} = \left(\frac{x^2}{5b} - \frac{ax}{3b^2} + \frac{a^2}{b^3}\right)2\sqrt{x} - \frac{a^3}{b^3}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{x^3dx\sqrt{x}}{X} = \left(\frac{x^3}{7b} - \frac{ax^2}{5b^2} + \frac{a^2x}{3b^3} - \frac{a^3}{b^4}\right)2\sqrt{x} - \frac{a^4}{b^4}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{x^4dx\sqrt{x}}{X} = \left(\frac{x^4}{9b} - \frac{ax^3}{7b^2} + \frac{a^2x^2}{5b^3} - \frac{a^3x}{3b^4} + \frac{a^4}{b^5}\right)2\sqrt{x} - \frac{a^5}{b^5}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{x^5dx\sqrt{x}}{X} = \left(\frac{x^5}{11b} - \frac{ax^4}{9b^2} + \frac{a^2x^3}{7b^3} - \frac{a^3x^2}{5b^4} + \frac{a^4x}{3b^5} - \frac{a^5}{b^6}\right)2\sqrt{x} + \frac{a^6}{b^6}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{dx\sqrt{x}}{X^2} = -\frac{\sqrt{x}}{bX} + \frac{1}{2b}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{xdx\sqrt{x}}{X^2} = \frac{2x\sqrt{x}}{bX} - \frac{3a}{b}\int \frac{dx\sqrt{x}}{X^2}$$

$$\int \frac{x^2dx\sqrt{x}}{X^2} = \left(\frac{x^2}{3b} - \frac{5ax}{3b^2}\right)\frac{2\sqrt{x}}{X} + \frac{5a^2}{b^2}\int \frac{dx\sqrt{x}}{X^2}$$

$$\int \frac{x^3dx\sqrt{x}}{X^2} = \left(\frac{x^3}{5b} - \frac{7ax^2}{15b^2} + \frac{7a^2x}{3b^3}\right)\frac{2\sqrt{x}}{X} - \frac{7a^3}{b^3}\int \frac{dx\sqrt{x}}{X^2}$$

$$\int \frac{x^4dx\sqrt{x}}{X^2} = \left(\frac{x^4}{7b} - \frac{9ax^3}{35b^2} + \frac{3a^2x^2}{5b^3} - \frac{3a^3x}{b^4}\right)\frac{2\sqrt{x}}{X} + \frac{9a^4}{b^4}\int \frac{dx\sqrt{x}}{X^2}$$

$$\int \frac{x^5dx\sqrt{x}}{X^2} = \left(\frac{x^5}{9b} - \frac{11ax^4}{63b^2} + \frac{11a^2x^3}{35b^3} - \frac{11a^3x^2}{15b^4} + \frac{11a^4x}{3b^5}\right)\frac{2\sqrt{x}}{X} - \frac{11a^5}{b^5}\int \frac{dx\sqrt{x}}{X^2}$$

$$\int \frac{x^6dx\sqrt{x}}{X^2} = \left(\frac{x^6}{11b} - \frac{13ax^5}{99b^2} + \frac{13a^2x^4}{63b^3} - \frac{13a^3x^3}{35b^4} + \frac{13a^4x^2}{15b^5} - \frac{13a^5x}{3b^6}\right)\frac{2\sqrt{x}}{X} + \frac{13a^6}{b^6}\int \frac{dx\sqrt{x}}{X^2}$$

TAB. LXXXIV.

$$\int \frac{x^m dx\sqrt{x}}{(a+bx)^3}, \int \frac{x^m dx\sqrt{x}}{(a+bx)^4}$$

$$a + bx = X$$

$$\int \frac{dx\sqrt{x}}{X^3} = \left(-\frac{1}{2bX^2} + \frac{1}{4abX}\right)\sqrt{x} + \frac{1}{8ab}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{xdx\sqrt{x}}{X^3} = -\frac{2x\sqrt{x}}{bX^2} + \frac{3a}{b}\int \frac{dx\sqrt{X}}{X^3}$$

$$\int \frac{x^2dx\sqrt{x}}{X^3} = \left(\frac{x^2}{b} + \frac{5ax}{b^2}\right)\frac{2\sqrt{x}}{X^2} - \frac{15a^2}{b^2}\int \frac{dx\sqrt{x}}{X^3}$$

$$\int \frac{x^3dx\sqrt{x}}{X^3} = \left(\frac{x^3}{3b} - \frac{7ax^2}{3b^2} - \frac{35a^2x}{3b^3}\right)\frac{2\sqrt{x}}{X^2} + \frac{35a^3}{b^3}\int \frac{dx\sqrt{x}}{X^3}$$

$$\int \frac{x^4dx\sqrt{x}}{X^3} = \left(\frac{x^4}{5b} - \frac{3ax^3}{5b^2} + \frac{21a^2x^2}{5b^3} + \frac{21a^3x}{b^4}\right)\frac{2\sqrt{x}}{X^2} - \frac{63a^4}{b^4}\int \frac{dx\sqrt{x}}{X^3}$$

$$\int \frac{x^5dx\sqrt{x}}{X^3} = \left(\frac{x^5}{7b} - \frac{11ax^4}{35b^2} + \frac{33a^2x^3}{35b^3} - \frac{33a^3x^2}{5b^4} - \frac{33a^4x}{b^5}\right)\frac{2\sqrt{x}}{X^2} + \frac{99a^5}{b^5}\int \frac{dx\sqrt{x}}{X^3}$$

$$\int \frac{dx\sqrt{x}}{X^4} = \left(-\frac{1}{3bX^3} + \frac{1}{12abX^2} + \frac{1}{8a^2bX}\right)\sqrt{x} + \frac{1}{16a^2b}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{xdx\sqrt{x}}{X^4} = -\frac{2x\sqrt{x}}{3bX^3} + \frac{a}{b}\int \frac{dx\sqrt{x}}{X^4}$$

$$\int \frac{x^2dx\sqrt{x}}{X^4} = \left(-\frac{x^2}{b} - \frac{5ax}{3b^2}\right)\frac{2\sqrt{x}}{X^3} + \frac{5a^2}{b^2}\int \frac{dx\sqrt{x}}{X^4}$$

$$\int \frac{x^3dx\sqrt{x}}{X^4} = \left(\frac{x^3}{b} + \frac{7ax^2}{b^2} + \frac{35a^2x}{3b^3}\right)\frac{2\sqrt{x}}{X^3} - \frac{35a^3}{b^3}\int \frac{dx\sqrt{x}}{X^4}$$

$$\int \frac{x^4dx\sqrt{x}}{X^4} = \left(\frac{x^4}{3b} - \frac{3ax^3}{b^2} - \frac{21a^2x^2}{b^3} - \frac{35a^3x}{b^4}\right)\frac{2\sqrt{x}}{X^3} + \frac{105a^4}{b^4}\int \frac{dx\sqrt{x}}{X^4}$$

$$\int \frac{x^5dx\sqrt{x}}{X^4} = \left(\frac{x^5}{5b} - \frac{11ax^4}{15b^2} + \frac{33a^2x^3}{5b^3} + \frac{231a^3x^2}{5b^4} + \frac{77a^4x}{b^5}\right)\frac{2\sqrt{x}}{X^3} - \frac{231a^5}{b^5}\int \frac{dx\sqrt{x}}{X^4}$$

TAB. LXXXV.

$$\int \frac{dx}{(a + bx^2)^n \sqrt{x}}$$

$$a + bx^2 = X$$

$$\int \frac{dx}{X\sqrt{x}} = \int \frac{dx}{X\sqrt{x}} \; ^*$$

$$\int \frac{dx}{X^2\sqrt{x}} = \frac{\sqrt{x}}{2aX} + \frac{3}{4a}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{dx}{X^3\sqrt{x}} = \left(\frac{1}{4aX^2} + \frac{7}{16a^2X}\right)\sqrt{x} + \frac{21}{32a^2}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{dx}{X^4\sqrt{x}} = \left(\frac{1}{6aX^3} + \frac{11}{48a^2X^2} + \frac{77}{192a^3X}\right)\sqrt{x} + \frac{77}{128a^3}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{dx}{X^5\sqrt{x}} = \left(\frac{1}{8aX^4} + \frac{5}{32a^2X^3} + \frac{55}{256a^3X^2} + \frac{385}{1024a^4X}\right)\sqrt{x} + \frac{1155}{2048a^4}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{dx}{X^6\sqrt{x}} = \left(\frac{1}{10aX^5} + \frac{19}{160a^2X^4} + \frac{19}{128a^3X^3} + \frac{209}{1024a^4X^2} + \frac{1463}{4096a^5X}\right)\sqrt{x} + \frac{4389}{8192a^5}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{bx}{X^7\sqrt{x}} = \left(\frac{1}{12aX^6} + \frac{23}{240a^2X^5} + \frac{437}{3840a^3X^4} + \frac{437}{4352a^4X^3} + \frac{4807}{24576a^5X^2} + \frac{33649}{98304a^6X}\right)\sqrt{x} + \frac{100947}{196608a^6}\int \frac{dx}{X\sqrt{x}}$$

* If a and b have the same signs, then is

$$\int \frac{dx}{X\sqrt{x}} = \frac{1}{bk^3\sqrt{2}}\left[\log. \frac{x + k\sqrt{2x} + k^2}{\sqrt{X}} + \text{arc tang.} \frac{k\sqrt{2x}}{k^2 - x}\right]$$

where $k = \sqrt[4]{\frac{a}{b}}$.

If a and b have different signs, then is

$$\int \frac{dx}{X\sqrt{x}} = \frac{1}{2bk^3}\left[\log. \frac{k - \sqrt{x}}{k + \sqrt{x}} - 2 \text{ arc tang.} \frac{\sqrt{x}}{k}\right]$$

where $k = \sqrt[4]{-\frac{a}{b}}$.

TAB. LXXXVI.

$$\int \frac{x^m dx\sqrt{x}}{a+bx^2}$$

$$a + bx^2 = X$$

$$\int \frac{dx\sqrt{x}}{X} = \int \frac{dx\sqrt{x}}{X} \;^*$$

$$\int \frac{xdx\sqrt{x}}{X} = \frac{2\sqrt{x}}{b} - \frac{a}{b}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{x^2dx\sqrt{x}}{X} = \frac{2x\sqrt{x}}{3b} - \frac{a}{b}\int \frac{dx\sqrt{x}}{X}$$

$$\int \frac{x^3dx\sqrt{x}}{X} = \left(\frac{x^2}{5b} - \frac{a}{b^2}\right)2\sqrt{x} + \frac{a^2}{b^2}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{x^4dx\sqrt{x}}{X} = \left(\frac{x^3}{7b} - \frac{ax}{3b^2}\right)2\sqrt{x} + \frac{a^2}{b^2}\int \frac{dx\sqrt{x}}{X}$$

$$\int \frac{x^5dx\sqrt{x}}{X} = \left(\frac{x^4}{9b} - \frac{ax^2}{5b^2} + \frac{a^2}{b^3}\right)2\sqrt{x} - \frac{a^3}{b^3}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{x^6dx\sqrt{x}}{X} = \left(\frac{x^5}{11b} - \frac{ax^3}{7b^2} + \frac{a^2x}{3b^3}\right)2\sqrt{x} - \frac{a^3}{b^3}\int \frac{dx\sqrt{x}}{X}$$

$$\int \frac{x^7dx\sqrt{x}}{X} = \left(\frac{x^6}{13b} - \frac{ax^4}{9b^2} + \frac{a^2x^2}{5b^3} - \frac{a^3}{b^4}\right)2\sqrt{x} + \frac{a^4}{b^4}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{x^8dx\sqrt{x}}{X} = \left(\frac{x^7}{15b} - \frac{ax^5}{11b^2} + \frac{a^2x^3}{7b^3} - \frac{a^3x}{3b^4}\right)2\sqrt{x} + \frac{a^4}{b^4}\int \frac{dx\sqrt{x}}{X}$$

$$\int \frac{x^9dx\sqrt{x}}{X} = \left(\frac{x^8}{17b} - \frac{ax^6}{13b^2} + \frac{a^2x^4}{9b^3} - \frac{a^3x^2}{5b^4} + \frac{a^4}{b^5}\right)2\sqrt{x} - \frac{a^5}{b^5}\int \frac{dx}{X\sqrt{x}}$$

* If a and b have tne same signs, then is

$$\int \frac{dx\sqrt{x}}{X} = \frac{1}{bk\sqrt{2}}\left[-\log.\frac{x+k^2+k\sqrt{2x}}{\sqrt{X}} + \text{arc tang.}\frac{k\sqrt{2x}}{k^2-x}\right]$$

whence $k = \sqrt[4]{\frac{b}{a}}$.

If a and b have different signs, then is

$$\int \frac{dx\sqrt{x}}{X} = \frac{1}{2bk}\left[\log.\frac{k-\sqrt{x}}{k+\sqrt{x}} + 2\,\text{arc tang.}\frac{\sqrt{x}}{k}\right]$$

whence $k = \sqrt[4]{-\frac{a}{b}}$

TAB. LXXXVII.

$$\int \frac{x^m dx\sqrt{x}}{(a+bx^2)^2}, \quad \int \frac{x^m dx\sqrt{x}}{(a+bx^2)^3}$$

$$a + bx^2 = X$$

$$\int \frac{dx\sqrt{x}}{X^2} = \frac{x\sqrt{x}}{2aX} + \frac{1}{4a}\int \frac{dx\sqrt{x}}{X}$$

$$\int \frac{x dx\sqrt{x}}{X^2} = -\frac{\sqrt{x}}{2bX} + \frac{1}{4b}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{x^2 dx\sqrt{x}}{X^2} = -\frac{x\sqrt{x}}{2bX} + \frac{3}{4b}\int \frac{dx\sqrt{x}}{X}$$

$$\int \frac{x^3 dx\sqrt{x}}{X^2} = \left(\frac{2x^2}{b} + \frac{5a}{2b^2}\right)\frac{\sqrt{x}}{X} - \frac{5a}{4b^2}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{x^4 dx\sqrt{x}}{X^2} = \left(\frac{2x^3}{3b} + \frac{7ax}{6b^2}\right)\frac{\sqrt{x}}{X} - \frac{7a}{4b^2}\int \frac{dx\sqrt{x}}{X}$$

$$\int \frac{x^5 dx\sqrt{x}}{X^2} = \left(\frac{2x^4}{5b} - \frac{18ax^2}{5b^2} - \frac{9a^2}{2b^3}\right)\frac{\sqrt{x}}{X} + \frac{9a^2}{4b^3}\int \frac{dx}{X\sqrt{x}}.$$

$$\int \frac{x^6 dx\sqrt{x}}{X^2} = \left(\frac{2x^5}{7b} - \frac{22ax^3}{21b^2} - \frac{11a^2x}{6b^3}\right)\frac{\sqrt{x}}{X} + \frac{11a^2}{4b^3}\int \frac{dx\sqrt{x}}{X}$$

$$\int \frac{x^7 dx\sqrt{x}}{X^2} = \left(\frac{2x^6}{9b} - \frac{26ax^4}{45b^2} + \frac{26a^2x^2}{5b^3} + \frac{13a^3}{2b^4}\right)\frac{\sqrt{x}}{X} - \frac{13a^3}{4b^4}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{dx\sqrt{x}}{X^3} = \left(\frac{1}{4aX^2} + \frac{5}{16a^2X}\right)x\sqrt{x} + \frac{5}{32a^2}\int \frac{dx\sqrt{x}}{X}$$

$$\int \frac{x dx\sqrt{x}}{X^3} = \frac{(bx^2 - 3a)\sqrt{x}}{16abX^2} + \frac{3}{32ab}\int \frac{dx}{X\sqrt{x}}$$

$$\int \frac{x^2 dx\sqrt{x}}{X^3} = -\frac{2x\sqrt{x}}{5bX^2} + \frac{3a}{5b}\int \frac{dx\sqrt{x}}{X^3}$$

$$\int \frac{x^3 dx\sqrt{x}}{X^3} = -\frac{2x^2\sqrt{x}}{3bX^2} + \frac{5a}{3b}\int \frac{x dx\sqrt{X}}{X^3}$$

$$\int \frac{x^4 dx\sqrt{x}}{X^3} = \left(-\frac{x^3}{b} - \frac{7ax}{5b^2}\right)\frac{2\sqrt{x}}{X^2} + \frac{21a^2}{5b^2}\int \frac{dx\sqrt{x}}{X^3}$$

$$\int \frac{x^5 dx\sqrt{x}}{X^3} = \left(\frac{x^4}{b} + \frac{3ax^2}{b^2}\right)\frac{2\sqrt{x}}{X^2} - \frac{15a^2}{b^2}\int \frac{x dx\sqrt{x}}{X^3}$$

$$\int \frac{x^6 dx\sqrt{x}}{X^3} = \left(\frac{x^5}{3b} + \frac{11ax^3}{3b^2} + \frac{77a^2x}{15b^3}\right)\frac{2\sqrt{x}}{X^2} - \frac{77a^3}{5b^3}\int \frac{dx\sqrt{x}}{X^3}$$

TAB. LXXXVIII

$$\int\frac{dx}{(a+bx)x^m\sqrt{x}}\,,\quad \int\frac{dx}{(a+bx)^2x^m\sqrt{x}}$$

$$a+bx=X$$

$$\int\frac{dx}{Xx\sqrt{x}}=-\frac{2}{a\sqrt{x}}-\frac{b}{a}\int\frac{dx}{X\sqrt{x}}$$

$$\int\frac{dx}{Xx^2\sqrt{x}}=\left(-\frac{1}{3ax}+\frac{b}{a^2}\right)\frac{2}{\sqrt{x}}+\frac{b^2}{a^2}\int\frac{dx}{X\sqrt{x}}$$

$$\int\frac{dx}{Xx^3\sqrt{x}}=\left(-\frac{1}{5ax^2}+\frac{b}{3a^2x}-\frac{b^2}{a^3}\right)\frac{2}{\sqrt{x}}-\frac{b^3}{a^3}\int\frac{dx}{X\sqrt{x}}$$

$$\int\frac{dx}{Xx^4\sqrt{x}}=\left(-\frac{1}{7ax^3}+\frac{b}{5a^2x^2}-\frac{b^2}{3a^3x}+\frac{b^3}{a^4}\right)\frac{2}{\sqrt{x}}+\frac{b^4}{a^4}\int\frac{dx}{X\sqrt{x}}$$

$$\int\frac{dx}{Xx^5\sqrt{x}}=\left(-\frac{1}{9ax^4}+\frac{b}{7a^2x^3}-\frac{b^2}{5a^3x^2}+\frac{b^3}{3a^4x}-\frac{b^4}{a^5}\right)\frac{2}{\sqrt{x}}-\frac{b^5}{a^5}\int\frac{dx}{X\sqrt{x}}$$

$$\int\frac{dx}{Xx^6\sqrt{x}}=\left(-\frac{1}{11ax^5}+\frac{b}{9a^2x^4}-\frac{b^2}{7a^3x^3}+\frac{b^3}{5a^4x^3}-\frac{b^4}{3a^5x}+\frac{b^5}{a^6}\right)\frac{2}{\sqrt{x}}$$
$$+\frac{b^6}{a^6}\int\frac{dx}{X\sqrt{x}}$$

$$\int\frac{dx}{X^2x\sqrt{x}}=-\frac{2}{aX\sqrt{x}}-\frac{3b}{a}\int\frac{dx}{X^2\sqrt{x}}$$

$$\int\frac{dx}{X^2x^2\sqrt{x}}=\left(-\frac{1}{3ax}+\frac{5b}{3a^2}\right)\frac{2}{X\sqrt{x}}+\frac{5b^2}{a^2}\int\frac{dx}{X^2\sqrt{x}}$$

$$\int\frac{dx}{X^2x^3\sqrt{x}}=\left(-\frac{1}{5ax^2}+\frac{7b}{15a^2x}-\frac{7b^2}{3a^3}\right)\frac{2}{X\sqrt{x}}-\frac{7b^3}{a^3}\int\frac{dx}{X^2\sqrt{x}}$$

$$\int\frac{dx}{X^2x^4\sqrt{x}}=\left(-\frac{1}{7ax^3}+\frac{9b}{35a^2x^2}-\frac{3b^2}{5a^3x}+\frac{3b^3}{a^4}\right)\frac{2}{X\sqrt{x}}+\frac{9b^4}{a^4}\int\frac{dx}{X^2\sqrt{x}}$$

$$\int\frac{dx}{X^2x^5\sqrt{x}}=\left(-\frac{1}{9ax^4}+\frac{11b}{63a^2x^3}-\frac{11b^2}{35a^3x^2}+\frac{11b^3}{15a^4x}-\frac{11b^4}{3a^5}\right)\frac{2}{X\sqrt{x}}$$
$$-\frac{11b^5}{a^5}\int\frac{dx}{X^2\sqrt{x}}$$

$$\int\frac{dx}{X^2x^6\sqrt{x}}=\left(-\frac{1}{11ax^5}+\frac{13b}{99a^2x^4}-\frac{13b^2}{63a^3x^3}+\frac{13b^3}{35a^4x^2}-\frac{13b^4}{15a^5x}\right.$$
$$\left.+\frac{13b^5}{3a^6}\right)\frac{2}{X\sqrt{x}}+\frac{13b^6}{a^6}\int\frac{dx}{X^2\sqrt{x}}$$

TAB. LXXXIX.

$$\int\frac{dx}{(a+bx)^3x^m\sqrt{x}},\ \int\frac{dx}{(a+bx)^4x^m\sqrt{x}}$$

$$a+bx=X$$

$$\int\frac{dx}{X^3x\sqrt{x}}=-\frac{2}{aX^2\sqrt{x}}-\frac{5b}{a}\int\frac{dx}{X^3\sqrt{x}}$$

$$\int\frac{dx}{X^3x^2\sqrt{x}}=\left(-\frac{1}{3ax}+\frac{7b}{3a^2}\right)\frac{2}{X^2\sqrt{x}}+\frac{35b^2}{3a^2}\int\frac{dx}{X^3\sqrt{x}}$$

$$\int\frac{dx}{X^3x^3\sqrt{x}}=\left(-\frac{1}{5ax^2}+\frac{3b}{5a^2x}-\frac{21b^2}{5a^3}\right)\frac{2}{X^2\sqrt{x}}+\frac{21b^3}{a^3}\int\frac{dx}{X^3\sqrt{x}}$$

$$\int\frac{dx}{X^3x^4\sqrt{x}}=\left(-\frac{1}{7ax^3}+\frac{11b}{35a^2x^2}-\frac{33b^2}{35a^3x}+\frac{33b^3}{5a^4}\right)\frac{2}{X^2\sqrt{x}}+\frac{33b^4}{a^4}\int\frac{dx}{X^3\sqrt{x}}$$

$$\int\frac{dx}{X^3x^5\sqrt{x}}=\left(-\frac{1}{9ax^4}+\frac{13b}{63a^2x^3}-\frac{143b^2}{315a^3x^2}+\frac{143b^3}{105a^4x}-\frac{143b^4}{15a^5}\right)\frac{2}{X^2\sqrt{x}}$$
$$-\frac{143b^5}{3a^5}\int\frac{dx}{X^3\sqrt{x}}$$

$$\int\frac{dx}{X^3x^6\sqrt{x}}=\left(-\frac{1}{11ax^5}+\frac{5b}{33a^2x^4}-\frac{65b^2}{231a^3x^3}+\frac{13b^3}{21a^4x^2}-\frac{13b^4}{7a^5x}\right.$$
$$\left.+\frac{13b^5}{a^6}\right)\frac{2}{X^2\sqrt{x}}+\frac{65b^6}{a^6}\int\frac{dx}{X^3\sqrt{x}}$$

$$\int\frac{dx}{X^4x\sqrt{x}}=-\frac{2}{aX^3\sqrt{x}}-\frac{7b}{a}\int\frac{dx}{X^4\sqrt{x}}$$

$$\int\frac{dx}{X^4x^2\sqrt{x}}=\left(-\frac{1}{3ax}+\frac{3b}{a^2}\right)\frac{2}{X^3\sqrt{x}}+\frac{21b^2}{a^2}\int\frac{dx}{X^4\sqrt{x}}$$

$$\int\frac{dx}{X^4x^3\sqrt{x}}=\left(-\frac{1}{5ax^2}+\frac{11b}{15a^2x}-\frac{33b^2}{5a^3}\right)\frac{2}{X^3\sqrt{x}}-\frac{231b^3}{5a^3}\int\frac{dx}{X^4\sqrt{x}}$$

$$\int\frac{dx}{X^4x^4\sqrt{x}}=\left(-\frac{1}{7ax^3}+\frac{13b}{35a^4x^2}-\frac{143b^2}{105a^3x}+\frac{429b^3}{35a^4}\right)\frac{2}{X^3\sqrt{x}}$$
$$+\frac{429b^4}{5a^4}\int\frac{dx}{X^4\sqrt{x}}$$

$$\int\frac{dx}{X^4x^5\sqrt{x}}=\left(-\frac{9}{9ax^3}+\frac{5b}{21a^2x^3}-\frac{13b^2}{21a^3x^2}+\frac{143b^3}{63a^4x}-\frac{143b^4}{7a^5}\right)\frac{2}{X^3\sqrt{x}}$$
$$-\frac{143b^5}{a^3}\int\frac{dx}{X^4\sqrt{x}}$$

TAB. XC.

$$\int\frac{dx}{(f+gx)^n\sqrt{(a+bx)}}$$

$$f+gx=X,\ a+bx=X',\ bf-ag=k$$

$$\int\frac{dx}{X\sqrt{X'}}=\left\{\begin{array}{c}\pm\dfrac{2}{\sqrt{gk}}\text{ arc tang }\sqrt{\dfrac{gX'}{k}}\\ \text{or}\\ \dfrac{1}{\sqrt{-gk}}\log\dfrac{bf-2ag-bgx+2\sqrt{-gk}\,.\,\sqrt{X'}}{X}\end{array}\right\}^*$$

$$\int\frac{dx}{X^2\sqrt{X'}}=\frac{\sqrt{X'}}{kX}+\frac{b}{2k}\int\frac{dx}{X\sqrt{X'}}$$

$$\int\frac{dx}{X^3\sqrt{X'}}=\left(\frac{1}{2kX^2}+\frac{3b}{4k^2X}\right)\sqrt{X'}+\frac{3b^2}{8k^2}\int\frac{dx}{X\sqrt{X'}}$$

$$\int\frac{dx}{X^4\sqrt{X'}}=\left(\frac{1}{3kX^3}+\frac{5b}{12k^2X^2}+\frac{5b^2}{8k^3X}\right)\sqrt{X'}+\frac{5b^3}{16k^3}\int\frac{dx}{X\sqrt{X'}}$$

$$\int\frac{dx}{X^5\sqrt{X'}}=\left(\frac{1}{4kX^4}+\frac{7b}{24k^2X^3}+\frac{35b^2}{96k^3X^2}+\frac{35b^3}{64k^4X}\right)\sqrt{X'}$$
$$+\frac{35b^4}{128k^4}\int\frac{dx}{X\sqrt{X'}}$$

$$\int\frac{dx}{X^6\sqrt{X'}}=\left(\frac{1}{5kX^5}+\frac{9b}{40k^2X^4}+\frac{21b^2}{80k^3X^3}+\frac{21b^3}{64k^4X^2}+\frac{63b^4}{128k^5X}\right)\sqrt{X'}$$
$$+\frac{63b^5}{256k^5}\int\frac{dx}{X\sqrt{X'}}$$

$$\int\frac{dx}{X^7\sqrt{X'}}=\left(\frac{1}{6kX^6}+\frac{11b}{60k^2X^5}+\frac{33b^2}{160k^3X^4}+\frac{77b^3}{320k^4X^3}+\frac{77b^4}{256k^5X^2}\right.$$
$$\left.+\frac{231b^5}{512k^6X}\right)\sqrt{X'}+\frac{231b^6}{1024k^6}\int\frac{dx}{X\sqrt{X'}}$$

* The first expression is taken with the sign $+$, when g and k are both positive, and with the sign $-$, when g and k are both negative. The second is taken when g and k have different signs. When $k=0$, then $\int\frac{dx}{X\sqrt{X'}}$, becomes $\frac{b}{g}\int\frac{dx}{(a+bx)^{\frac{3}{2}}}=-\frac{2}{g\sqrt{(a+bx)}}$.

TAB. XCI.

$$\int\frac{x^m dx}{(f+gx)\sqrt{(a+bx)}},\quad \int\frac{x^m dx}{(f+gx)^2\sqrt{(a+bx)}}$$

$$f+gx=X,\quad a+bx=X'$$

$$\int\frac{x\,dx}{X\sqrt{X'}}=\frac{1}{g}\int\frac{dx}{\sqrt{X'}}-\frac{f}{g}\int\frac{dx}{X\sqrt{X'}}$$

$$\int\frac{x^2dx}{X\sqrt{X'}}=\frac{1}{g}\int\frac{x\,dx}{\sqrt{X'}}-\frac{f}{g^2}\int\frac{dx}{\sqrt{X'}}+\frac{f^2}{g^2}\int\frac{dx}{X\sqrt{X'}}$$

$$\int\frac{x^3dx}{X\sqrt{X'}}=\frac{1}{g}\int\frac{x^2dx}{\sqrt{X'}}-\frac{f}{g^2}\int\frac{x\,dx}{\sqrt{X'}}+\frac{f^2}{g^3}\int\frac{dx}{\sqrt{X'}}-\frac{f^3}{g^3}\int\frac{dx}{X\sqrt{X'}}$$

$$\int\frac{x^4dx}{X\sqrt{X'}}=\frac{1}{g}\int\frac{x^3dx}{\sqrt{X'}}-\frac{f}{g^2}\int\frac{x^2dx}{\sqrt{X'}}+\frac{f^2}{g^3}\int\frac{x\,dx}{\sqrt{X'}}-\frac{f^3}{g^4}\int\frac{dx}{\sqrt{X'}}$$
$$+\frac{f^4}{g^4}\int\frac{dx}{X\sqrt{X'}}$$

$$\int\frac{x^5dx}{X\sqrt{X'}}=\frac{1}{g}\int\frac{x^4dx}{\sqrt{X'}}-\frac{f}{g^2}\int\frac{x^3dx}{\sqrt{X'}}+\frac{f^2}{g^3}\int\frac{x^2dx}{\sqrt{X'}}-\frac{f^3}{g^4}\int\frac{x\,dx}{\sqrt{X'}}$$
$$+\frac{f^4}{g^5}\int\frac{dx}{\sqrt{X'}}-\frac{f^5}{g^5}\int\frac{dx}{X\sqrt{X'}}$$

$$\int\frac{x\,dx}{X^2\sqrt{X'}}=\frac{1}{g}\int\frac{dx}{X\sqrt{X'}}-\frac{f}{g}\int\frac{dx}{X^2\sqrt{X'}}$$

$$\int\frac{x^2dx}{X^2\sqrt{X'}}=\frac{1}{g^2}\int\frac{dx}{\sqrt{X'}}-\frac{2f}{g^2}\int\frac{dx}{X\sqrt{X'}}+\frac{f^2}{g^4}\int\frac{dx}{X^2\sqrt{X'}}$$

$$\int\frac{x^3dx}{X^2\sqrt{X'}}=\frac{1}{g^2}\int\frac{x\,dx}{\sqrt{X'}}-\frac{2f}{g^5}\int\frac{dx}{\sqrt{X'}}+\frac{3f^2}{g^3}\int\frac{dx}{X\sqrt{X'}}-\frac{f^3}{g^3}\int\frac{dx}{X^2\sqrt{X'}}$$

$$\int\frac{x^4dx}{X^2\sqrt{X'}}=\frac{1}{g^2}\int\frac{x^2dx}{\sqrt{X'}}-\frac{2f}{g^3}\int\frac{x\,dx}{\sqrt{X'}}+\frac{3f^2}{g^4}\int\frac{dx}{\sqrt{X'}}-\frac{4f^3}{g^4}\int\frac{dx}{X\sqrt{X'}}$$
$$+\frac{f^4}{g^4}\int\frac{dx}{X^2\sqrt{X'}}$$

$$\int\frac{x^5dx}{X^2\sqrt{X'}}=\frac{1}{g^2}\int\frac{x^3dx}{\sqrt{X'}}-\frac{2f}{g^3}\int\frac{x^2dx}{\sqrt{X'}}+\frac{3f^2}{g^4}\int\frac{x\,dx}{\sqrt{X'}}-\frac{4f^3}{g^5}\int\frac{dx}{\sqrt{X'}}$$
$$+\frac{5f^4}{g^5}\int\frac{dx}{X\sqrt{X'}}-\frac{f^5}{g^5}\int\frac{dx}{X^2\sqrt{X'}}$$

$$\int\frac{x^6dx}{X^2\sqrt{X'}}=\frac{1}{g^2}\int\frac{x^4dx}{\sqrt{X'}}-\frac{2f}{g^3}\int\frac{x^3dx}{\sqrt{X'}}+\frac{3f^2}{g^4}\int\frac{x^2dx}{\sqrt{X'}}-\frac{4f^3}{g^5}\int\frac{x\,dx}{\sqrt{X'}}$$
$$+\frac{5f^4}{g^6}\int\frac{dx}{\sqrt{X'}}-\frac{6f^5}{g^6}\int\frac{dx}{X\sqrt{X'}}-\frac{f^6}{g^6}\int\frac{dx}{X^2\sqrt{X'}}$$

TAB. XCII.

$$\int\frac{x^m dx}{(f+gx)^3\sqrt{(a+bx)}},\quad \int\frac{x^m dx}{(f+gx)^4\sqrt{(a+bx)}}$$

$$f+gx=X,\quad a+bx=X'$$

$$\int\frac{x\,dx}{X^3\sqrt{X'}}=\frac{1}{g}\int\frac{dx}{X^2\sqrt{X'}}-\frac{f}{g}\int\frac{dx}{X^3\sqrt{X'}}$$

$$\int\frac{x^2dx}{X^3\sqrt{X'}}=\frac{1}{g^2}\int\frac{dx}{X\sqrt{X'}}-\frac{2f}{g^2}\int\frac{dx}{X^2\sqrt{X'}}+\frac{f^2}{g^2}\int\frac{dx}{X^3\sqrt{X'}}$$

$$\int\frac{x^3dx}{X^3\sqrt{X'}}=\frac{1}{g^3}\int\frac{dx}{\sqrt{X'}}-\frac{3f}{g^3}\int\frac{dx}{X\sqrt{X'}}+\frac{3f^2}{g^3}\int\frac{dx}{X^2\sqrt{X'}}-\frac{f^3}{g^3}\int\frac{dx}{X^3\sqrt{X'}}$$

$$\int\frac{x^4dx}{X^3\sqrt{X'}}=\frac{1}{g^4}\int\frac{x\,dx}{\sqrt{X'}}-\frac{4f}{g^4}\int\frac{dx}{\sqrt{X'}}+\frac{6f^2}{g^4}\int\frac{dx}{X\sqrt{X'}}-\frac{4f^3}{g^4}\int\frac{dx}{X^2\sqrt{X'}}$$
$$+\frac{f^4}{g^4}\int\frac{dx}{X^3\sqrt{X'}}$$

$$\int\frac{x^5dx}{X^3\sqrt{X'}}=\frac{1}{g^5}\int\frac{x^2dx}{\sqrt{X'}}-\frac{5f}{g^5}\int\frac{x\,dx}{\sqrt{X'}}+\frac{10f^2}{g^5}\int\frac{dx}{\sqrt{X'}}-\frac{10f^3}{g^5}\int\frac{dx}{X\sqrt{X'}}$$
$$+\frac{5f^4}{g^5}\int\frac{dx}{X^2\sqrt{X'}}-\frac{f^5}{g^5}\int\frac{dx}{X^3\sqrt{X'}}$$

$$\int\frac{x\,dx}{X^4\sqrt{X'}}=\frac{1}{g}\int\frac{dx}{X^3\sqrt{X'}}-\frac{f}{g}\int\frac{dx}{X^4\sqrt{X'}}$$

$$\int\frac{x^2dx}{X^4\sqrt{X'}}=\frac{1}{g^2}\int\frac{dx}{X^2\sqrt{X'}}-\frac{2f}{g^2}\int\frac{dx}{X^3\sqrt{X'}}+\frac{f^2}{g^2}\int\frac{dx}{X^4\sqrt{X'}}$$

$$\int\frac{x^3dx}{X^4\sqrt{X'}}=\frac{1}{g^3}\int\frac{dx}{X\sqrt{X'}}-\frac{3f}{g^3}\int\frac{dx}{X^2\sqrt{X'}}+\frac{3f^2}{g^3}\int\frac{dx}{X^3\sqrt{X'}}$$
$$-\frac{f^3}{g^3}\int\frac{dx}{X^4\sqrt{X'}}$$

$$\int\frac{x^4dx}{X^4\sqrt{X'}}=\frac{1}{g^4}\int\frac{dx}{\sqrt{X'}}-\frac{4f}{g^4}\int\frac{dx}{X\sqrt{X'}}+\frac{6f^2}{g^4}\int\frac{dx}{X^2\sqrt{X'}}-\frac{4f^3}{g^4}\int\frac{dx}{X^3\sqrt{X'}}$$
$$+\frac{f^4}{g^4}\int\frac{dx}{X^4\sqrt{X'}}$$

$$\int\frac{x^5dx}{X^4\sqrt{X'}}=\frac{1}{g^5}\int\frac{x\,dx}{\sqrt{X'}}-\frac{5f}{g^5}\int\frac{dx}{\sqrt{X'}}+\frac{10f^2}{g^5}\int\frac{dx}{X\sqrt{X'}}-\frac{10f^3}{g^5}\int\frac{dx}{X^2\sqrt{X'}}$$
$$+\frac{5f^4}{g^5}\int\frac{dx}{X^3\sqrt{X'}}-\frac{f^5}{g^5}\int\frac{dx}{X^4\sqrt{X'}}$$

TAB. XCIII.

$$\int \frac{x^m dx}{(f+gx)\sqrt{(a+bx^2)}}$$

$$a+bx^2=X,\ f+gx=X',\ ag^2+bf^2=k$$

$$\int \frac{dx}{X'\sqrt{X}} = \left\{ \begin{array}{l} \pm \frac{1}{\sqrt{k}} \log \frac{ag-bfx \mp \sqrt{k}.\sqrt{X}}{X'} \\ \text{or} \\ \frac{1}{\sqrt{-k}} \text{ arc tang } \frac{ag-bfx}{\sqrt{-k}.\sqrt{X}} \end{array} \right\}^*$$

$$\int \frac{xdx}{X'\sqrt{X}} = \frac{1}{g}\int \frac{dx}{\sqrt{X}} - \frac{f}{g}\int \frac{dx}{X'\sqrt{X}}$$

$$\int \frac{x^2dx}{X'\sqrt{X}} = \frac{1}{g}\int \frac{xdx}{\sqrt{X}} - \frac{f}{g^2}\int \frac{dx}{\sqrt{X}} + \frac{f^2}{g^2}\int \frac{dx}{X'\sqrt{X}}$$

$$\int \frac{x^3dx}{X'\sqrt{X}} = \frac{1}{g}\int \frac{x^2dx}{\sqrt{X}} - \frac{f}{g^2}\int \frac{xdx}{\sqrt{X}} + \frac{f^2}{g^3}\int \frac{dx}{\sqrt{X}} - \frac{f^3}{g^3}\int \frac{dx}{X'\sqrt{X}}$$

$$\int \frac{x^4dx}{X'\sqrt{X}} = \frac{1}{g}\int \frac{x^3dx}{\sqrt{X}} - \frac{f}{g^2}\int \frac{x^2dx}{\sqrt{X}} + \frac{f^2}{g^3}\int \frac{xdx}{\sqrt{X}} - \frac{f^3}{g^4}\int \frac{dx}{\sqrt{X}} + \frac{f^4}{g^4}\int \frac{dx}{X'\sqrt{X}}$$

$$\int \frac{x^5dx}{X'\sqrt{X}} = \frac{1}{g}\int \frac{x^4dx}{\sqrt{X}} - \frac{f}{g^2}\int \frac{x^3dx}{\sqrt{X}} + \frac{f^2}{g^3}\int \frac{x^2dx}{\sqrt{X}} - \frac{f^3}{g^4}\int \frac{xdx}{\sqrt{X}} + \frac{f^4}{g^5}\int \frac{dx}{\sqrt{X}} - \frac{f^5}{g^5}\int \frac{dx}{X'\sqrt{X}}$$

* The first expression is real, when k is positive; the second, when k is negative. With regard to the signs $\pm$ $\mp$ which occur in the first expression, the upper are taken together; and so, in like manner, the lower; otherwise it is immaterial which are used. Moreover

$$\text{arc tang } \frac{ag-bfx}{\sqrt{-k}.\sqrt{X}} = \text{arc sin } \frac{ag-bfx}{(f+gx)\sqrt{-ab}}$$
$$= \text{arc cos } \frac{\sqrt{-k}.\sqrt{X}}{(f+gx)\sqrt{-ab}} = \&c.$$

The factor $\sqrt{-ab}$, which occurs in the sine and cosine, is necessarily real, because a and b can neither be both positive nor both negative at the same time; for in the first case k would be positive, and therefore the logarithmic form valid; in the second case $\sqrt{(a+bx^2)}$ would be necessarily imaginary.

TAB. XCIV.

$$\int\frac{x^m dx}{(f+gx^2)\sqrt{(a+bx^2)}},\quad \int\frac{x^m dx\,\sqrt{(a+bx^2)}}{f+gx^2}$$

$$a+bx^2=X,\ f+gx^2=X'$$

$$\left.\begin{aligned}\int\frac{dx}{X'\sqrt{X}}&=\int\frac{dx}{X'\sqrt{X}}\\ \int\frac{xdx}{X'\sqrt{X}}&=\int\frac{xdx}{X'\sqrt{X}}\end{aligned}\right\}$$ (see the following page)

$$\int\frac{x^2dx}{X'\sqrt{X}}=\frac{1}{g}\int\frac{dx}{\sqrt{X}}-\frac{f}{g}\int\frac{dx}{X'\sqrt{X}}$$

$$\int\frac{x^3dx}{X'\sqrt{X}}=\frac{1}{g}\int\frac{xdx}{\sqrt{X}}-\frac{f}{g}\int\frac{xdx}{X'\sqrt{X}}$$

$$\int\frac{x^4dx}{X'\sqrt{X}}=\frac{1}{g}\int\frac{x^2dx}{\sqrt{X}}-\frac{f}{g^2}\int\frac{dx}{\sqrt{X}}+\frac{f^2}{g^2}\int\frac{dx}{X'\sqrt{X}}$$

$$\int\frac{x^5dx}{X'\sqrt{X}}=\frac{1}{g}\int\frac{x^3dx}{\sqrt{X}}-\frac{f}{g^2}\int\frac{xdx}{\sqrt{X}}+\frac{f^2}{g^2}\int\frac{xdx}{X'\sqrt{X}}$$

$$\int\frac{x^6dx}{X'\sqrt{X}}=\frac{1}{g}\int\frac{x^4dx}{\sqrt{X}}-\frac{f}{g^2}\int\frac{x^2dx}{\sqrt{X}}+\frac{f^2}{g^3}\int\frac{dx}{\sqrt{X}}-\frac{f^3}{g^3}\int\frac{dx}{X'\sqrt{X}}$$

$$\int\frac{x^7dx}{X'\sqrt{X}}=\frac{1}{g}\int\frac{x^5dx}{\sqrt{X}}-\frac{f}{g^2}\int\frac{x^3dx}{\sqrt{X}}+\frac{f^2}{g^3}\int\frac{xdx}{\sqrt{X}}+\frac{f^3}{g^3}\int\frac{xdx}{X'\sqrt{X}}$$

$$\int\frac{dx\sqrt{X}}{X'}=\frac{b}{g}\int\frac{dx}{\sqrt{X}}+\left(a-\frac{bf}{g}\right)\int\frac{dx}{X'\sqrt{X}}$$

$$\int\frac{xdx\sqrt{X}}{X'}=\frac{b}{g}\int\frac{xdx}{\sqrt{X}}+\left(a-\frac{bf}{g}\right)\int\frac{xdx}{X'\sqrt{X}}$$

$$\int\frac{x^2dx\sqrt{X}}{X'}=\frac{b}{g}\int\frac{x^2dx}{\sqrt{X}}+\left(\frac{a}{g}-\frac{bf}{g^2}\right)\int\frac{dx}{\sqrt{X}}-\left(\frac{af}{g}-\frac{bf^2}{g^2}\right)\int\frac{dx}{X'\sqrt{X}}$$

$$\int\frac{x^3dx\sqrt{X}}{X'}=\frac{b}{g}\int\frac{x^3dx}{\sqrt{X}}+\left(\frac{a}{g}-\frac{bf}{g^2}\right)\int\frac{xdx}{\sqrt{X}}-\left(\frac{af}{g}-\frac{bf^2}{g^2}\right)\int\frac{xdx}{X'\sqrt{X}}$$

$$\int\frac{x^4dx\sqrt{X}}{X'}=\frac{b}{g}\int\frac{x^4dx}{\sqrt{X}}+\left(\frac{a}{g}-\frac{bf}{g^2}\right)\int\frac{x^2dx}{\sqrt{X}}-\left(\frac{af}{g^2}-\frac{bf^2}{g^3}\right)\int\frac{dx}{\sqrt{X}}$$
$$+\left(\frac{af^2}{g^2}-\frac{bf^3}{g^3}\right)\int\frac{dx}{X'\sqrt{X}}$$

$$\int\frac{x^5dx\sqrt{X}}{X'}=\frac{b}{g}\int\frac{x^5dx}{\sqrt{X}}+\left(\frac{a}{g}-\frac{bf}{g^2}\right)\int\frac{x^3dx}{\sqrt{X}}-\left(\frac{af}{g^2}-\frac{bf^2}{g^3}\right)\int\frac{xdx}{\sqrt{X}}$$
$$+\left(\frac{af^2}{g^2}-\frac{bf^3}{g^3}\right)\int\frac{xdx}{X'\sqrt{X}}$$

Note on the preceding Table.

$$\text{I. } \int \frac{dx}{X'\sqrt{X}}$$

In general, whatever may be the signs of $a, b, f, g,$

$$\int \frac{dx}{X'\sqrt{X}} = \frac{1}{\sqrt{(bf^2 - afg)}} \log \frac{f\sqrt{(a+bx^2)} + x\sqrt{(bf^2 - afg)}}{\sqrt{(f+gx^2)}}$$

$$\text{or} \int \frac{dx}{X'\sqrt{X}} = \frac{1}{\sqrt{(afg - bf^2)}} \text{ arc tang } \frac{x\sqrt{(afg - bf^2)}}{f\sqrt{(a+bx^2)}}$$

The first form is real, when $bf^2 - afg$ is positive; the second, when $bf^2 - afg$ is negative. For $\sqrt{(f+gx^2)}$ we may also put $\sqrt{-(f+gx^2)}$, if f and g be negative. Moreover

$$\text{arc tang } \frac{x\sqrt{(afg - bf^2)}}{f\sqrt{(a+bx^2)}} = \text{arc cos } \sqrt{\frac{af + bfx^2}{af + agx^2}}$$

$$= \text{arc sin } x\sqrt{\frac{ag - bf}{af + agx^2}} = \text{\&c.}$$

$$\text{II. } \int \frac{xdx}{X'\sqrt{X}}$$

For $a, b, f, g,$ we have either

$$\int \frac{xdx}{X'\sqrt{X}} = \frac{1}{\sqrt{(ag^2 - bfg)}} \log \frac{g\sqrt{(a+bx^2)} - \sqrt{(ag^2 - bfg)}}{\sqrt{(f+gx^2)}}$$

$$\text{or} \int \frac{xdx}{X'\sqrt{X}} = \frac{1}{\sqrt{(bfg - ag^2)}} \text{ arc tang } \frac{g\sqrt{(a+bx^2)}}{\sqrt{(bfg - ag^2)}}.$$

The first form is real when $ag^2 - bfg$ is positive; the second, when $ag^2 - bfg$ is negative. With regard to $\sqrt{(f+gx^2)}$ the observation already made applies also in this case. Moreover

$$\text{arc tang } \frac{g\sqrt{(a+bx^2)}}{\sqrt{(bfg - ag^2)}} = \text{arc cos } \sqrt{\frac{bf - ag}{bf + bgx^2}}$$

$$= \text{arc sin} \sqrt{\frac{ag + bgx^2}{bf + bgx^2}} = \text{\&c}$$

TAB. XCV.

$$\int\frac{dx}{(fx^m+gx^{m+1})\sqrt{(a+bx)}},\ \int\frac{dx}{(fx^m+gx^{m+1})\sqrt{(a+bx^2)}}$$

$$a+bx=X,\ a+bx^2=X',\ f+gx=Z$$

$$\int\frac{dx}{xZ\sqrt{X}}=\frac{1}{f}\int\frac{dx}{x\sqrt{X}}-\frac{g}{f}\int\frac{dx}{Z\sqrt{X}}$$

$$\int\frac{dx}{x^2Z\sqrt{X}}=\frac{1}{f}\int\frac{dx}{x^2\sqrt{X}}-\frac{g}{f^2}\int\frac{dx}{x\sqrt{X}}+\frac{g^2}{f^2}\int\frac{dx}{Z\sqrt{X}}$$

$$\int\frac{dx}{x^3Z\sqrt{X}}=\frac{1}{f}\int\frac{dx}{x^3\sqrt{X}}-\frac{g}{f^2}\int\frac{dx}{x^2\sqrt{X}}+\frac{g^2}{f^3}\int\frac{dx}{x\sqrt{X}}-\frac{g^3}{f^3}\int\frac{dx}{Z\sqrt{X}}$$

$$\int\frac{dx}{x^4Z\sqrt{X}}=\frac{1}{f}\int\frac{dx}{x^4\sqrt{X}}-\frac{g}{f^2}\int\frac{dx}{x^3\sqrt{X}}+\frac{g^2}{f^3}\int\frac{dx}{x^2\sqrt{X}}-\frac{g^3}{f^4}\int\frac{dx}{x\sqrt{X}}+\frac{g^4}{f^4}\int\frac{dx}{Z\sqrt{X}}$$

....

$$\int\frac{dx}{x^mZ\sqrt{X}}=\frac{1}{f}\int\frac{dx}{x^m\sqrt{X}}-\frac{g}{f^2}\int\frac{dx}{x^{m-1}\sqrt{X}}+\frac{g^2}{f^3}\int\frac{dx}{x^{m-2}\sqrt{X}}-\&c.\ \pm\frac{g^{m-1}}{f^m}\int\frac{dx}{x\sqrt{X}}\mp\frac{g^m}{f^m}\int\frac{dx}{Z\sqrt{X}}$$

$$\int\frac{dx}{xZ\sqrt{X'}}=\frac{1}{f}\int\frac{dx}{x\sqrt{X'}}-\frac{g}{f}\int\frac{dx}{Z\sqrt{X'}}$$

$$\int\frac{dx}{x^2Z\sqrt{X'}}=\frac{1}{f}\int\frac{dx}{x^2\sqrt{X'}}-\frac{g}{f^2}\int\frac{dx}{x\sqrt{X'}}+\frac{g^2}{f^2}\int\frac{dx}{Z\sqrt{X'}}$$

$$\int\frac{dx}{x^3Z\sqrt{X'}}=\frac{1}{f}\int\frac{dx}{x^3\sqrt{X'}}-\frac{g}{f^2}\int\frac{dx}{x^2\sqrt{X'}}+\frac{g^2}{f^3}\int\frac{dx}{x\sqrt{X'}}-\frac{g^3}{f^3}\int\frac{dx}{Z\sqrt{X'}}$$

$$\int\frac{dx}{x^4Z\sqrt{X'}}=\frac{1}{f}\int\frac{dx}{x^4\sqrt{X'}}-\frac{g}{f^2}\int\frac{dx}{x^3\sqrt{X'}}+\frac{g^2}{f^3}\int\frac{dx}{x^2\sqrt{X'}}-\frac{g^3}{f^4}\int\frac{dx}{x\sqrt{X'}}+\frac{g^4}{f^4}\int\frac{dx}{Z\sqrt{X'}}$$

....

$$\int\frac{dx}{x^mZ\sqrt{X'}}=\frac{1}{f}\int\frac{dx}{x^m\sqrt{X'}}-\frac{g}{f^2}\int\frac{dx}{x^{m-1}\sqrt{X'}}+\frac{g^2}{f^3}\int\frac{dx}{x^{m-2}\sqrt{X'}}-\&c.\ \pm\frac{g^{m-1}}{f^m}\int\frac{dx}{x\sqrt{X'}}\mp\frac{g^m}{f^m}\int\frac{dx}{Z\sqrt{X'}}$$

TAB. XCVI.

$$\int \frac{x^m dx}{(f+gx)\sqrt{(a+bx+cx^2)}}$$

$$a+bx+cx^2 = X,\ f+gx = Z$$
$$ag^2 - bfg + cf^2 = k$$

$$\int \frac{dx}{Z\sqrt{X}} = \begin{cases} \pm \dfrac{1}{\sqrt{k}} \log \dfrac{2ag - bf + (bg - 2cf)x \mp 2\sqrt{k}\,.\,\sqrt{X}}{f+gx} \;* \\ \text{or} \\ \dfrac{1}{\sqrt{-k}} \text{arc tang} \dfrac{2ag - bf + (bg - 2cf)x}{2\sqrt{-k}\,.\,\sqrt{X}} \end{cases}$$

$$\int \frac{x dx}{Z\sqrt{X}} = \frac{1}{g}\int\frac{dx}{\sqrt{X}} - \frac{f}{g}\int\frac{dx}{Z\sqrt{X}}$$

$$\int \frac{x^2 dx}{Z\sqrt{X}} = \frac{1}{g}\int\frac{x dx}{\sqrt{X}} - \frac{f}{g^2}\int\frac{dx}{\sqrt{X}} + \frac{f^2}{g^2}\int\frac{dx}{Z\sqrt{X}}$$

$$\int \frac{x^3 dx}{Z\sqrt{X}} = \frac{1}{g}\int\frac{x^2 dx}{\sqrt{X}} - \frac{f}{g^2}\int\frac{x dx}{\sqrt{X}} + \frac{f^2}{g^3}\int\frac{dx}{\sqrt{X}} - \frac{f^3}{g^3}\int\frac{dx}{Z\sqrt{X}}$$

$$\int \frac{x^4 dx}{Z\sqrt{X}} = \frac{1}{g}\int\frac{x^3 dx}{\sqrt{X}} - \frac{f}{g^2}\int\frac{x^2 dx}{\sqrt{X}} + \frac{f^2}{g^3}\int\frac{x dx}{\sqrt{X}} - \frac{f^3}{g^4}\int\frac{dx}{\sqrt{X}} + \frac{f^4}{g^4}\int\frac{dx}{Z\sqrt{X}}$$

$$\int \frac{x^5 dx}{Z\sqrt{X}} = \frac{1}{g}\int\frac{x^4 dx}{\sqrt{X}} - \frac{f}{g^2}\int\frac{x^3 dx}{\sqrt{X}} + \frac{f^2}{g^3}\int\frac{x^2 dx}{\sqrt{X}} - \frac{f^3}{g^4}\int\frac{x dx}{\sqrt{X}} + \frac{f^4}{g^5}\int\frac{dx}{\sqrt{X}} - \frac{f^5}{g^5}\int\frac{dx}{Z\sqrt{X}}$$

* The first expression is real, when k is positive; the second when k is negative. As to the signs $\pm$, $\mp$ in the first form, the upper are taken together; so also are the lower. Otherwise it is immaterial which are used. Moreover

$$\text{arc tang} \frac{2ag - bf + (bg - 2cf)x}{2\sqrt{-k}\,.\,\sqrt{X}} = \text{arc cos} \frac{2\sqrt{-k}\,.\,\sqrt{X}}{(f+gx)\sqrt{(b^2 - 4ac)}}$$
$$= \text{arc sin} \frac{2ag - bf + (bg - 2cf)x}{(f+gx)\sqrt{(b^2 - 4ac)}} = \&c.$$

The root of $\sqrt{(b^2 - 4ac)}$, which here occurs in the sine and cosine, is real, when $ag^2 - bfg + cf^2$ is negative; for otherwise $\sqrt{(a+bx+cx^2)}$ could not be real.

TABLE

of some more general Formulæ.

$$a + bx^n = X$$

$$\int x^m dx X^p = \frac{x^{m+1}X^p}{m+1} - \frac{pnb}{m+1}\int x^{m+n} dx X^{p-1}$$

$$\int \frac{x^m dx}{X^p} = -\frac{x^{m-n+1}}{(p-1)\,nbX^{p-1}} + \frac{m-n+1}{(p-1)\,nb}\int \frac{x^{m-n}dx}{X^{p-1}}$$

$$\int x^m dx X^p = \frac{x^{m-n+1}X^{p+1}}{(m+np+1)b} - \frac{(m-n+1)\,a}{(m+np+1)\,b}\int x^{m-n}dx X^p$$

$$\int \frac{x^m dx}{X^p} = \frac{x^{m-n+1}}{(m-np+1)bX^{p-1}} - \frac{(m-n+1)\,a}{(m-np+1)\,b}\int \frac{x^{m-n}dx}{X^p}$$

$$\int x^m dx X^p = \frac{x^{m+1}X^p}{m+np+1} + \frac{pna}{m+np+1}\int x^m dx X^{p-1}$$

$$\int \frac{dx X^p}{x^m} = -\frac{X^p}{(m-np-1)\,x^{m-1}} - \frac{pna}{m-np-1}\int \frac{dx X^{p-1}}{x^m}$$

$$\int \frac{dx X^p}{x^m} = -\frac{X^{p+1}}{(m-1)\,ax^{m-1}} - \frac{(m-n-np-1)\,b}{(m-1)\,a}\int \frac{dx X^p}{x^{m-n}}$$

$$\int \frac{dx}{x^m X^p} = -\frac{1}{(m-1)\,ax^{m-1}X^{p-1}} - \frac{(m-n+np-1)\,b}{(m-1)\,a}\int \frac{dx}{x^{m-n}X^p}$$

$$\int \frac{x^m dx}{X^p} = \frac{x^{m+1}}{(p-1)naX^{p-1}} - \frac{m+n-np+1}{(p-1)\,na}\int \frac{x^m dx}{X^{p-1}}$$

$$\int \frac{dx}{x^m X^p} = \frac{1}{(p-1)\,nax^{m-1}X^{p-1}} + \frac{m-n+np-1}{(p-1)\,na}\int \frac{dx}{x^m X^{p-1}}$$

$$\int \frac{dx}{X^p} = \frac{x}{(p-1)\,naX^{p-1}} + \frac{np-n-1}{(p-1)\,na}\int \frac{dx}{X^{p-1}}$$

$$\int dx X^p = \frac{xX^p}{np+1} + \frac{pna}{np+1}\int dx X^{p-1}$$

TABLE

Of some more general Formulæ.

$$ax^k + bx^{k+n} = X$$

$$\int x^m \mathrm{d}x X^p = \frac{x^{m+1} X^p}{m+pk+1} - \frac{pnb}{m+pk+1} \int x^{m+k+n} \mathrm{d}x X^{p-1}$$

$$\int \frac{x^m \mathrm{d}x}{X^p} = -\frac{x^{m-k-n+1}}{(p-1)nbX^{p-1}} + \frac{m-pk-n+1}{(p-1)nb} \int \frac{x^{m-k-n}\mathrm{d}x}{X^{p-1}}$$

$$\int x^m \mathrm{d}x X^p = \frac{x^{m-k-n+1} X^{p+1}}{(m+pk+np+1)b} - \frac{(m+pk-n+1)a}{(m+pk+np+1)b} \int x^{m-n} \mathrm{d}x X^p$$

$$\int \frac{x^m \mathrm{d}x}{X^p} = \frac{x^{m-k-n+1}}{(m-pk-np+1)bX^{p-1}} - \frac{(m-pk-n+1)a}{(m-pk-np+1)b} \int \frac{x^{m-n}\mathrm{d}x}{X^p}$$

$$\int x^m \mathrm{d}x X^p = \frac{x^{m+1} X^p}{m+pk+np+1} + \frac{pna}{m+pk+np+1} \int x^{m+k} \mathrm{d}x X^{p-1}$$

$$\int \frac{\mathrm{d}x X^p}{x^m} = -\frac{X^p}{(m-pk-np-1)x^{m-1}} - \frac{pna}{m-pk-np-1} \int \frac{\mathrm{d}x X^{p-1}}{x^{m-k}}$$

$$\int \frac{\mathrm{d}x X^p}{x^m} = -\frac{X^{p+1}}{(m-pk-1)ax^{m+k-1}} - \frac{(m-n-pk-np-1)b}{(m-pk-1)a} \int \frac{\mathrm{d}x X^p}{x^{m-n}}$$

$$\int \frac{\mathrm{d}x}{x^m X^p} = -\frac{1}{(m+pk-1)ax^{m+k-1}X^{p-1}} - \frac{(m-n+pk+np-1)b}{(m+pk-1)a} \int \frac{\mathrm{d}x}{x^{m-n}X^p}$$

$$\int \frac{x^m \mathrm{d}x}{X^p} = \frac{x^{m-k+1}}{(p-1)naX^{p-1}} - \frac{m+n-pk-np+1}{(p-1)na} \int \frac{x^{m-k}\mathrm{d}x}{X^{p-1}}$$

$$\int \frac{\mathrm{d}x}{x^m X^p} = \frac{1}{(p-1)nax^{m+k-1}X^{p-1}} + \frac{m-n+pk+np-1}{(p-1)na} \int \frac{\mathrm{d}x}{x^{m+k}X^{p-1}}$$

$$\int \frac{\mathrm{d}x}{X^p} = \frac{1}{(p-1)nax^{k-1}X^{p-1}} + \frac{pk+np-n-1}{(p-1)na} \int \frac{\mathrm{d}x}{x^k X^{p-1}}$$

$$\int \mathrm{d}x X^p = \frac{xX^p}{pk+np+1} + \frac{pna}{pk+np+1} \int x^k \mathrm{d}x X^{p-1}$$

TABLE

Of some more general Formulæ.

$$a + bx = X$$

$$\int \frac{x^m dx}{\sqrt{X}} = \left(\frac{X^m}{2m+1} - \frac{{}^{m}AaX^{m-1}}{2m-1} + \frac{{}^{m}Ba^2X^{m-2}}{2m-3} - \frac{{}^{m}Ca^3X^{m-3}}{2m-5} + \ldots\ldots\right.$$

$$\left.\ldots\ldots \pm \frac{{}^{m}\overset{-2}{M}a^{m-2}X^2}{5} \mp \frac{{}^{m}\overset{-1}{M}a^{m-1}X}{3} \pm \frac{{}^{m}Ma^m}{1}\right) \frac{2\sqrt{X}}{b^{m+1}}$$

$$\int \frac{x^m dx}{X^{\frac{3}{2}}} = \left(\frac{X^m}{2m-1} - \frac{{}^{m}AaX^{m-1}}{2m-3} + \frac{{}^{m}Ba^2X^{m-2}}{2m-5} - \frac{{}^{m}Ca^3X^{m-3}}{2m-7} + \ldots\ldots\right.$$

$$\left.\ldots\ldots \pm \frac{{}^{m}\overset{-2}{M}a^{m-2}X^2}{3} \mp \frac{{}^{m}\overset{-1}{M}a^{m-1}X}{1} \pm \frac{{}^{m}Ma^m}{-1}\right) \frac{2}{b^{m+1}\sqrt{X}}$$

$$\int \frac{x^m dx}{X^{\frac{5}{2}}} = \left(\frac{X^m}{2m-3} - \frac{{}^{m}AaX^{m-1}}{2m-5} + \frac{{}^{m}Ba^2X^{m-2}}{2m-7} - \frac{{}^{m}Ca^3X^{m-3}}{2m-9} + \ldots\ldots\right.$$

$$\left.\ldots\ldots \pm \frac{{}^{m}\overset{-2}{M}a^{m-2}X^2}{1} \mp \frac{{}^{m}\overset{-1}{M}a^{m-1}X}{-1} \pm \frac{{}^{m}Ma^m}{-3}\right) \frac{2}{b^{m+1}X\sqrt{X}}$$

$$\int \frac{x^m dx}{X^{\frac{7}{2}}} = \left(\frac{X^m}{2m-5} - \frac{{}^{m}AaX^{m-1}}{2m-7} + \frac{{}^{m}Ba^2X^{m-2}}{2m-9} - \frac{{}^{m}Ca^3X^{m-3}}{2m-11} + \ldots\ldots\right.$$

$$\left.\ldots\ldots \pm \frac{{}^{m}\overset{-2}{M}a^{m-2}X^2}{-1} \mp \frac{{}^{m}\overset{-1}{M}a^{m-1}X}{-3} \pm \frac{{}^{m}Ma^m}{-5}\right) \frac{2}{b^{m+1}X^2\sqrt{X}}$$

$$\int \frac{x^m dx}{X^{\frac{n}{2}}} = \left(\frac{X^m}{2m-n+2} - \frac{{}^{m}AaX^{m-1}}{2m-n} + \frac{{}^{m}Ba^2X^{m-2}}{2m-n-2} - \ldots\ldots\right.$$

$$\left.\ldots\ldots \pm \frac{{}^{m}\overset{-2}{M}a^{m-2}X^2}{-(n-6)} \mp \frac{{}^{m}\overset{-1}{M}a^{m-1}X}{-(n-4)} \pm \frac{{}^{m}Ma^m}{-(n-2)}\right) \frac{2}{b^{m+1}X^{\frac{n-2}{2}}}$$

$$\int x^m dx\sqrt{X} = \left(\frac{X^m}{2m+3} - \frac{{}^{m}AaX^{m-1}}{2m+1} + \frac{{}^{m}Ba^2X^{m-2}}{2m-1} - \frac{{}^{m}Ca^3X^{m-3}}{2m-3} + \ldots\ldots\right.$$

$$\left.\ldots\ldots \pm \frac{{}^{m}\overset{-2}{M}a^{m-2}X^2}{7} \mp \frac{{}^{m}\overset{-1}{M}a^{m-1}X}{5} \pm \frac{{}^{m}Ma^m}{3}\right) \frac{2X\sqrt{X}}{b^{m+1}}$$

TABLE

Of some more general Formulæ

$$a+bx=X$$

$$\int x^m dx X^{\frac{3}{2}} = \left(\frac{X^m}{2m+5} - \frac{{}^mAaX^{m-1}}{2m+3} + \frac{{}^mBa^2X^{m-2}}{2m+1} - \frac{{}^mCa^3X^{m-3}}{2m-1} + \ldots\right.$$

$$\left.\ldots\ldots \pm \frac{{}^m\overset{-2}{M}a^{m-2}X^2}{9} \mp \frac{{}^m\overset{-1}{M}a^{m-1}X}{7} \pm \frac{{}^mMa^m}{5}\right)\frac{2X^2\sqrt{X}}{b^{m+1}}$$

$$\int x^m dx X^{\frac{5}{2}} = \left(\frac{X^m}{2m+7} - \frac{{}^mAaX^{m-1}}{2m+5} + \frac{{}^mBa^2X^{m-2}}{2m+3} - \frac{{}^mCa^3X^{m-3}}{2m+1} + \ldots\right.$$

$$\left.\ldots\ldots \pm \frac{{}^m\overset{-2}{M}a^{m-2}X^2}{11} \mp \frac{{}^m\overset{-1}{M}a^{m-1}X}{9} \pm \frac{{}^mMa^m}{7}\right)\frac{2X^3\sqrt{X}}{b^{m+1}}$$

$$\int x^m dx X^{\frac{n}{2}} = \left(\frac{X^m}{2m+n+2} - \frac{{}^mAaX^{m-1}}{2m+n} + \frac{{}^mBa^2X^{m-2}}{2m+n-2} - \frac{{}^mCa^3X^{m-3}}{2m+n-4} + \ldots\right.$$

$$\left.\ldots\ldots \pm \frac{{}^m\overset{-2}{M}a^{m-2}X^2}{n+6} \mp \frac{{}^m\overset{-1}{M}a^{m-1}X}{n+4} \pm \frac{{}^mMa^m}{n+2}\right)\frac{2X^{\frac{n+2}{2}}}{b^{m+1}}$$

$$\int \frac{x^m dx}{X^{\frac{p}{q}}} = \left(\frac{X^m}{qm-p+q} - \frac{{}^mAaX^{m-1}}{qm-p} + \frac{{}^mBa^2X^{m-2}}{qm-p-q} - \frac{{}^mCa^3X^{m-3}}{qm-p-2q}\right.$$

$$+ \frac{{}^mDa^4X^{m-4}}{qm-p-3q} \ldots\ldots$$

$$\left.\ldots\ldots \pm \frac{{}^m\overset{-2}{M}a^{m-2}X^2}{-(p-3q)} \mp \frac{{}^m\overset{-1}{M}a^{m-1}X}{-(p-2q)} \pm \frac{{}^mMa^m}{-(p-q)}\right)\frac{q}{b^{m+1}X^{\frac{p}{q}-1}}$$

$$\int x^m dx X^{\frac{p}{q}} = \left(\frac{X^m}{qm+p+q} - \frac{{}^mAaX^{m-1}}{qm+p} + \frac{{}^mBa^2X^{m-2}}{qm+p-q} - \frac{{}^mCa^3X^{m-3}}{qm+p-2q}\right.$$

$$+ \frac{{}^mDa^4X^{m-4}}{qm+p-3q} - \ldots\ldots$$

$$\left.\ldots\ldots \pm \frac{{}^m\overset{-2}{M}a^{m-2}X^2}{p+3q} \mp \frac{{}^m\overset{-1}{M}a^{m-1}X}{p+2q} \pm \frac{{}^mMa^m}{p+q}\right)\frac{qX^{\frac{p}{q}+1}}{b^{m+1}}$$

TABLE

Of some more general Formulæ.

$$a + bx = X$$

$$\int \frac{dx}{x^m\sqrt{X}} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \cdots\cdots \pm \frac{K}{x^2} \mp \frac{L}{x}\right)\sqrt{X} \mp \frac{Lb}{2}\int \frac{dx}{x\sqrt{X}}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(2m-3)b}{(2m-4)a}A,\ C = \frac{(2m-5)b}{(2m-6)a}B,$$

$$D = \frac{(2m-7)b}{(2m-8)a}C,\ E = \frac{(2m-9)b}{(2m-10)a}D,\ \ldots\ldots L = \frac{3b}{2a}K.$$

$$\int \frac{dx}{x^m X^{\frac{3}{2}}} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \cdots\cdots \pm \frac{K}{x^2} \mp \frac{L}{x}\right)\frac{1}{\sqrt{X}} \mp \frac{3Lb}{2}\int \frac{dx}{xX^{\frac{3}{2}}}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(2m-1)b}{(2m-4)a}A,\ C = \frac{(2m-3)b}{(2m-6)a}B,$$

$$D = \frac{(2m-5)b}{(2m-8)a}C,\ E = \frac{(2m-7)b}{(2m-10)a}D,\ \ldots\ldots L = \frac{5b}{2a}K.$$

$$\int \frac{dx}{x^m X^{\frac{5}{2}}} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \cdots\cdots \pm \frac{K}{x^2} \mp \frac{L}{x}\right)\frac{1}{X\sqrt{X}} \mp \frac{5bL}{2}\int \frac{dx}{xX^{\frac{5}{2}}}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(2m+1)b}{(2m-4)a}A,\ C = \frac{(2m-1)b}{(2m-6)a}B,$$

$$D = \frac{(2m-3)b}{(2m-8)a}C,\ E = \frac{(2m-5)b}{(2m-10)a}D,\ \ldots\ldots L = \frac{7b}{2a}K.$$

$$\int \frac{dx}{x^m X^{\frac{n}{2}}} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \cdots\cdots \pm \frac{K}{x^2} \mp \frac{L}{x}\right)\frac{1}{X^{\frac{n-2}{2}}} \mp \frac{nbL}{2}\int \frac{dx}{xX^{\frac{n}{2}}}$$

TABLE

Of some more general Formulæ.

$$a + bx = X$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(2m+n-4)b}{(2m-4)a}A,\ C = \frac{(2m+n-6)b}{(2m-6)a}B,$$

$$D = \frac{(2m+n-8)b}{(2m-8)a}C,\ E = \frac{(2m+n-10)b}{(2m-10)a}D, \ldots L = \frac{(n+2)b}{2a}K.$$

$$\int \frac{dx}{x^m X^{\frac{p}{q}}} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \ldots\ldots\right.$$

$$\left.\ldots\ldots \pm \frac{K}{x^2} \mp \frac{L}{x}\right)\frac{1}{X^{\frac{p-q}{q}}} \mp \frac{pbL}{q}\int \frac{dx}{xX^{\frac{p}{q}}}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(qm+p-5q)b}{(m-2)qa}A,\ C = \frac{(qm+p-3q)b}{(m-3)qa}B,$$

$$D = \frac{(qm+p-4q)b}{(m-4)qa}C,\ E = \frac{(qm+p-5q)b}{(m-5)qa}D, \ldots\ldots L = \frac{(p+q)b}{qa}K.$$

$$\int \frac{dx\sqrt{X}}{x^m} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \ldots\ldots\right.$$

$$\left.\ldots\ldots \pm \frac{K}{x^2} \mp \frac{L}{x}\right)X\sqrt{X} \pm \frac{bL}{2}\int \frac{dx\sqrt{X}}{x}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(2m-5)b}{(2m-4)a}A,\ C = \frac{(2m-7)b}{(2m-6)a}B,$$

$$D = \frac{(2m-9)b}{(2m-8)a}C,\ E = \frac{(2m-11)b}{(2m-10)a}D, \ldots\ldots L = \frac{b}{2a}K$$

$$\int \frac{dxX^{\frac{3}{2}}}{x^m} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \ldots\ldots\right.$$

$$\left.\ldots\ldots \pm \frac{K}{x^2} \mp \frac{L}{x}\right)X^2\sqrt{X} \pm \frac{3bL}{2}\int \frac{dxX^{\frac{3}{2}}}{x}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(2m-7)b}{(2m-4)a}A,\ C = \frac{(2m-9)b}{(2m-6)a}B,$$

$$D = \frac{(2m-11)b}{(2m-8)a}C,\ E = \frac{(2m-13)b}{(2m-10)a}D, \ldots\ldots L = \frac{-b}{2a}K.$$

TABLE

Of some more general Formulæ

$$a + bx^n = X$$

$$\int \frac{dx X^{\frac{5}{2}}}{x^m} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \ldots\ldots\ldots\ldots \right.$$

$$\left. \ldots\ldots\ldots\ldots \pm \frac{K}{x^2} \mp \frac{L}{x} \right) X^3 \sqrt{X} \pm \frac{5bL}{2} \int \frac{dx X^{\frac{5}{2}}}{x}$$

$$A = -\frac{1}{(m-1)a}, \ B = \frac{(2m-9)b}{(2m-4)a} A, \ C = \frac{(2m-11)b}{(2m-6)a} B,$$

$$D = \frac{(2m-13)b}{(2m-8)a} C, \ E = \frac{(2m-15)b}{(2m-10)a} D, \ldots\ldots L = \frac{-3b}{2a} K.$$

$$\int \frac{dx X^{\frac{n}{2}}}{x^m} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \ldots\ldots\ldots\ldots \right.$$

$$\left. \ldots\ldots\ldots\ldots\ldots \pm \frac{K}{x^2} \mp \frac{L}{x} \right) X^{\frac{n+2}{2}} \pm \frac{nbL}{2} \int \frac{dx X^{\frac{n}{2}}}{x}$$

$$A = -\frac{1}{(m-1)a}, \ B = \frac{(2m-n-4)b}{(2m-4)a} A, \ C = \frac{(2m-n-6)b}{(2m-6)a} B,$$

$$D = \frac{(2m-n-8)b}{(2m-8)a} C, \ \ldots\ldots\ldots\ldots\ldots L = \frac{-(n-2)b}{2a} K.$$

$$\int \frac{dx X^{\frac{p}{q}}}{x^m} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \ldots\ldots\ldots \right.$$

$$\left. \ldots\ldots\ldots\ldots\ldots \pm \frac{K}{x^2} \mp \frac{L}{x} \right) X^{\frac{p+q}{q}} \pm \frac{pbL}{q} \int \frac{dx X^{\frac{p}{q}}}{x}$$

$$A = -\frac{1}{(m-1)a}, \ B = \frac{(qm-p-2q)b}{(m-2)qa} A, \ C = \frac{(qm-p-3q)b}{(m-3)qa} B.$$

$$D = \frac{(qm-p-4q)b}{(m-4)qa} C, \ E = \frac{(qm-p-5q)b}{(m-5)qa} D, \ldots\ldots L = \frac{(q-p)b}{qa} K.$$

TABLE

Of some more general Formulæ.

$$a + bx = X$$

$$\int \frac{dx X^{\frac{2n+1}{2}}}{x} = \left(\frac{X^n}{2n+1} + \frac{aX^{n-1}}{2n-1} + \frac{a^2X^{n-2}}{2n-3} + \frac{a^3X^{n-3}}{2n-5} + \dots\dots\dots\dots \right.$$

$$\left. \dots\dots\dots\dots + \frac{a^{n-2}X^2}{5} + \frac{a^{n-1}X}{3} + \frac{a^n}{1}\right) 2\sqrt{X} + a^{n+1}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx X^{\frac{p}{q}}}{x} = \frac{qX^{\frac{p}{q}}}{p} + \frac{qaX^{\frac{p}{q}-1}}{p-q} + \frac{qa^2X^{\frac{p}{q}-2}}{p-2q} + \frac{qa^3X^{\frac{p}{q}-3}}{p-3q} + \dots\dots\dots\dots$$

$$\dots\dots\dots\dots + \frac{qa^{i-1}X^{\frac{p}{q}-i+1}}{p-(i-1)q} + a^i \int \frac{dx X^{\frac{p}{q}-i}}{x}$$

$$\int \frac{dx}{xX^{\frac{2n+1}{2}}} = \left[\frac{1}{(2n-1)aX^{n-1}} + \frac{1}{(2n-3)a^2X^{n-2}} + \frac{1}{(2n-5)a^3X^{n-3}} + \dots\dots \right.$$

$$\left. \dots\dots\dots\dots + \frac{1}{5a^{n-2}X^2} + \frac{1}{3a^{n-1}X} + \frac{1}{a^n}\right] \frac{2}{\sqrt{X}} + \frac{1}{a^n}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dx}{xX^{\frac{p}{q}}} = \frac{q}{(p-q)aX^{\frac{p}{q}-1}} + \frac{q}{(p-2q)a^2X^{\frac{p}{q}-2}} + \frac{q}{(p-3q)a^3X^{\frac{p}{q}-3}} + \dots\dots$$

$$\dots\dots\dots\dots + \frac{q}{(p-iq)a^iX^{\frac{p}{q}-i}} + \frac{1}{a^i}\int \frac{dx}{xX^{\frac{p}{q}-i}}$$

$$\int \frac{x^m dx\sqrt{X}}{X^n} = \frac{2x^m\sqrt{X}}{(2m-2n+3)bX^{n-1}} - \frac{(2m+1)a}{(2m-2n+3)b}\int \frac{x^{m-1}dx\sqrt{x}}{X^n}$$

$$\int \frac{x^m dx\sqrt{x}}{X^n} = \left(Ax^m - Bx^{m-1} + Cx^{m-2} - Dx^{m-3} + \dots\dots\dots\dots \right.$$

$$\left. \dots\dots\dots\dots \pm Kx^2 \mp Lx\right)\frac{2\sqrt{x}}{X^{n-1}} \pm \frac{3aL}{2}\int \frac{dx\sqrt{x}}{X^n}$$

$$A = \frac{1}{(2m-2n+3)b},\quad B = \frac{(2m+1)\,a}{(2m-2n+1)\,b}A,\quad C = \frac{(2m-1)a}{(2m-2n-1)b}B,$$

$$D = \frac{(2m-3)\,a}{(2m-2n-3)\,b}C,\ \dots\dots\dots\dots\dots\ L = \frac{5a}{(5-2n)\,b}K.$$

TABLE

Of some more general Formulæ.

$$a+bx=X,\quad ad-bc=k$$

$$\int\frac{\mathrm{d}x}{X^p\sqrt{(c+dx)}}=\frac{\sqrt{(c+dx)}}{(p-1)kX^{p-1}}+\frac{(2p-3)d}{(2p-2)k}\int\frac{\mathrm{d}x}{X^{p-1}\sqrt{(c+dx)}}$$

$$\int\frac{\mathrm{d}x}{X^p\sqrt{(c+dx)}}=\left(\frac{A}{X^{p-1}}+\frac{B}{X^{p-2}}+\frac{C}{X^{p-3}}+\frac{D}{X^{p-4}}+\frac{E}{X^{p-5}}+\ldots\ldots\right.$$

$$\left.\ldots\ldots\ldots\ldots+\frac{K}{X^2}+\frac{L}{X}\right)\sqrt{(c+dx)}+\frac{dL}{2}\int\frac{\mathrm{d}x}{X\sqrt{(c+dx)}}$$

$$A=\frac{1}{(p-1)k},\ B=\frac{(2p-3)d}{(2p-4)k}A,\ C=\frac{(2p-5)d}{(2p-6)k}B,$$

$$D=\frac{(2p-7)d}{(2p-8)k}C,\ E=\frac{(2p-9)d}{(2p-10)k}D,\ldots\ldots\ldots L=\frac{3d}{2k}K.$$

$$a+bx^2=X$$

$$\int x^m\mathrm{d}xX^{\frac{n}{2}}=\frac{x^{m+1}X^{\frac{n}{2}}}{m+1}-\frac{nb}{m+1}\int x^{m+2}\,\mathrm{d}xX^{\frac{n}{2}-1}$$

$$\int\frac{x^m\mathrm{d}x}{X^{\frac{n}{2}}}=-\frac{x^{m-1}}{(n-2)\,bX^{\frac{n}{2}-1}}+\frac{m-1}{(n-2)b}\int\frac{x^{m-2}\mathrm{d}x}{X^{\frac{n}{2}-1}}$$

$$\int x^m\mathrm{d}xX^{\frac{n}{2}}=\frac{x^{m-1}X^{\frac{n}{2}+1}}{(m+n+1)b}-\frac{(m-1)a}{(m+n+1)b}\int x^{m-2}\mathrm{d}xX^{\frac{n}{2}}$$

$$\int\frac{x^m\mathrm{d}x}{X^{\frac{n}{2}}}=\frac{x^{m-1}}{(m-n+1)\,bX^{\frac{n}{2}-1}}-\frac{(m-1)a}{(m-n+1)b}\int\frac{x^{m-2}\mathrm{d}x}{X^{\frac{n}{2}}}$$

$$\int x^m\mathrm{d}xX^{\frac{n}{2}}=\frac{x^{m+1}X^{\frac{n}{2}}}{m+n+1}+\frac{na}{m+n+1}\int x^m\,\mathrm{d}xX^{\frac{n}{2}-1}$$

$$\int\frac{\mathrm{d}xX^{\frac{n}{2}}}{x^m}=-\frac{X^{\frac{n}{2}}}{(m-n-1)\,x^{m-1}}-\frac{na}{m-n-1}\int\frac{\mathrm{d}xX^{\frac{n}{2}-1}}{x^m}$$

TABLE

of some more general Formulæ.

$$a + bx^2 = X$$

$$\int \frac{dx X^{\frac{n}{2}}}{x^m} = -\frac{X^{\frac{n}{2}+1}}{(m-1)\, a x^{m-1}} - \frac{m-n-3)\, b}{(m-1)\, a} \int \frac{dx X^{\frac{n}{2}}}{x^{m-2}}$$

$$\int \frac{dx}{x^m X^{\frac{n}{2}}} = -\frac{1}{(m-1) a x^{m-1} X^{\frac{n}{2}-1}} - \frac{(m+n-3)\, b}{(m-1)\, a} \int \frac{dx}{x^{m-2} X^{\frac{n}{2}}}$$

$$\int \frac{x^m dx}{X^{\frac{n}{2}}} = \frac{x^{m+1}}{(n-2)\, a X^{\frac{n}{2}-1}} - \frac{m-n+3}{(n-2)\, a} \int \frac{x^m dx}{X^{\frac{n}{2}-1}}$$

$$\int \frac{dx}{x^m X^{\frac{n}{2}}} = \frac{1}{(n-2) a x^{m-1} X^{\frac{n}{2}-1}} + \frac{m+n-3}{(n-2)\, a} \int \frac{dx}{x^m X^{\frac{n}{2}-1}}$$

$$\int \frac{dx}{X^{\frac{n}{2}}} = \frac{x}{(n-2)\, a X^{\frac{n}{2}-1}} + \frac{n-3}{(n-2)\, a} \int \frac{dx}{X^{\frac{n}{2}-1}}$$

$$\int dx X^{\frac{n}{2}} = \frac{x X^{\frac{n}{2}}}{n+1} + \frac{na}{n+1} \int dx X^{\frac{n}{2}-1}$$

$$\int \frac{dx}{x X^{\frac{n}{2}}} = \frac{1}{(n-2) a X^{\frac{n}{2}-1}} + \frac{1}{a} \int \frac{dx}{x X^{\frac{n}{2}-1}}$$

$$\int \frac{dx X^{\frac{n}{2}}}{x} = \frac{X^{\frac{n}{2}}}{n} + a \int \frac{dx X^{\frac{n}{2}-1}}{x}$$

$$\int \frac{dx}{X^{\frac{2n+1}{2}}} = \left(\frac{A}{X^{n-1}} + \frac{B}{X^{n-2}} + \frac{C}{X^{n-3}} + \ldots\ldots\ldots\ldots + \frac{K}{X} + L\right) \frac{x}{\sqrt{X}}$$

$$A = \frac{1}{(2n-1)\, a},\ B = \frac{2n-2}{(2n-3)\, a} A,\ C = \frac{2n-4}{(2n-5)\, a} B,$$

$$D = \frac{2n-6}{(2n-7)\, a} C,\ E = \frac{2n-8}{(2n-9)\, a} D, \ldots\ldots\ldots L = \frac{2}{a} K.$$

$$\int \frac{x dx}{X^n} = -\frac{1}{(n-1)\, 2b X^{n-1}}$$

TABLE

Of some more general Formulæ.

$$a + bx^2 = X$$

$$\int dx X^{\frac{2n+1}{2}} = \Big(AX^n + BX^{n-1} + CX^{n-2} + DX^{n-3} + \ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots + KX + L\Big)x\sqrt{X} + La\int\frac{dx}{\sqrt{X}}$$

$$A = \frac{1}{2n+2},\ B = \frac{(2n+1)a}{2n}A,\ C = \frac{(2n-1)a}{2n-2}B,$$

$$D = \frac{(2n-3)a}{2n-4}C,\ E = \frac{(2n-5)a}{2n-6}D, \ldots\ldots\ldots L = \frac{3a}{2}K.$$

$$\int x dx X^n = \frac{X^{n+1}}{(n+1)2b}$$

$$\int\frac{dx X^{\frac{2n+1}{2}}}{x} = \Big(\frac{X^n}{2n+1} + \frac{aX^{n-1}}{2n-1} + \frac{a^2X^{n-2}}{2n-3} + \frac{a^3X^{n-3}}{2n-5} + \ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots + \frac{a^{n-2}X^2}{5} + \frac{a^{n-1}X}{3} + \frac{a^n}{1}\Big)\sqrt{X} + a^{n+1}\int\frac{dx}{x\sqrt{X}}$$

$$\int\frac{dx}{xX^{\frac{2n+1}{2}}} = \Big[\frac{1}{(2n-1)aX^{n-1}} + \frac{1}{(2n-3)a^2X^{n-2}} + \frac{1}{(2n-5)a^3X^{n-3}} + \ldots\ldots$$

$$\ldots\ldots\ldots\ldots + \frac{1}{5a^{n-2}X^2} + \frac{1}{3a^{n-1}X} + \frac{1}{a^n}\Big]\frac{1}{\sqrt{X}} + \frac{1}{a^n}\int\frac{dx}{x\sqrt{X}}$$

$$\int\frac{x^m dx}{X^{\frac{n}{2}}} = \Big(Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \ldots\ldots$$

$$\ldots\ldots \pm Kx^{m-2i+3} \mp Lx^{m-2i+1}\Big)X^{-\frac{n}{2}+1} \pm (m-2i+1)aL\int\frac{x^{m-2i}dx}{X^{\frac{n}{2}}}$$

$$A = \frac{1}{(m-n+1)b},\ B = \frac{(m-1)a}{(m-n-1)b}A,\ C = \frac{(m-3)a}{(m-n-3)b}B,$$

$$D = \frac{(m-5)a}{(m-n-5)b}C,\ E = \frac{(m-7)a}{(m-n-7)b}D, \ldots L = \frac{(m-2i+3)a}{(m-n-2i+3)b}K.$$

$$\int\frac{x^{2m+1}dx}{\sqrt{X}} = \Big(Ax^{2m} - Bx^{2m-2} + Cx^{2m-4} - Dx^{2m-6} + \ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots \pm Kx^4 \mp Lx^2\Big)\sqrt{X} \pm 2aL\int\frac{xdx}{\sqrt{X}}.$$

TABLE

of some more general Formulæ.

$$a + bx^2 = X$$

$$A = \frac{1}{(2m+1)b},\ B = \frac{2ma}{(2m-1)b}A,\ C = \frac{(2m-2)a}{(2m-3)b}B,$$
$$D = \frac{(2m-4)a}{(2m-5)b}C,\ E = \frac{(2m-6)a}{(2m-7)b}D, \ldots L = \frac{4a}{3b}K.$$

$$\int\frac{x^{2m}dx}{\sqrt{X}} = \Big(Ax^{2m-1} - Bx^{2m-3} + Cx^{2m-5} - Dx^{2m-7} + Ex^{2m-9} - \ldots\ldots$$
$$\ldots\ldots \pm Kx^3 \mp Lx\Big)\sqrt{X} \pm aL\int\frac{dx}{\sqrt{X}}$$

$$A = \frac{1}{2mb},\ B = \frac{(2m-1)a}{(2m-2)b}A,\ C = \frac{(2m-3)a}{(2m-4)b}B,$$
$$D = \frac{(2m-5)a}{(2m-6)b}C,\ E = \frac{(2m-7)a}{(2m-8)b}D, \ldots\ldots L = \frac{3a}{2b}K.$$

$$\int\frac{x^{2m+1}dx}{X^{\frac{3}{2}}}\Big(Ax^{2m} - Bx^{2m-2} + Cx^{2m-4} - Dx^{2m-6} + Ex^{2m-8} - \ldots\ldots$$
$$\ldots\ldots \pm Kx^4 \mp Lx^2\Big)\frac{1}{\sqrt{X}} \pm 2aL\int\frac{xdx}{X^{\frac{3}{2}}}$$

$$A = \frac{1}{(2m-1)b},\ B = \frac{2ma}{(2m-3)b}A,\ C = \frac{(2m-2)a}{(2m-5)b}B,$$
$$D = \frac{(2m-4)a}{(2m-7)b}C,\ E = \frac{(2m-6)a}{(2m-9)b}D, \ldots\ldots L = \frac{4a}{b}K.$$

$$\int\frac{x^{2m}dx}{X^{\frac{3}{2}}} = \Big(Ax^{2m-1} - Bx^{2m-3} + Cx^{2m-5} - Dx^{2m-7} + Ex^{2m-9} - \ldots\ldots$$
$$\ldots\ldots \pm Kx^5 \mp Lx^3\Big)\frac{1}{\sqrt{X}} \pm 3aL\int\frac{x^2dx}{X^{\frac{3}{2}}}$$

$$A = \frac{1}{(2m-2)b},\ B = \frac{(2m-1)a}{(2m-4)b}A,\ C = \frac{(2m-3)a}{(2m-6)b}B,$$
$$D = \frac{(2m-5)a}{(2m-8)b}C,\ E = \frac{(2m-7)a}{(2m-10)b}D, \ldots\ldots L = \frac{5a}{2b}K.$$

$$\int\frac{dx}{x^m X^{\frac{n}{2}}} = \Big(\frac{A}{x^{m-1}} - \frac{B}{x^{m-3}} + \frac{C}{x^{m-5}} - \frac{D}{x^{m-7}} + \frac{E}{x^{m-9}} - \ldots\ldots$$
$$\ldots\ldots \pm\frac{K}{x^{m-2i+3}} \mp \frac{L}{x^{m-2i+1}}\Big)X^{-\frac{n}{2}+1} \mp (m+n-2i-1)bL\int\frac{dx}{x^{m-2i}X^{\frac{n}{2}}}$$

TABLE

Of some more general Formulæ.

$$a + bx^2 = X$$

$$A = -\frac{1}{(m-1)a}, \; B = \frac{(m+n-3)b}{(m-3)a}A, \; C = \frac{(m+n-5)b}{(m-5)a}B,$$

$$D = \frac{(m+n-7)b}{(m-7)a}C, \; E = \frac{(m+n-9)b}{(m-9)a}D, \ldots L = \frac{(m+n-2i+1)b}{(m-2i+1)a}K.$$

$$\int \frac{dx}{x^{2m+1}\sqrt{X}} = \left(\frac{A}{x^{2m}} - \frac{B}{x^{2m-2}} + \frac{C}{x^{2m-4}} - \frac{D}{x^{2m-6}} + \frac{E}{x^{2m-8}} - \ldots\ldots\right.$$

$$\left.\ldots\ldots\ldots\ldots \pm \frac{K}{x^4} \mp \frac{L}{x^2}\right)\sqrt{X} \mp bL \int \frac{dx}{x\sqrt{X}}$$

$$A = -\frac{1}{2ma}, \; B = \frac{(2m-1)b}{(2m-2)a}A, \; C = \frac{(2m-3)b}{(2m-4)a}B,$$

$$D = \frac{(2m-5)b}{(2m-6)a}C, \; E = \frac{(2m-7)b}{(2m-8)a}D, \ldots\ldots L = \frac{3b}{2a}K.$$

$$\int \frac{dx}{x^{2m}\sqrt{X}} = \left(\frac{A}{x^{2m-1}} - \frac{B}{x^{2m-3}} + \frac{C}{x^{2m-5}} - \frac{D}{x^{2m-7}} + \frac{E}{x^{2m-9}} - \ldots\ldots\right.$$

$$\left.\ldots\ldots\ldots\ldots \pm \frac{K}{x^3} \mp \frac{L}{x}\right)\sqrt{X}$$

$$A = -\frac{1}{(2m-1)a}, \; B = \frac{(2m-2)b}{(2m-3)a}A, \; C = \frac{(2m-4)b}{(2m-5)a}B,$$

$$D = \frac{(2m-6)b}{(2m-7)a}C, \; E = \frac{(2m-8)b}{(2m-9)a}D, \ldots\ldots L = \frac{2b}{a}K$$

$$\int \frac{dx}{x^{2m+1}X^{\frac{3}{2}}} = \left(\frac{A}{x^{2m}} - \frac{B}{x^{2m-2}} + \frac{C}{x^{2m-4}} - \frac{D}{x^{2m-6}} + \frac{E}{x^{2m-8}} - \ldots\ldots\right.$$

$$\left.\ldots\ldots\ldots\ldots \pm \frac{K}{x^4} \mp \frac{L}{x^2}\right)\frac{1}{\sqrt{X}} \mp 3bL \int \frac{dx}{xX^{\frac{3}{2}}}$$

$$A = \frac{1}{2ma}, \; B = \frac{(2m+1)b}{(2m-2)a}A, \; C = \frac{(2m-1)b}{(2m-4)a}B,$$

$$D = \frac{(2m-3)b}{(2m-6)a}C, \; E = \frac{(2m-5)a}{(2m-8)b}D, \ldots\ldots L = \frac{5b}{2a}K.$$

TABLE

of some more general Formulæ.

$$a + bx^2 = X$$

$$\int \frac{dx}{x^{2m} X^{\frac{3}{2}}} = \left(\frac{A}{x^{2m-1}} - \frac{B}{x^{2m-3}} + \frac{C}{x^{2m-5}} - \frac{D}{x^{2m-7}} + \frac{E}{x^{2m-9}} - \dots\dots\dots \right.$$

$$\left. \dots\dots\dots\dots \pm \frac{K}{x^3} \mp \frac{L}{x} \right) \frac{1}{\sqrt{X}} \mp 2bL \int \frac{dx}{X^{\frac{3}{2}}}$$

$$A = -\frac{1}{(2m-1)a}, \; B = \frac{2mb}{(2m-3)a} A, \; C = \frac{(2m-2)b}{(2m-5)a} B,$$

$$D = \frac{(2m-4)b}{(2m-7)a} C, \; E = \frac{(2m-6)b}{(2m-9)a} D, \dots\dots L = \frac{4b}{a} K$$

$$\int x^m dx X^{\frac{n}{2}} = \left(Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \dots \right.$$

$$\left. \dots\dots \pm Kx^{m-2i+3} \mp Lx^{m-2i+1} \right) X^{\frac{n}{2}+1} \pm (m-2i+1)aL \int x^{m-2i} dx X^{\frac{n}{2}}$$

$$A = \frac{1}{(m+n+1)b}, \; B = \frac{(m-1)a}{(m+n-1)b} A, \; C = \frac{(m-3)a}{(m+n-3)b} B,$$

$$D = \frac{(m-5)a}{(m+n-5)b} C, \; E = \frac{(m-7)a}{(m+n-7)b} D, \dots L = \frac{(m-2i+3)a}{(m+n-2i+3)b} K.$$

$$\int x^{2m+1} dx \sqrt{X} = \left(Ax^{2m} - Bx^{2m-2} + Cx^{2m-4} - Dx^{2m-6} + Ex^{2m-8} - \dots \right.$$

$$\left. \dots\dots\dots \pm Kx^4 \mp Lx^2 \right) X\sqrt{X} \pm 2aL \int x dx \sqrt{X}$$

$$A = \frac{1}{(2m+3)b}, \; B = \frac{2ma}{(2m+1)b} A, \; C = \frac{(2m-2)a}{(2m-1)b} B,$$

$$D = \frac{(2m-4)a}{(2m-3)b} C, \; E = \frac{(2m-6)a}{(2m-5)b} D, \dots\dots L = \frac{4a}{5b} K.$$

$$\int x^{2m} dx \sqrt{X} = \left(Ax^{2m-1} - Bx^{2m-3} + Cx^{2m-5} - Dx^{2m-7} + Ex^{2m-9} \dots\dots \right.$$

$$\left. \dots\dots\dots \pm Kx^3 \mp Lx \right) X\sqrt{X} \pm aL \int dx \sqrt{X}$$

$$A = \frac{1}{(2m+2)b}, \; B = \frac{(2m-1)a}{2mb} A, \; C = \frac{(2m-3)a}{(2m-2)b} B,$$

$$D = \frac{(2m-5)a}{(2m-4)b} C, \; E = \frac{(2m-7)a}{(2m-6)b} D, \dots\dots L = \frac{3a}{4b} K.$$

TABLE

of some more general Formulæ.

$$a + bx^2 = X$$

$$\int x^{2m+1} X^{\frac{3}{2}} = \Big(Ax^{2m} - Bx^{2m-2} + Cx^{2m-4} - Dx^{2m-6} + Ex^{2m-8} - \ldots\ldots$$

$$\ldots\ldots\ldots\ldots \pm Kx^4 \mp Lx^2\Big) X^2\sqrt{X} \pm 2aL \int x\mathrm{d}x X^{\frac{3}{2}}$$

$$A = \frac{1}{(2m+5)\,a}, \; B = \frac{2ma}{(2m+3)\,b} A, \; C = \frac{(2m-2)\,a}{(2m+1)\,b} B,$$

$$D = \frac{(2m-4)\,a}{(2m-1)\,b} C, \; E = \frac{(2m-6)\,a}{(2m-3)\,b} D, \ldots\ldots L = \frac{4a}{7b} K.$$

$$\int x^{2m} X^{\frac{3}{2}} = \Big(Ax^{2m-1} - Bx^{2m-3} + Cx^{2m-5} - Dx^{2m-7} + Ex^{2m-9} - \ldots$$

$$\ldots\ldots\ldots\ldots \pm Kx^3 \mp Lx\Big) X^2\sqrt{X} \pm aL \int \mathrm{d}x X^{\frac{3}{2}}$$

$$A = \frac{1}{(2m+4)\,b}, \; B = \frac{(2m-1)\,a}{(2m+2)\,b} A, \; C = \frac{(2m-3)\,a}{2mb} B,$$

$$D = \frac{(2m-5)\,a}{(2m-2)\,b} C, \; E = \frac{(2m-7)\,a}{(2m-4)\,b} D, \ldots\ldots L = \frac{3a}{6b} K.$$

$$\int \frac{\mathrm{d}x X^{\frac{n}{2}}}{x^m} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-3}} + \frac{C}{x^{m-5}} - \frac{D}{x^{m-7}} + \frac{E}{x^{m-9}} - \ldots\ldots\ldots\right.$$

$$\left.\ldots\ldots\ldots \pm \frac{K}{x^{m-2i+3}} \mp \frac{L}{x^{m-2i+1}}\right) X^{\frac{n}{2}+1} \mp (m-n-2i-1)\,bL \int \frac{\mathrm{d}x X^{\frac{n}{2}}}{x^{m-2i}}$$

$$A = -\frac{1}{(m-1)\,a}, \; B = \frac{(m-n-3)\,b}{(m-3)\,a} A, \; C = \frac{(m-n-5)\,b}{(m-5)\,a} B,$$

$$D = \frac{(m-n-7)\,b}{(m-7)\,a} C, \; E = \frac{(m-n-9)\,b}{(m-9)\,a} D, \ldots L = \frac{(m-n-2i+1)\,b}{(m-2i+1)\,a} K.$$

$$\int \frac{\mathrm{d}x\sqrt{X}}{x^{2m+1}} = \left(\frac{A}{x^{2m}} - \frac{B}{x^{2m-2}} + \frac{C}{x^{2m-4}} - \frac{D}{x^{2m-6}} + \frac{E}{x^{2m-8}} - \ldots\ldots\ldots\ldots\right.$$

$$\left.\ldots\ldots\ldots\ldots \pm \frac{K}{x^4} \mp \frac{L}{x^2}\right) X\sqrt{X} \pm bL \int \frac{\mathrm{d}x\sqrt{X}}{x}$$

TABLE

Of some more general Formulæ.

$$a + bx^2 = X$$

$$A = -\frac{1}{2ma},\ B = \frac{(2m-3)\,b}{(2m-2)\,a}A,\ C = \frac{(2m-5)\,b}{(2m-4)\,a}B,$$

$$D = \frac{(2m-7)\,b}{(2m-6)\,a}C,\ E = \frac{(2m-9)\,b}{(2m-8)\,a}D, \ldots\, L = \frac{b}{2a}K.$$

$$\int\frac{\mathrm{d}x\sqrt{X}}{x^{2m}} = \left(\frac{A}{x^{2m-1}} - \frac{B}{x^{2m-3}} + \frac{C}{x^{2m-5}} - \frac{D}{x^{2m-7}} + \frac{E}{x^{2m-9}} - \ldots\ldots\right.$$

$$\left.\ldots\ldots \pm \frac{K}{x^5} \mp \frac{L}{x^3}\right) X\sqrt{X}$$

$$A = -\frac{1}{(2m-1)\,a},\ B = \frac{(2m-4)\,b}{(2m-3)\,a}A,\ C = \frac{(2m-6)\,b}{(2m-5)\,a}B,$$

$$D = \frac{(2m-8)\,b}{(2m-7)\,a}C,\ E = \frac{(2m-10)\,b}{(2m-9)\,a}D, \ldots\ldots L = \frac{2b}{3a}K.$$

$$\int\frac{\mathrm{d}xX^{\frac{3}{2}}}{x^{2m+1}} = \left(\frac{A}{x^{2m}} - \frac{B}{x^{2m-2}} + \frac{C}{x^{2m-4}} - \frac{D}{x^{2m-6}} + \frac{E}{x^{2m-8}} - \ldots\ldots\right.$$

$$\left.\ldots\ldots \pm \frac{K}{x^4} \mp \frac{L}{x^2}\right) X^2\sqrt{X} \pm 3bL\int\frac{\mathrm{d}xX^{\frac{3}{2}}}{x}$$

$$A = -\frac{1}{2ma},\ B = \frac{(2m-5)\,b}{(2m-2)\,a}A,\ C = \frac{(2m-7)\,b}{(2m-4)\,a}B,$$

$$D = \frac{(2m-9)\,b}{(2m-6)\,a}C,\ E = \frac{(2m-11)\,b}{(2m-8)\,a}D, \ldots\ldots L = \frac{-b}{2a}K$$

$$\int\frac{\mathrm{d}xX^{\frac{3}{2}}}{x^{2m}} = \left(\frac{A}{x^{2m-1}} - \frac{B}{x^{2m-3}} + \frac{C}{x^{2m-5}} - \frac{D}{x^{2m-7}} + \frac{E}{x^{2m-9}} - \ldots\ldots\right.$$

$$\left.\ldots\ldots \pm \frac{K}{x^3} \mp \frac{L}{x}\right) X^2\sqrt{X} \pm 4bL\int\mathrm{d}xX^{\frac{3}{2}}$$

$$A = -\frac{1}{(2m-1)\,a},\ B = \frac{(2m-6)\,b}{(2m-3)\,a}A,\ C = \frac{(2m-8)\,b}{(2m-5)\,a}B,$$

$$D = \frac{(2m-10)\,b}{(2m-7)\,a}C,\ E = \frac{(2m-12)\,b}{(2m-9)\,a}D, \ldots\ldots L = \frac{-2b}{a}K$$

TABLE

of some more general Formulæ.

$$ax + bx^2 = X$$

$$\int x^m dx X^{\frac{n}{2}} = \frac{2x^{m+1}X^{\frac{n}{2}}}{2m+n+2} - \frac{nb}{2m+n+2}\int x^{m+2}dx X^{\frac{n}{2}-1}$$

$$\int \frac{x^m dx}{X^{\frac{n}{2}}} = -\frac{2x^{m-1}}{(n-2)bX^{\frac{n}{2}-1}} + \frac{2m-n}{(n-2)b}\int \frac{x^{m-2}dx}{X^{\frac{n}{2}-1}}$$

$$\int x^m dx X^{\frac{n}{2}} = \frac{x^{m-1}X^{\frac{n}{2}-1}}{(m+n+1)b} - \frac{(2m+n)a}{(m+n+1)2b}\int x^{m-1}dx X^{\frac{n}{2}}$$

$$\int \frac{x^m dx}{X^{\frac{n}{2}}} = \frac{x^{m-1}}{(m-n+1)bX^{\frac{n}{2}}} - \frac{(2m-n)a}{(m-n+1)2b}\int \frac{x^{m-1}dx}{X^{\frac{n}{2}}}$$

$$\int x^m dx X^{\frac{n}{2}} = \frac{x^{m+1}X^{\frac{n}{2}}}{m+n+1} + \frac{na}{2(m+n+1)}\int x^{m+1}dx X^{\frac{n}{2}-1}$$

$$\int \frac{dx X^{\frac{n}{2}}}{x^m} = -\frac{X^{\frac{n}{2}}}{(m-n-1)x^{m-1}} - \frac{na}{2(m-n-1)}\int \frac{dx X^{\frac{n}{2}-1}}{x^{m-1}}$$

$$\int \frac{dx X^{\frac{n}{2}}}{x^m} = -\frac{2X^{\frac{n}{2}+1}}{(2m-n-2)ax^m} - \frac{(m-n-2)2b}{(2m-n-2)a}\int \frac{dx X^{\frac{n}{2}}}{x^{m-1}}$$

$$\int \frac{dx}{x^m X^{\frac{n}{2}}} = -\frac{2}{(2m+n-2)ax^m X^{\frac{n}{2}-1}} - \frac{(m+n-2)2b}{(2m+n-2)a}\int \frac{dx}{x^{m-1}X^{\frac{n}{2}}}$$

$$\int \frac{x^m dx}{X^{\frac{n}{2}}} = \frac{2x^m}{(n-2)aX^{\frac{n}{2}-1}} - \frac{2(m-n+2)}{(n-2)a}\int \frac{x^{m-1}dx}{X^{\frac{n}{2}-1}}$$

$$\int \frac{dx}{x^m X^{\frac{n}{2}}} = \frac{2}{(n-2)ax^m X^{\frac{n}{2}-1}} + \frac{2(m+n-2)}{(n-2)a}\int \frac{dx}{x^{m+1}X^{\frac{n}{2}-1}}$$

$$\int \frac{dx}{X^{\frac{n}{2}}} = -\frac{2(2bx+a)}{(n-2)a^2 X^{\frac{n}{2}-1}} - \frac{(n-3)4b}{(n-2)a^2}\int \frac{dx}{X^{\frac{n}{2}-1}}$$

$$\int dx X^{\frac{n}{2}} = \frac{(2bx+a)X^{\frac{n}{2}}}{(n+1)2b} - \frac{na^2}{(n+1)4b}\int dx X^{\frac{n}{2}-1}$$

TABLE

of some more general Formulæ.

$$ax + bx^2 = X$$

$$\int x^m dx X^{\frac{n}{2}} = \left(Ax^{m-1} - Bx^{m-2} + Cx^{m-3} - Dx^{m-4} + Ex^{m-5} - \ldots\right.$$

$$\left.\ldots \pm Kx^{m-i+1} \mp Lx^{m-i}\right) X^{\frac{n}{2}+1} \pm \left(m+\frac{n}{2}-i+1\right) aL \int x^{m-i} dx X^{\frac{n}{2}}$$

$$A = \frac{1}{(m+n+1)b},\ B = \frac{(2m+n)a}{(m+n)2b} A,\ C = \frac{(2m+n-2)a}{(m+n-1)2b} B,$$

$$D = \frac{(2m+n-4)a}{(m+n-2)2b} C,\ E = \frac{(2m+n-6)a}{(m+n-3)2b} D, \ldots\ldots$$

$$\ldots\ldots L = \frac{(2m+n-2i+4)a}{(m+n-i+2)2b} K.$$

$$\int x^m dx \sqrt{X} = \left(Ax^{m-1} - Bx^{m-2} + Cx^{m-3} - Dx^{m-4} + Ex^{m-5} - \ldots\right.$$

$$\left.\ldots \pm Kx \mp L\right) X\sqrt{X} \pm \frac{3aL}{2} \int dx \sqrt{X}$$

$$A = \frac{1}{(m+2)b},\ B = \frac{(2m+1)a}{(m+1)2b} A,\ C = \frac{(2m-1)a}{2mb} B,$$

$$D = \frac{(2m-3)a}{(m-1)2b} C,\ E = \frac{(2m-5)a}{(m-2)2b} D, \ldots\ldots L = \frac{5a}{6b} K.$$

$$\int x^m dx X^{\frac{3}{2}} = \left(Ax^{m-1} - Bx^{m-2} + Cx^{m-3} - Dx^{m-4} + Ex^{m-5} - \ldots\right.$$

$$\left.\ldots \pm Kx \mp L\right) X^2\sqrt{X} \pm \frac{5aL}{2} \int dx X^{\frac{3}{2}}$$

$$A = \frac{1}{(m+4)b},\ B = \frac{(2m+3)a}{(m+3)2b} A,\ C = \frac{(2m+1)a}{(m+2)2b} B,$$

$$D = \frac{(2m-1)a}{(m+1)2b} C,\ E = \frac{(2m-3)a}{2mb} D \ldots\ldots L = \frac{7a}{10b} K.$$

$$\int \frac{dx X^{\frac{n}{2}}}{x^m} = \left(\frac{A}{x^m} - \frac{B}{x^{m-1}} + \frac{C}{x^{m-2}} - \frac{D}{x^{m-3}} + \frac{E}{x^{m-4}} - \ldots\right.$$

$$\left.\ldots \pm \frac{K}{x^{m-i+2}} \mp \frac{L}{x^{m-i+1}}\right) X^{\frac{n}{2}+1} \mp (m-n-i-1) bL \int \frac{dx X^{\frac{n}{2}}}{x^{m-i}}$$

TABLE

Of some more general Formulæ.

$$ax + bx^2 = X$$

$$A = -\frac{2}{(2m-n-2)a},\ B = \frac{(m-n-2)\,2b}{(2m-n-4)\,a}A,\ C = \frac{(m-n-3)\,2b}{(2m-n-6)\,a}B,$$

$$D = \frac{(m-n-4)\,2b}{(2m-n-8)\,a}C,\ E = \frac{(m-n-5)2b}{(2m-n-10)a}D, \ldots\ldots$$

$$\ldots\ldots L = \frac{(m-n-i)\,2b}{(2m-n-2i)\,a}K.$$

$$\int\frac{dx\sqrt{X}}{x^m} = \left(\frac{A}{x^m} - \frac{B}{x^{m-1}} + \frac{C}{x^{m-2}} - \frac{D}{x^{m-3}} + \frac{E}{x^{m-4}} - \ldots\ldots\right.$$

$$\left.\ldots\ldots \pm \frac{K}{x^5} \mp \frac{L}{x^4}\right)X\sqrt{X} \mp bL\int\frac{dx\sqrt{X}}{x^3}$$

$$A = -\frac{2}{(2m-3)\,a},\ B = \frac{(m-3)\,2b}{(2m-5)\,a}A,\ C = \frac{(m-4)\,2b}{(2m-7)\,a}B,$$

$$D = \frac{(m-5)\,2b}{(2m-9)\,a}C,\ E = \frac{(m-6)\,2b}{(2m-11)\,a}D, \ldots\ldots L = \frac{4b}{5a}K.$$

$$\int\frac{dxX^{\frac{3}{2}}}{x^m} = \left(\frac{A}{x^m} - \frac{B}{x^{m-1}} + \frac{C}{x^{m-2}} - \frac{D}{x^{m-3}} + \frac{E}{x^{m-4}} - \ldots\ldots\right.$$

$$\left.\ldots\ldots \pm \frac{K}{x^7} \mp \frac{L}{x^6}\right)X^2\sqrt{X} \mp bL\int\frac{dxX^{\frac{3}{2}}}{x^5}$$

$$A = -\frac{2}{(2m-5)\,a},\ B = \frac{(m-5)\,2b}{(2m-7)\,a}A,\ C = \frac{(m-6)\,2b}{(2m-9)\,a}B,$$

$$D = \frac{(m-7)\,2b}{(2m-11)\,a}C,\ E = \frac{(m-8)\,2b}{(2m-13)a}D, \ldots\ldots L = \frac{4b}{7a}K.$$

$$\int\frac{x^m dx}{X^{\frac{n}{2}}} = \left(Ax^{m-1} - Bx^{m-2} + Cx^{m-3} - Dx^{m-4} + Ex^{m-5} - \ldots\ldots\right.$$

$$\left.\ldots\ldots \pm Kx^{m-i+1} \mp Lx^{m-i}\right)X^{-\frac{n}{2}+1} \pm \left(m - \frac{n}{2} - i + 1\right)aL\int\frac{x^{m-i}dx}{X^{\frac{n}{2}}}$$

$$A = \frac{1}{(m-n+1)\,b},\ B = \frac{(2m-n)\,a}{(m-n)\,2b}A,\ C = \frac{(2m-n-2)\,a}{(m-n-1)\,2b}B,$$

$$D = \frac{(2m-n-4)\,a}{(m-n-2)\,2b}C,\ E = \frac{(2m-n-6)\,a}{(m-n-3)\,2b}D, \ldots\ldots$$

$$\ldots\ldots L = \frac{(2m-n-2i+4)\,a}{(m-n-i+2)\,2b}K.$$

TABLE

of some more general Formulæ.

$$ax + bx^2 = X$$

$$\int \frac{x^m \mathrm{d}x}{\sqrt{X}} = \Big(Ax^{m-1} - Bx^{m-2} + Cx^{m-3} - Dx^{m-4} + Ex^{m-5} - \ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots\ldots \pm Kx \mp L\Big)\sqrt{X} \pm \frac{aL}{2}\int\frac{\mathrm{d}x}{\sqrt{X}}$$

$$A = \frac{1}{mb},\ B = \frac{(2m-1)\,a}{(m-1)\,2b}A,\ C = \frac{(2m-3)\,a}{(m-2)\,2b}B,$$

$$D = \frac{(2m-5)\,a}{(m-3)\,2b}C,\ E = \frac{(2m-7)\,a}{(m-4)\,2b}D, \ldots\ldots L = \frac{3a}{2b}K.$$

$$\int \frac{x^m \mathrm{d}x}{X^{\frac{3}{2}}} = \Big(Ax^{m-1} - Bx^{m-2} + Cx^{m-3} - Dx^{m-4} + Ex^{m-5} - \ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots\ldots \pm Kx^3 \mp Lx^2\Big)\frac{1}{\sqrt{X}} \pm \frac{3aL}{2}\int\frac{x^2\mathrm{d}x}{X^{\frac{3}{2}}}$$

$$A = \frac{1}{(m-2)\,b},\ B = \frac{(2m-3)\,a}{(m-3)\,2b}A,\ C = \frac{(2m-5)\,a}{(m-4)\,2b}B,$$

$$D = \frac{(2m-7)\,a}{(m-5)\,2b}C,\ E = \frac{(2m-9)\,a}{(m-6)\,2b}D, \ldots\ldots\ L = \frac{5a}{2b}K.$$

$$\int \frac{\mathrm{d}x}{x^m X^{\frac{n}{2}}} = \Big(\frac{A}{x^m} - \frac{B}{x^{m-1}} + \frac{C}{x^{m-2}} - \frac{D}{x^{m-3}} + \frac{E}{x^{m-4}} - \ldots\ldots\ldots\ldots$$

$$\ldots\ldots \pm \frac{K}{x^{m-i+2}} \mp \frac{L}{x^{m-i+1}}\Big)X^{-\frac{n}{2}+1} \mp (m+n-i-1)\,bL\int\frac{\mathrm{d}x}{x^{m-i}X^{\frac{n}{2}}}$$

$$A = -\frac{2}{(2m+n-2)\,a},\ B = \frac{(m+n-2)\,2b}{(2m+n-4)\,a}A,\ C = \frac{(m+n-3)\,2b}{(2m+n-6)\,a}B,$$

$$D = \frac{(m+n-4)\,2b}{(2m+n-8)\,a}C,\ E = \frac{(m+n-5)\,2b}{(2m+n-10)\,a}D, \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ L = \frac{(m+n-i)\,2b}{(2m+n-2i)\,a}K.$$

TABLE

of some more general Formulæ.

$$ax + bx^2 = X$$

$$\int \frac{\mathrm{d}x}{x^m\sqrt{X}} = \left(\frac{A}{x^m} - \frac{B}{x^{m-1}} + \frac{C}{x^{m-2}} - \frac{D}{x^{m-3}} + \frac{E}{x^{m-4}} - \ldots\ldots \right.$$

$$\left. \ldots\ldots\ldots\ldots\ldots\ldots \pm \frac{K}{x^2} \mp \frac{L}{x}\right)\sqrt{X}$$

$$A = -\frac{2}{(2m-1)a},\ B = \frac{(m-1)\,2b}{(2m-3)\,a}A,\ C = \frac{(m-2)\,2b}{(2m-5)\,a}B,$$

$$D = \frac{(m-3)\,2b}{(2m-7)\,a}C,\ E = \frac{(m-4)\,2b}{(2m-9)\,a}D, \ldots L = \frac{2b}{a}K.$$

$$\int \frac{\mathrm{d}x}{x^m X^{\frac{3}{2}}} = \left(\frac{A}{x^m} - \frac{B}{x^{m-1}} + \frac{C}{x^{m-2}} - \frac{D}{x^{m-3}} + \frac{E}{x^{m-4}} - \ldots\ldots\ldots\ldots\ldots \right.$$

$$\left. \ldots\ldots\ldots\ldots \pm \frac{K}{x^2} \mp \frac{L}{x}\right)\frac{1}{\sqrt{X}} \mp 2bL \int \frac{\mathrm{d}x}{X^{\frac{3}{2}}}$$

$$A = -\frac{2}{(2m+1)a},\ B = \frac{(m+1)\,2b}{(2m-1)\,a}A,\ C = \frac{2mb}{(2m-3)\,a}B,$$

$$D = \frac{(m-1)\,2b}{(2m-5)\,a}C,\ E = \frac{(m-2)\,2b}{(2m-7)\,a}D, \ldots\ldots L = \frac{6b}{3a}K.$$

$$ax + bx^2 = X,\ 2bx + a = U$$

$$\int \frac{\mathrm{d}x}{X^{\frac{n}{2}}} = \left(\frac{A}{X^{\frac{n-3}{2}}} - \frac{B}{X^{\frac{n-5}{2}}} + \frac{C}{X^{\frac{n-7}{2}}} - \frac{D}{X^{\frac{n-9}{2}}} + \frac{E}{X^{\frac{n-11}{2}}} - \ldots\ldots\ldots\ldots \right.$$

$$\left. \ldots\ldots \pm \frac{K}{X^{\frac{n-2i+1}{2}}} \mp \frac{L}{X^{\frac{n-2i-1}{2}}}\right)\frac{2U}{\sqrt{X}} \mp (n-2i-1)\,4bL \int \frac{\mathrm{d}x}{X^{\frac{n}{2}-i}}$$

$$A = -\frac{1}{(n-2)\,a^2},\ B = \frac{(n-3)\,4b}{(n-4)\,a^2}A,\ C = \frac{(n-5)\,4b}{(n-6)\,a^2}B,$$

$$D = \frac{(n-7)\,4b}{(n-8)\,a^2}C,\ E = \frac{(n-9)\,4b}{(n-10)\,a^2}D, \ldots\ldots L = \frac{(n-2i+1)\,4b}{(n-2i)\,a^2}K.$$

TABLE

of some more general Formulæ.

$$ax + bx^2 = X,\ 2bx + a = U$$

$$\int \frac{dx}{X^{\frac{n}{2}}} = \left(\frac{A}{X^{\frac{n-3}{2}}} - \frac{B}{X^{\frac{n-5}{2}}} + \frac{C}{X^{\frac{n-7}{2}}} - \frac{D}{X^{\frac{n-9}{2}}} + \frac{E}{X^{\frac{n-11}{2}}} - \dots\dots\dots\dots \right.$$

$$\left. \dots\dots \pm \frac{K}{X^2} \mp \frac{I}{X} \right) \frac{2U}{\sqrt{X}} \mp 8bL \int \frac{dx}{X^{\frac{3}{2}}}$$

$$A = -\frac{1}{(n-2)a^2},\ B = \frac{(n-3)4b}{(n-4)a^2} A,\ C = \frac{(n-5)4b}{(n-6)a^2} B,$$

$$D = \frac{(n-7)4b}{(n-8)a^2} C,\ E = \frac{(n-9)4b}{(n-10)a^2} D, \dots\dots L = \frac{4 \cdot 4b}{3a^2} K.$$

$$a + bx + cx^2 = X$$

$$\int x^m dx X^p = \frac{x^{m+1} X^p}{m+1} - \frac{pb}{m+1} \int x^{m+1} dx X^{p-1} - \frac{2pc}{m+1} \int x^{m+2} dx X^{p-1}$$

$$\int x^m dx X^p = \frac{x^{m-1} X^{p+1}}{(m+2p+1)c} - \frac{(m-1)a}{(m+2p+1)c} \int x^{m-2} dx X^p$$

$$- \frac{(m+p)b}{(m+2p+1)c} \int x^{m-1} dx X^p$$

$$\int \frac{x^m dx}{X^p} = \frac{x^{m-1}}{(m-2p+1)cX^{p-1}} - \frac{(m-1)a}{(m-2p+1)c} \int \frac{x^{m-2} dx}{X^p}$$

$$- \frac{(m-p)b}{(m-2p+1)c} \int \frac{x^{m-1} dx}{X^p}$$

$$\int \frac{dx X^p}{X^m} = -\frac{X^p}{(m-1)x^{m-1}} + \frac{pb}{m-1} \int \frac{dx X^{p-1}}{x^{m-1}} + \frac{2pc}{m-1} \int \frac{dx X^{p-1}}{x^{m-2}}$$

$$\int x^m dx X^p = \frac{x^{m+1} X^p}{m+2p+1} + \frac{2pa}{m+2p+1} \int x^m dx X^{p-1}$$

$$+ \frac{pb}{m+2p+1} \int x^{m+1} dx X^{p-1}$$

$$\int \frac{dx X^p}{x^m} = -\frac{X^p}{(m-2p-1)x^{m-1}} - \frac{2pa}{m-2p-1} \int \frac{dx X^{p-1}}{x^{m}}$$

$$- \frac{pb}{m-2p-1} \int \frac{dx X^{p-1}}{x^{n-1}}$$

TABLE

of some more general Formulæ.

$$a+bx+cx^2=X,\ 4ac-b^2=k$$

$$\int\frac{dxX^p}{x^m}=-\frac{X^{p+1}}{(m-1)ax^{m-1}}-\frac{(m-p-2)b}{(m-1)a}\int\frac{dxX^p}{x^{m-1}}$$
$$-\frac{(m-2p-3)c}{(m-1)a}\int\frac{dxX^p}{x^{m-2}}$$

$$\int\frac{dx}{x^mX^p}=-\frac{1}{(m-1)ax^{m-1}X^{p-1}}-\frac{(m+p-2)b}{(m-1)a}\int\frac{dx}{x^{m-1}X^p}$$
$$-\frac{(m+2p-3)c}{(m-1)a}\int\frac{dx}{x^{m-2}X^p}$$

$$\int\frac{dx}{X^p}=\frac{2cx+b}{(p-1)kX^{p-1}}+\frac{(2p-3)2c}{(p-1)k}\int\frac{dx}{X^{p-1}}$$

$$\int dxX^p=\frac{(2cx+b)X^p}{(2p+1)2c}+\frac{pk}{(2p+1)2c}\int dxX^{p-1}$$

$$\int\frac{dx}{X^{\frac{n}{2}}}=\Big(\frac{A}{X^{\frac{n-3}{2}}}+\frac{B}{X^{\frac{n-5}{2}}}+\frac{C}{X^{\frac{n-7}{2}}}+\frac{D}{X^{\frac{n-9}{2}}}+\frac{E}{X^{\frac{n-11}{2}}}+\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots+\frac{K}{X^{\frac{n-2i+1}{2}}}+\frac{L}{X^{\frac{n-2i-1}{2}}}\Big)\frac{2(2cx+b)}{\sqrt{X}}$$
$$+(n-2i-1)4cL\int\frac{dx}{X^{\frac{n}{2}-i}}$$

$$A=\frac{1}{(n-2)k},\ B=\frac{(n-3)4c}{(n-4)k}A,\ C=\frac{(n-5)4c}{(n-6)k}B,$$
$$D=\frac{(n-7)4c}{(n-8)k}C,\ E=\frac{(n-9)4c}{(n-10)k}D,\ \ldots\ldots L=\frac{(n-2i+1)4c}{(n-2i)k}K.$$

$$\int\frac{dx}{X^{\frac{n}{2}}}=\Big(\frac{A}{X^{\frac{n-3}{2}}}+\frac{B}{X^{\frac{n-5}{2}}}+\frac{C}{X^{\frac{n-7}{2}}}+\frac{D}{X^{\frac{n-9}{2}}}+\frac{E}{X^{\frac{n-11}{2}}}+\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots+\frac{K}{X^2}+\frac{L}{X}\Big)\frac{2(2cx+b)}{\sqrt{X}}+8cL\int\frac{dx}{X^{\frac{1}{2}}}$$

TABLE

of some more general Formulæ.

$$a + bx + cx^2 = X,\quad 4ac - b^2 = k$$

$$A = \frac{1}{(n-2)k},\ B\frac{(n-3)4c}{(n-4)k}A,\ C = \frac{(n-5)4c}{(n-6)k}B,$$

$$D = \frac{(n-7)4c}{(n-8)k}C,\ E = \frac{(n-9)4c}{(n-10)k}D,\ \ldots\ldots\ldots\ L\frac{4.4c}{3k}K.$$

$$\int dxX^{\frac{n}{2}} = \left(AX^{\frac{n-1}{2}} + BX^{\frac{n-3}{2}} + CX^{\frac{n-5}{2}} + DX^{\frac{n-7}{2}} + EX^{\frac{n-9}{2}} + \ldots\ldots\right.$$

$$\left.\ldots\ldots\ldots\ldots + KX^{\frac{n-2i+3}{2}} + LX^{\frac{n-2i+1}{2}}\right)(2cx + b)\sqrt{X}$$

$$+\frac{n-2i+2}{2}kL\int dxX^{\frac{n}{2}-i}$$

$$A = \frac{1}{(n+1)2c},\ B = \frac{nk}{(n-1)4c}A,\ C = \frac{n-2)k}{(n-3)4c}B,$$

$$D = \frac{(n-4)k}{(n-5)4c}C,\ E = \frac{(n-6)k}{(n-7)4c}D,\ \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots\ L = \frac{(n-2i+4)k}{(n-2i+3)4c}K.$$

$$\int dxX^{\frac{n}{2}} = \left(AX^{\frac{n-1}{2}} + BX^{\frac{n-3}{2}} + CX^{\frac{n-5}{2}} + DX^{\frac{n-7}{2}} + EX^{\frac{n-9}{2}} + \ldots\ldots\right.$$

$$\left.\ldots\ldots + KX^2 + LX\right)(2cx + b)\sqrt{X} + \frac{3kL}{2}\int dx\sqrt{X}$$

$$A = \frac{1}{(n+1)2c},\ B = \frac{nk}{(n-1)4c}A,\ C = \frac{(n-2)k}{(n-3)4c}B,$$

$$D = \frac{(n-4)k}{(n-5)4c}C,\ E = \frac{(n-6)k}{(n-7)4c}D,\ \ldots\ldots\ L = \frac{5k}{4.4c}K.$$

$$\int\frac{xdx}{X^{\frac{n}{2}}} = -\frac{1}{(n-2)cX^{\frac{n}{2}-1}} - \frac{b}{2c}\int\frac{dx}{X^{\frac{n}{2}}}$$

$$\int xdxX^{\frac{n}{2}} = \frac{X^{\frac{n}{2}+1}}{(n+2)c} - \frac{b}{2c}\int dxX^{\frac{n}{2}}$$

TABLE

of some more general Formulæ.

$$a + bx + cx^2 = X.$$

$$\int \frac{dx}{xX^{\frac{n}{2}}} = \frac{1}{(n-2)\,aX^{\frac{n-2}{2}}} + \frac{1}{a}\int \frac{dx}{xX^{\frac{n-2}{2}}} - \frac{b}{2a}\int \frac{dx}{X^{\frac{n}{2}}}$$

$$\int \frac{dxX^{\frac{n}{2}}}{x} = \frac{X^{\frac{n}{2}}}{n} + a\int \frac{dxX^{\frac{n-2}{2}}}{x} + \frac{b}{2}\int dxX^{\frac{n-2}{2}}$$

$$\int \frac{dx}{xX^{\frac{2n+1}{2}}} = \left[\frac{1}{(2n-1)\,aX^{n-1}} + \frac{1}{(2n-3)\,a^2X^{n-2}} + \frac{1}{(2n-5)\,a^3X^{n-3}} + \cdots\right.$$

$$\left.\cdots\cdots + \frac{1}{5a^{n-2}X^2} + \frac{1}{3a^{n-1}X} + \frac{1}{a^n}\right]\frac{1}{\sqrt{X}}$$

$$-\frac{b}{2a}\int \frac{dx}{X^{\frac{2n+1}{2}}} - \frac{b}{2a^2}\int \frac{dx}{X^{\frac{2n-1}{2}}} - \frac{b}{2a^3}\int \frac{dx}{X^{\frac{2n-3}{2}}} - \cdots\cdots$$

$$\cdots\cdots \frac{b}{2a^{n-1}}\int \frac{dx}{X^{\frac{5}{2}}} - \frac{b}{2a^n}\int \frac{dx}{X^{\frac{3}{2}}} + \frac{1}{a^n}\int \frac{dx}{x\sqrt{X}}$$

$$\int \frac{dxX^{\frac{2n+1}{2}}}{x} = \left(\frac{X^n}{2n+1} + \frac{aX^{n-1}}{2n-1} + \frac{a^2X^{n-2}}{2n-3} + \frac{a^3X^{n-3}}{2n-5} + \cdots\cdots\right.$$

$$\left.\cdots\cdots + \frac{a^{n-2}X^2}{5} + \frac{a^{n-1}X}{5} + \frac{a^n}{1}\right)\sqrt{X}$$

$$+\frac{b}{2}\int dxX^{\frac{2n-1}{2}} + \frac{ab}{2}\int dxX^{\frac{2n-3}{2}} + \frac{a^2b}{2}\int dxX^{\frac{2n-5}{2}} + \cdots\cdots$$

$$\cdots\cdots + \frac{a^{n-2}b}{2}\int dxX^{\frac{3}{2}} + \frac{a^{n-1}b}{2}\int dx\sqrt{X}$$

$$+\frac{a^nb}{2}\int \frac{dx}{\sqrt{X}} + a^{n+1}\int \frac{dx}{x\sqrt{X}}$$

Values of the Definite Integrals.

$$\int' \frac{x^m dx}{\sqrt{(a^2-x^2)}}, \quad \int' x^m dx \sqrt{(a^2-x^2)},$$

from $x=0$ to $x=a$.

$$a^2 - x^2 = X, \quad \pi = 3{,}14159\ldots\ldots$$

$\int' \frac{dx}{\sqrt{X}} = \frac{\pi}{2}$	$\int' \frac{x dx}{\sqrt{X}} = a$
$\int' \frac{x^2 dx}{\sqrt{X}} = \frac{1}{2} \cdot \frac{\pi a^2}{2}$	$\int' \frac{x^3 dx}{\sqrt{X}} = \frac{2}{3} \cdot a^3$
$\int' \frac{x^4 dx}{\sqrt{X}} = \frac{1.3}{2.4} \cdot \frac{\pi a^4}{2}$	$\int' \frac{x^5 dx}{\sqrt{X}} = \frac{2.4}{3.5} \cdot a^5$
$\int' \frac{x^6 dx}{\sqrt{X}} = \frac{1.3.5}{2.4.6} \cdot \frac{\pi a^6}{2}$	$\int' \frac{x^7 dx}{\sqrt{X}} = \frac{2.4.6}{3.5.7} \cdot a^7$
$\int' \frac{x^8 dx}{\sqrt{X}} = \frac{1.3.5.7}{2.4.6.8} \cdot \frac{\pi a^8}{2}$	$\int' \frac{x^9 dx}{\sqrt{X}} = \frac{2.4.6.8}{3.5.7.9} \cdot a^9$
$\int' \frac{x^{10} dx}{\sqrt{X}} = \frac{1.3.5.7.9}{2.4.6.8.10} \cdot \frac{\pi a^{10}}{2}$	$\int' \frac{x^{11} dx}{\sqrt{X}} = \frac{2.4.6.8.10}{3.5.7.9.11} \cdot a^{11}$

$$\int' \frac{x^{2r} dx}{\sqrt{X}} = \frac{1.3.5.7\ldots\ldots\ldots(2r-3)(2r-1)}{2.4.6.8\ldots\ldots\ldots\ldots(2r-2)\,2r} \cdot \frac{\pi a^{2r}}{2}$$

$$\int' \frac{x^{2r+1} dx}{\sqrt{X}} = \frac{2.4.6.8\ldots\ldots\ldots\ldots\ldots(2r-2)\,2r}{3.5.7.9\ldots\ldots\ldots(2r-1)(2r+1)} \cdot a^{2r+1}$$

$\int' dx\sqrt{X} = \frac{\pi a^2}{4}$	$\int' x dx\sqrt{X} = \frac{a^3}{3}$
$\int' x^2 dx\sqrt{X} = \frac{1}{4} \cdot \frac{\pi a^4}{4}$	$\int' x^3 dx\sqrt{X} = \frac{2}{5} \cdot \frac{a^5}{3}$
$\int' x^4 dx\sqrt{X} = \frac{1.3}{4.6} \cdot \frac{\pi a^6}{4}$	$\int' x^5 dx\sqrt{X} = \frac{2.4}{5.7} \cdot \frac{a^7}{3}$
$\int' x^6 dx\sqrt{X} = \frac{1.3.5}{4.6.8} \cdot \frac{\pi a^8}{4}$	$\int' x^7 dx\sqrt{X} = \frac{2.4.6}{5.7.9} \cdot \frac{a^9}{3}$
$\int' x^8 dx\sqrt{X} = \frac{1.3.5.7}{4.6.8.10} \cdot \frac{\pi a^{10}}{4}$	$\int' x^9 dx\sqrt{X} = \frac{2.4.6.8}{5.7.9.11} \cdot \frac{a^{11}}{3}$

$$\int' x^{2r} dx\sqrt{X} = \frac{1.3.5.7\ldots\ldots(2r-3)(2r-1)}{4.6.8.10\ldots\ldots\ldots 2r(2r+2)} \cdot \frac{\pi a^{2r+2}}{4}$$

$$\int' x^{2r+1} dx\sqrt{X} = \frac{2.4.6.8\ldots\ldots\ldots\ldots(2r-2)\,2r}{5.7.9.11\ldots\ldots(2r+1)(2r+3)} \cdot \frac{a^{2r+3}}{3}$$

Values of the Definite Integrals

$$\int x^m dx\,(a^2-x^2)^{\frac{3}{2}};\quad \int x^m dx\,(a^2-x^2)^{\frac{5}{2}},$$

from $x=0$ to $x=a$.

$a^2-x^2=X$, $\pi=3{,}14159\ldots\ldots$

$$\int dx X^{\frac{3}{2}}=\frac{3\pi a^4}{16}$$
$$\int x^2dx X^{\frac{3}{2}}=\frac{1}{6}\cdot\frac{3\pi a^6}{16}$$
$$\int x^4dx X^{\frac{3}{2}}=\frac{1.3}{6.8}\cdot\frac{3\pi a^8}{16}$$
$$\int x^6dx X^{\frac{3}{2}}=\frac{1.3.5}{6.8.10}\cdot\frac{3\pi a^{10}}{16}$$
$$\int x^8dx X^{\frac{3}{2}}=\frac{1.3.5.7}{6.8.10.12}\cdot\frac{3\pi a^{12}}{16}$$

$$\int x dx X^{\frac{3}{2}}=\frac{a^5}{5}$$
$$\int x^3dx X^{\frac{3}{2}}=\frac{2}{7}\cdot\frac{a^7}{5}$$
$$\int x^5dx X^{\frac{3}{2}}=\frac{2.4}{7.9}\cdot\frac{a^9}{5}$$
$$\int x^7dx X^{\frac{3}{2}}=\frac{2.4.6}{7.9.11}\cdot\frac{a^{11}}{5}$$
$$\int x^9dx X^{\frac{3}{2}}=\frac{2.4.6.8}{7.9.11.13}\cdot\frac{a^{13}}{5}$$

$$\int x^{2r}dx X^{\frac{3}{2}}=\frac{1.3.5.7\ldots\ldots(2r-3)(2r-1)}{6.8.10.12\ldots\ldots(2r+2)(2r+4)}\cdot\frac{3\pi a^{2r+4}}{16}$$
$$\int x^{2r+1}dx X^{\frac{3}{2}}=\frac{2.4.6.8\ldots\ldots\ldots\ldots(2r-2)2r}{7.9.11.13\ldots\ldots(2r+3)(2r+5)}\cdot\frac{a^{2r+5}}{5}$$

$$\int dx X^{\frac{5}{2}}=\frac{5\pi a^6}{32}$$
$$\int x^2dx X^{\frac{5}{2}}=\frac{1}{8}\cdot\frac{5\pi a^8}{32}$$
$$\int x^4dx X^{\frac{5}{2}}=\frac{1.3}{8.10}\cdot\frac{5\pi a^{10}}{32}$$
$$\int x^6dx X^{\frac{5}{2}}=\frac{1.3.5}{8.10.12}\cdot\frac{5\pi a^{12}}{32}$$
$$\int x^8dx X^{\frac{5}{2}}=\frac{1.3.5.7}{8.10.12.14}\cdot\frac{5\pi a^{14}}{32}$$

$$\int x dx X^{\frac{5}{2}}=\frac{a^7}{7}$$
$$\int x^3dx X^{\frac{5}{2}}=\frac{2}{9}\cdot\frac{a^9}{7}$$
$$\int x^5dx X^{\frac{5}{2}}=\frac{2.4}{9.11}\cdot\frac{a^{11}}{7}$$
$$\int x^7dx X^{\frac{5}{2}}=\frac{2.4.6}{9.11.13}\cdot\frac{a^{13}}{7}$$
$$\int x^9dx X^{\frac{5}{2}}=\frac{2.4.6.8}{9.11.13.15}\cdot\frac{a^{15}}{7}$$

$$\int x^{2r}dx X^{\frac{5}{2}}=\frac{1.3.5.7\ldots\ldots(2r-3)(2r-1)}{8.10.12.14\ldots\ldots(2r+4)(2r+6)}\cdot\frac{5\pi a^{2r+6}}{32}$$
$$\int x^{2r+1}dx X^{\frac{5}{2}}=\frac{2.4.6.8\ldots\ldots\ldots\ldots(2r-2)2r}{9.11.13.15\ldots\ldots(2r+5)(2r+7)}\cdot\frac{a^{2r+7}}{7}$$

Values of the Definite Integrals

$$\int' dx\,(a^2-x^2)^{\frac{n}{2}},\quad \int' x^m dx\,(a^2-x^2)^{\frac{n}{2}},$$

from $x=0$ to $x=a$.

$$a^2-x^2=X,\quad \pi=3{,}14159\ldots\ldots$$

$$\int' dx\sqrt{X}=\frac{1}{2}\cdot\frac{\pi a^2}{2}$$

$$\int' dxX^{\frac{3}{2}}=\frac{1\cdot3}{2\cdot4}\cdot\frac{\pi a^4}{2}$$

$$\int' dxX^{\frac{5}{2}}=\frac{1\cdot3\cdot5}{2\cdot4\cdot6}\cdot\frac{\pi a^6}{2}$$

$$\int' dxX^{\frac{7}{2}}=\frac{1\cdot3\cdot5\cdot7}{2\cdot4\cdot6\cdot8}\cdot\frac{\pi a^8}{2}$$

$$\int' dxX^{\frac{9}{2}}=\frac{1\cdot3\cdot5\cdot7\cdot9}{2\cdot4\cdot6\cdot8\cdot10}\cdot\frac{\pi a^{10}}{2}$$

....

$$\int' dxX^{\frac{n}{2}}=\frac{1\cdot3\cdot5\cdot7\ \ldots\ldots\ldots\ldots\ (n-2)\cdot n}{2\cdot4\cdot6\cdot8\ \ldots\ldots\ (n-1)(n+1)}\cdot\frac{\pi a^{n+1}}{2}$$

$$\int' x^2dxX^{\frac{n}{2}}=\frac{1}{n+3}\cdot a^2\int' dxX^{\frac{n}{2}}$$

$$\int' x^4dxX^{\frac{n}{2}}=\frac{1\cdot3}{(n+3)(n+5)}\cdot a^4\int' dxX^{\frac{n}{2}}$$

$$\int' x^6dxX^{\frac{n}{2}}=\frac{1\cdot3\cdot5}{(n+3)(n+5)(n+7)}\cdot a^6\int' dxX^{\frac{n}{2}}$$

....

$$\int' xdxX^{\frac{n}{2}}=\frac{a^{n+2}}{n+2}$$

$$\int' x^3dxX^{\frac{n}{2}}=\frac{2}{n+4}\cdot\frac{a^{n+4}}{n+2}$$

$$\int' x^5dxX^{\frac{n}{2}}=\frac{2\cdot4}{(n+4)(n+6)}\cdot\frac{a^{n+6}}{n+2}$$

....

$$\int' x^{2r}dxX^{\frac{n}{2}}=\frac{1\cdot3\cdot5\cdot7\ \ldots\ldots\ldots\ldots\ (2r-1)}{(n+3)(n+5)(n+7)\ \ldots\ldots\ (n+2r+1)}\cdot a^{2r}\int' dxX^{\frac{n}{2}}$$

$$\int' x^{2r+1}dxX^{\frac{n}{2}}=\frac{2\cdot4\cdot6\cdot8\ \ldots\ldots\ldots\ldots\ 2r}{(n+4)(n+6)(n+8)\ \ldots\ldots\ (n+2r+2)}\cdot\frac{a^{n+2r+2}}{n+2}$$

Values of the Definite Integrals.

$$\int' \frac{x^m dx}{\sqrt{(a^4-x^4)}}, \int' x^m\, dx\, (a^4 - x^4)^{\frac{n}{2}},$$

from $x=0$ to $x=a$.

$a^2 - x^2 = X$, $\pi = 3{,}14159\ldots\ldots$

$$\int' \frac{dx}{\sqrt{(a^4-x^4)}} = \int' \frac{dx\,(a^2+x^2)^{-\frac{1}{2}}}{\sqrt{X}} = \frac{1}{a}\int' \frac{dx}{\sqrt{X}} + \frac{{}^{-\frac{1}{2}}A}{a^3}\int' \frac{x^2dx}{\sqrt{X}}$$

$$+ \frac{{}^{-\frac{1}{2}}B}{a^5}\int' \frac{x^4dx}{\sqrt{X}} + \frac{{}^{-\frac{1}{2}}C}{a^7}\int' \frac{x^6dx}{\sqrt{X}} + \frac{{}^{-\frac{1}{2}}D}{a^9}\int' \frac{x^8dx}{\sqrt{X}} + \&c.$$

$$= \frac{\pi}{2a}\left[1 - \left(\frac{1}{2}\right)^2 + \left(\frac{1\,.\,3}{2\,.\,4}\right)^2 - \left(\frac{1\,.\,3\,.\,5}{2\,.\,4\,.\,6}\right)^2 + \left(\frac{1\,.\,3\,.\,5\,.\,7}{2\,.\,4\,.\,6\,.\,8}\right)^2\right.$$

$$\left. - \left(\frac{1\,.\,3\,.\,5\,.\,7\,.\,9}{2\,.\,4\,.\,6\,.\,8\,.\,10}\right)^2 + \left(\frac{1\,.\,3\,.\,5\,.\,7\,.\,9\,.\,11}{2\,.\,4\,.\,6\,.\,8\,.\,10\,.\,12}\right)^2 - \&c.\right]$$

$$\int' dx\sqrt{(a^4-x^4)} = \int' dx\,(a^2+x^2)^{\frac{1}{2}}\sqrt{X} = a\int' dx\sqrt{X} + \frac{{}^{\frac{1}{2}}A}{a}\int' x^2dx\sqrt{X}$$

$$+ \frac{{}^{\frac{1}{2}}B}{a^3}\int' x^4dx\sqrt{X} + \frac{{}^{\frac{1}{2}}C}{a^5}\int' x^6dx\sqrt{X} + \&c.$$

$$= \frac{\pi a^3}{4}\left[1 + \frac{1}{2}\cdot\frac{1}{4} - \frac{1.1}{2.4}\cdot\frac{1.3}{4.6} + \frac{1.1.3}{2.4.6}\cdot\frac{1.3.5}{4.6.8} - \frac{1.1.3.5}{2.4.6.8}\cdot\frac{1.3.5.7}{4.6.8.10}\right.$$

$$\left. + \frac{1\,.\,1\,.\,3\,.\,5\,.\,7}{2\,.\,4\,.\,6\,.\,8\,.\,10}\cdot\frac{1\,.\,3\,.\,5\,.\,7\,.\,9}{4\,.\,6\,.\,8\,.\,10\,.\,12} - \&c.\right]$$

$$\int' \frac{x^m dx}{\sqrt{(a^4-x^4)}} = \frac{1}{a}\int' \frac{x^m dx}{\sqrt{X}} + \frac{{}^{-\frac{1}{2}}A}{a^3}\int' \frac{x^{m+2}dx}{\sqrt{X}} + \frac{{}^{-\frac{1}{2}}B}{a^5}\int' \frac{x^{m+4}dx}{\sqrt{X}}$$

$$+ \frac{{}^{-\frac{1}{2}}C}{a^7}\int' \frac{x^{m+6}dx}{\sqrt{X}} + \frac{{}^{-\frac{1}{2}}D}{a^9}\int' \frac{x^{m+8}dx}{\sqrt{X}} + \&c.$$

$$\int' x^m dx(a^4-x^4)^{\frac{n}{2}} = a^n\int' x^m dx X^{\frac{n}{2}} + {}^{\frac{n}{2}}A a^{n-2}\int' x^{m+2}dx X^{\frac{n}{2}}$$

$$+ {}^{\frac{n}{2}}B a^{n-4}\int' x^{m+4}dx X^{\frac{n}{2}} + {}^{\frac{n}{2}}C a^{n-6}\int' x^{m+6}dx X^{\frac{n}{2}} + \&c.$$

Values of the Definite Integrals

$$\int \frac{x^m dx(1+cx^h)^{\frac{p}{q}}}{\sqrt{(a^2-x^2)}},\quad \int x^m dx(1+cx^h)^{\frac{p}{q}}(a^2-x^2)^{\frac{n}{2}}$$

from $x=0$ to $x=a$

$$a^2-x^2=X$$

$$\int' \frac{dx(1+cx^h)^{\frac{p}{q}}}{\sqrt{X}} = \int' \frac{dx}{\sqrt{X}} + \frac{p}{q}Ac\int' \frac{x^h dx}{\sqrt{X}} + \frac{p}{q}Bc^2\int' \frac{x^{2h}dx}{\sqrt{X}} + \frac{p}{q}Cc^3\int' \frac{x^{3h}dx}{\sqrt{X}} + \frac{p}{q}Dc^4\int' \frac{x^{4h}dx}{\sqrt{X}} + \text{\&c.}$$

$$\int'(1+cx^h)^{\frac{p}{q}}X^{\frac{n}{2}} = \int' dxX^{\frac{n}{2}} + \frac{p}{q}Ac\int' x^h dxX^{\frac{n}{2}} + \frac{p}{q}Bc^2\int' x^{2h}dxX^{\frac{n}{2}} + \frac{p}{q}Cc^3\int' x^{3h}dxX^{\frac{n}{2}} + \frac{p}{q}Dc^4\int' x^{4h}dxX^{\frac{n}{2}} + \text{\&c.}$$

$$\int' \frac{dx}{(1+cx^h)^{\frac{p}{q}}\sqrt{X}} = \int' \frac{dx}{\sqrt{X}} + \frac{-p}{q}Ac\int' \frac{x^h dx}{\sqrt{X}} + \frac{-p}{q}Bc^2\int' \frac{x^{2h}dx}{\sqrt{X}} + \frac{-p}{q}Cc^3\int' \frac{x^{3h}dx}{\sqrt{X}} + \frac{-p}{q}Dc^4\int' \frac{x^{4h}dx}{\sqrt{X}} + \text{\&c.}$$

$$\int' \frac{dxX^{\frac{n}{2}}}{(1+cx^h)^{\frac{p}{q}}} = \int' dxX^{\frac{n}{2}} + \frac{-p}{q}Ac\int' x^h dxX^{\frac{n}{2}} + \frac{-p}{q}Bc^2\int' x^{2h}dxX^{\frac{n}{2}} + \frac{-p}{q}Cc^3\int' x^{3h}dxX^{\frac{n}{2}} + \frac{-p}{q}Dc^4\int' x^{4h}dxX^{\frac{n}{2}} + \text{\&c.}$$

$$\int' \frac{x^m dx(1+cx^h)^{\frac{p}{q}}}{\sqrt{X}} = \int' \frac{x^m dx}{\sqrt{X}} + \frac{p}{q}Ac\int' \frac{x^{m+h}dx}{\sqrt{X}} + \frac{p}{q}Bc^2\int' \frac{x^{m+2h}dx}{\sqrt{X}} + \frac{p}{q}Cc^3\int' \frac{x^{m+3h}dx}{\sqrt{X}} + \frac{p}{q}Dc^4\int' \frac{x^{m+4h}dx}{\sqrt{X}} + \text{\&c.}$$

$$\int' x^m dx(1+cx^h)^{\frac{p}{q}}X^{\frac{n}{2}} = \int' x^m dxX^{\frac{n}{2}} + \frac{p}{q}Ac\int' x^{m+h}dxX^{\frac{n}{2}} + \frac{p}{q}Bc^2\int' x^{m+2h}dxX^{\frac{n}{2}} + \frac{p}{q}Cc^3\int' x^{m+3h}dxX^{\frac{n}{2}} + \text{\&c.}$$

$$\int' \frac{x^m dx}{(1+cx^h)^{\frac{p}{q}}\sqrt{X}} = \int' \frac{x^m dx}{\sqrt{X}} + \frac{-p}{q}Ac\int' \frac{x^{m+h}dx}{\sqrt{X}} + \frac{-p}{q}Bc^2\int' \frac{x^{m+2h}dx}{\sqrt{X}} + \frac{-p}{q}Cc^3\int' \frac{x^{m+3h}dx}{\sqrt{X}} + \frac{-p}{q}Dc^4\int' \frac{x^{m+4h}dx}{\sqrt{X}} + \text{\&c.}$$

$$\int' \frac{x^m dxX^{\frac{n}{2}}}{(1+cx^h)^{\frac{p}{q}}} = \int' x^m dxX^{\frac{n}{2}} + \frac{-p}{q}Ac\int' x^{m+h}dxX^{\frac{n}{2}} + \frac{-p}{q}Bc^2\int' x^{m+2h}dxX^{\frac{n}{2}} + \frac{-p}{q}Cc^3\int' x^{m+3h}dxX^{\frac{n}{2}} + \text{\&c.}$$

Relations between the values of Definite Integrals.

$1-x^n=X,\ \pi=3{,}14159\ldots\ldots,$

$m, n, p, r,$ any positive numbers.

The Integrals are taken from $x=0$ to $x=1$.

$$\int' \frac{x^m dx}{\sqrt{(x-x^2)}} = 2\int' \frac{x^{2m} dx}{\sqrt{(1-x^2)}}$$

$$\int' \frac{x^r dx}{\sqrt{(1-x^2)}} \times \int' \frac{x^{r+1} dx}{\sqrt{(1-x^2)}} = \frac{1}{r+1}\cdot\frac{\pi}{2}$$

$$\int' \frac{x^r dx}{\sqrt{(1-x^4)}} \times \int' \frac{x^{r+2} dx}{\sqrt{(1-x^4)}} = \frac{1}{2(r+1)}\cdot\frac{\pi}{2}$$

$$\int' \frac{x^r dx}{\sqrt{(1-x^{2n})}} \times \int' \frac{x^{r+n} dx}{\sqrt{(1-x^{2n})}} = \frac{1}{n(r+1)}\cdot\frac{\pi}{2}$$

$$\int' \frac{dx}{\sqrt{(1-x^{2n})}} \times \int' \frac{x^n dx}{\sqrt{(1-x^{2n})}} = \frac{1}{n}\cdot\frac{\pi}{2}$$

$$\int' x^{m-1}dx X^{\frac{p-n}{n}} = \int' x^{p-1}dx X^{\frac{m-n}{n}}$$

$$\int' x^{m-1}dx X^{\frac{p-n}{n}} = \frac{(m+p)(m+p+n)(m+p+2n)\ldots\ldots(m+p+in)}{m\ \ (m+n)\ \ (m+2n)\ldots\ldots\ldots\ldots(m+in)} \times \int' x^{m+(i+1)n-1}dx X^{\frac{p-n}{n}}$$

$$\frac{\int' x^{m-1}dx X^{\frac{p-n}{n}}}{\int' x^{r-1}dx X^{\frac{p-n}{n}}} = \frac{(m+p)\ (m+p+n)\ (m+p+2n)\ \text{in inf.}}{m\quad (m+n)\quad (m+2n)\quad \text{in inf.}} \times \frac{r\quad (r+n)\quad (r+2n)\quad \text{in inf.}}{(r+p)\ (r+p+n)\ (r+p+2n)\ \text{in inf.}}$$

$$\frac{\int' x^{m-1}dx X^{\frac{p-n}{n}}}{\int' x^{m+r-1}dx X^{\frac{p-n}{n}}} = \frac{\int' x^{m-1}dx X^{\frac{r-n}{n}}}{\int' x^{m+p-1}dx X^{\frac{r-n}{n}}}$$

$$\int' \frac{x^{m-1}dx}{1+x^n} = \frac{\pi}{n\sin\frac{m\pi}{n}}$$ [This and the following value hold only as long as $m-1$ is $\angle\ n$.]

$$\int' \frac{x^{m-1}dx}{\sqrt[n]{X^m}} = \int' x^{n-m-1}dx X^{\frac{m-n}{n}} = \frac{\pi}{n\sin\frac{m\pi}{n}}$$

The Developement into a Series of the Integral

$$\int x^m dx\,(a+bx^n)^p.$$

$$a + bx^n = X$$

$$\int x^m dx X^p = a^p x^{m+1}\left(A + Bx^n + Cx^{2n} + Dx^{3n} + Ex^{4n} + \&c.\right)$$

$$A = \frac{1}{m+1},\ B = \frac{pA}{m+n+1}\cdot\frac{b}{a},\ C = \frac{pB}{m+2n+1}\cdot\frac{b^2}{a^2},$$

$$D = \frac{pC}{m+3n+1}\cdot\frac{b^3}{a^3},\ E = \frac{pD}{m+4n+1}\cdot\frac{b^4}{a^4} + \&c.$$

$$\int x^m dx X^p = b^p x^{m+np+1}\left(A + \frac{B}{x^n} + \frac{C}{x^{2n}} + \frac{D}{x^{3n}} + \frac{E}{x^{4n}} + \&c.\right)$$

$$A = \frac{1}{m+np+1},\ B = \frac{pA}{m+(p-1)n+1}\cdot\frac{a}{b},$$

$$C = \frac{pB}{m+(p-2)n+1}\cdot\frac{a^2}{b^2},\ D = \frac{pC}{m+(p-3)n+1}\cdot\frac{a^3}{b^3}$$

$$E = \frac{pD}{m+(p-4)n+1}\cdot\frac{a^4}{b^4},\ \&c.$$

$$\int x^m dx X^p = x^{m+1}X^{p+1}\left(\frac{A}{x^n} - \frac{B}{x^{2n}} + \frac{C}{x^{3n}} - \frac{D}{x^{4n}} + \frac{E}{x^{5n}} - \&c\right)$$

$$A = \frac{1}{(m+np+1)b},\ B = \frac{(m-n+1)a}{(m-n+np+1)b}A,$$

$$C = \frac{(m-2n+1)a}{(m-2n+np+1)b}B,\ D = \frac{(m-3n+1)a}{(m-3n+np+1)b}C,$$

$$E = \frac{(m-4n+1)a}{(m-4n+np+1)b}D,\ \&c$$

$$\int x^m dx X^p = x^{m+1}X^{p+1}\left(A - Bx^n + Cx^{2n} - Dx^{3n} + Ex^{4n} - \&c.\right)$$

$$A = \frac{1}{(m+1)a},\ B = \frac{(m+n+np+1)b}{(m+n+1)a}A,\ C = \frac{(m+2n+np+1)b}{(m+2n+1)a}$$

$$D = \frac{(m+3n+np+1)b}{(m+3n+1)a}C,\ E\frac{(m+4n+np+1)b}{(m+4n+1)a}D,\ \&c.$$

The Developement into a Series of the Integral

$$\int x^m dx\,(a+bx^n)^p$$

$$a+bx^n = X$$

$$\int x^m dx X^p = -\,x^{m+1}X^{p+1}\left(A+BX+CX^2+DX^3+\&c.\right)$$

$$A=\frac{1}{(p+1)na},\ B=\frac{m+n+np+1}{(p+2)na}A,\ C=\frac{m+2n+np+1}{(p+3)na}$$

$$D=\frac{m+3n+np+1}{(p+4)na}C,\ E=\frac{m+4n+np+1}{(p+5)na}D,\ \&c.$$

$$\int x^m dx X^p = x^{m+1}X^p\left(A+\frac{B}{X}+\frac{C}{X^2}+\frac{D}{X^3}+\frac{E}{X^4}+\&c.\right)$$

$$A=\frac{1}{m+np+1},\ B=\frac{pna}{m-n+np+1}A,\ C=\frac{(p-1)na}{m-2n+np+1}B,$$

$$D=\frac{(p-2)na}{m-3n+np+1}C,\ E=\frac{(p-3)na}{m-4n+np+1}D,\ \&c.$$

$$\int x^m dx X^p = x^{m+1}X^p\left[A-B\left(\frac{x^n}{X}\right)+C\left(\frac{x^n}{X}\right)^2-D\left(\frac{x^n}{X}\right)^3\right.$$
$$\left.+E\left(\frac{x^n}{X}\right)^4-F\left(\frac{x^n}{X}\right)^5+\&c.\right]$$

$$A=\frac{1}{m+1},\ B=\frac{pnb}{m+n+1}A,\ C=\frac{(p-1)nb}{m+2n+1}B,$$

$$D=\frac{(p-2)nb}{m+3n+1}C,\ E=\frac{(p-3)nb}{m+4n+1}D,\ F=\frac{(p-4)nb}{m+5n+1}E\ \&c.$$

$$\int x^m dx X^p = x^{m-n+1}X^{p+1}\left[A-B\left(\frac{X}{x^n}\right)+C\left(\frac{X}{x^n}\right)^2-D\left(\frac{X}{x^n}\right)^3\right.$$
$$\left.+E\left(\frac{X}{x^n}\right)^4-F\left(\frac{X}{x^n}\right)^5+\&c.\right]$$

$$A=\frac{1}{(p+1)nb},\ B=\frac{m-n+1}{(p+2)nb}A,\ C=\frac{m-2n+1}{(p+3)nb}B,$$

$$D=\frac{m-3n+1}{(p+4)nb}C,\ E=\frac{m-4n+1}{(p+5)nb}D,\ F=\frac{m-5n+1}{(p+6)nb}E,\ \&c.$$

METHODS OF INTEGRATION.*

$F:[x, y, z, t, \&c.]$ a rational function of x, y, z, t, &c.

I. The Differential

$$dxF:[x, \sqrt[m]{x}, \sqrt[n]{x}, \sqrt[p]{x}, \sqrt[q]{x}, \&c.]$$

becomes rational, by putting $x = y^{mnpq\ldots}$; for then we have $\sqrt[m]{x} = y^{npq\ldots}$, $\sqrt[n]{x} = y^{mpq\ldots}$, $\sqrt[p]{x} = y^{mnq\ldots}$, $\sqrt[q]{x} = y^{mnp\ldots}$, &c. and $dx = (mnpq\ldots)y^{(mnpq\ldots)-1}dy$. To this belongs, for example, the differential $\frac{x^3+2\sqrt[5]{ax^2}+\sqrt[3]{x}}{bx+c\sqrt[4]{dx}}dx$; which is rationalized by putting $a = y^{60}$.

II. The Differential

$$dxF:[x, \sqrt[n]{(a+bx)}]$$

becomes rational by putting $a+bx=y^n$; for then $\sqrt[n]{(a+bx)}=y$, $x=\frac{y^n-a}{b}$, $dx=\frac{ny^{n-1}dy}{b}$. To this form, *e. g.* belong the Differentials $\frac{x^4dx}{cx^3+\sqrt[4]{(a+bx)^3}}$, $\frac{x^2dx\sqrt[5]{(a+bx)^3}}{cx+d\sqrt[5]{(a+bx)^2}}$, which become rational, when we make $a+bx=y^4$, and $a+bx=y^5$.

III. The Differential

$$dxF:\left[x, \sqrt[n]{\frac{a+bx}{f+gx}}\right]$$

is rationalized by putting $\frac{a+bx}{f+gx}=y^n$; for then $\sqrt[n]{\frac{a+bx}{f+gx}}=y$

* A differential is here considered as integrated, as soon as by any transformation it is rationalized, or reduced to such an irrational form as admits of being made rational.

$x = \frac{a - fy^n}{gy^n - b}$, $dx = \frac{n(bf - ag)y^{n-1}dy}{(gy^n - b)^2}$. To which forms belong e. g. the differentials $x^m dx \left(\frac{a + bx}{f + gx}\right)^{\frac{p}{q}}$, $\frac{dx\sqrt[n]{(a + bx)}}{x^m\sqrt[n]{(f+gx)}}$, $\frac{dx\sqrt[n]{(a+bx)}}{\sqrt[n]{(a+bx)} + \sqrt[n]{(f+gx)}}$. The differentials admit of the following forms:

$$x^m dx \left[\left(\frac{a+bx}{f+gx}\right)^{\frac{1}{n}}\right]^k, \ \frac{dx}{x^m}\sqrt[n]{\frac{a+bx}{f+gx}}, \ \frac{dx}{\sqrt[n]{\frac{a+bx}{f+gx}}+1}, \ \sqrt[n]{\frac{a+bx}{f+gx}}.$$

IV. The Differential

$$dxF : [x, (a+bx)^{\frac{m}{n}}, (a+bx)^{\frac{p}{q}}, (a+bx)^{\frac{r}{s}}, \&c.]$$

becomes rational, by making $a + bx = y^{nqs\cdots}$; for then $(a + bx)^{\frac{m}{n}} = y^{mqs\cdots}$, $(a + bx)^{\frac{p}{q}} = y^{nps\cdots}$, $(a + bx)^{\frac{r}{s}} = y^{nqr\cdots}$, &c., $x = \frac{y^{nqs\cdots} - a}{b}$, $dx = \frac{nqs\cdots}{b}y^{(nqs\cdots)-1}dy$.

V. In a similar manner the Differential

$$dxF : \left[x, \left(\frac{a+bx}{f+gx}\right)^{\frac{m}{n}}, \left(\frac{a+bx}{f+gx}\right)^{\frac{p}{q}}, \left(\frac{a+bx}{f+gx}\right)^{\frac{r}{s}}, \&c.\right]$$

is made rational, by putting $\frac{a+bx}{f+gx} = y^{nqs\cdots}$.

VI. To make the Differential

$$dxF : [x, \sqrt{(a + bx + cx^2)}]$$

rational, we must distinguish the two cases in which c is positive and negative.

First Case. The Differential $dxF : [x, \sqrt{(a + bx + cx^2)}]$ is rationalized by putting $a + bx + cx^2 = c(x + y)^2$; hence we obtain $x = \frac{a - cy^2}{2cy - b}$, $dx = -\frac{2c(cy^2 - by + a)dy}{(2cy - b)^2}$, $\sqrt{(a + bx + cx^2)} = \frac{(cy^2 - by + a)\sqrt{c}}{2cy - b}$.

Second Case. Let r, r' denote the two roots of the equation $a + bx - cx^2 = 0$; then $\sqrt{(a + bx - cx^2)} = \sqrt{c(x - r)(r' - x)}$ The Differential $dxF : [x, +(a + bx - cx^2)]$ is rationalized, by putting $\sqrt{c(x - r)(r' - x)} = (x - r)cy$; for hence we obtain $x = \frac{cry^2 + r'}{cy^2 + 1}$, $dx = \frac{(r - r')\,2cydy}{(cy^2 + 1)^2}$, $\sqrt{(a + bx - cx^2)} = \frac{(r' - r)cy}{cy^2 + 1}$.

The roots of the equation $a + bx - cx^2 = 0$ are necessarily real whether a and b be positive or negative; because otherwise $\sqrt{a + bx - cx^2}$ would be imaginary for every value of x.

VII. The Differentials

$$dxF : [x, \sqrt{(a + cx^2)}], \ dxF : [x, \sqrt{(bx + cx^2)}]$$

are comprehended in the above forms; for we thence obtain them by making $b = 0$, or $a = 0$.

VIII. To rationalize the Differential

$$dxF : [x, \sqrt{(a + bx)}, \sqrt{(a' + b'x)}]$$

we first make $a + bx = (a' + b'x)y^2$; this gives $x = \frac{a - a'y^2}{b'y^2 - b}$, $dx = \frac{(a'b - ab')\,2ydy}{(b'y^2 - b)^2}$, $\sqrt{(a + bx)} = \frac{y\sqrt{(ab' - a'b)}}{\sqrt{(b'y^2 - b)}}$, $\sqrt{(a' + b'x)} = \frac{\sqrt{(ab' - a'b)}}{\sqrt{(b'y^2 - b)}}$. By the substitution of these values, the given Differential is transformed into another of the form $dyF' : [y, \sqrt{(b'y^2 - b)}]$, where F' denotes any rational function other than F; and this differential can again be rationalized.

IX. The Differential

$$x^{m-1}dx(a + bx^n)^{\frac{p}{q}}$$

admits of being rationalized, in both cases, where $\frac{m}{n}$ or $\frac{m}{n} + \frac{p}{q}$ is a whole positive or negative number.

First Case. Let $a + bx^n = y^q$, then $(a + bx^n)^{\frac{p}{q}} = y^p$,

$$x^n = \frac{y^q - a}{b},\ x^m = \left(\frac{y^q - a}{b}\right)^{\frac{m}{n}},\ x^{m-1}dx = \frac{qy^{q-1}}{nb}\left(\frac{y^q - a}{b}\right)^{\frac{m-n}{n}}$$

By the substitution of these values, the above differential is transformed into $\frac{q}{nb}\, y^{p+q-1}\, dy \left(\frac{y^q - a}{b}\right)^{\frac{m-n}{n}}$, and is, therefore, rational when $\frac{m-n}{n}$ or $\frac{m}{n}$ is a whole number.

Second Case. Let $a + bx^n = x^n y^q$; then $x^n = \frac{a}{y^q - b}$,

$$a + bx^n = \frac{ay^q}{y^q - b},\ (a + bx^n)^{\frac{p}{q}} = \frac{a^{\frac{p}{q}}\, y^p}{(y^q - b)^{\frac{p}{q}}},\ x^m = \frac{a^{\frac{m}{n}}}{(y^q - b)^{\frac{m}{n}}},$$

$x^{m-1}dx = -\dfrac{qa^{\frac{m}{n}}\, y^{q-1}}{n(y^q - b)^{\frac{m}{n}+1}}$. The given differential is transformed into $\dfrac{qa^{\frac{m}{n}+\frac{p}{q}}\, y^{p+q-1}}{n(y^q - b)^{\frac{m}{n}+\frac{p}{q}+1}}$, and is, therefore, rational, when $\frac{m}{n} + \frac{p}{q}$ is a whole number.

X. In the same two cases, and by the same substitutions the differential

$$x^{m-1}dx\,(a + bx^n)^{\frac{p}{q}} F : [x^n]$$

is rationalized. To which belongs the differential $x^{m+in-1}dx(a+bx^n)^{\frac{p}{q}}$, and the still more general one $\frac{Px^{m-1}\,dx}{Q} \times (a + bx^n)^{\frac{p}{q}}$, where $P = A + Bx^n + Cx^{2n} + Dx^{3n} + \&c.$, $Q = A' + B'x^n + C'x^{2n} + D'x^{3n} + \&c.$

The differential

$$x^{m-1}dxF : [x^m, x^n, \sqrt[q]{(a + bx^n)}],$$

which comprises forms IX and X, may be rationalized, when $\frac{m}{n}$ is a whole positive or negative number; for by making

$\sqrt{(a + bx^n)} = y$, we get $x^n = \frac{y^q - a}{b}$, $x^m = \left(\frac{y^q - a}{b}\right)^{\frac{m}{n}}$

$x^{m-1} dx = \frac{qy^{q-1}}{nb} \left(\frac{y^q - a}{b}\right)^{\frac{m}{n}-1} dy.$

XII. Let X, X', X'', denote rational functions of x, then the differentials

$$\frac{Xdx}{X' + X'' \sqrt{(a + bx + cx^2)}},$$

$$\frac{Xdx}{X'\sqrt{(a + bx + cx^2)} + X''\sqrt{(a' + b'x + c'x^2)}},$$

may always be rationalized, by multiplying the former by $X' - X''\sqrt{(a + bx + cx^2)}$, and the latter by $X'\sqrt{(a + bx + cx^2)} - X''\sqrt{(a' + b'x + c'x^2)}$; from thence the former is transformed into $\frac{XX'dx}{X'^2 - X''^2(a + bx + cx^2)} - \frac{XX''dx\sqrt{(a + bx + cx^2)}}{X'^2 - X''^2(a + bx + cx^2)}$, of which we have merely to make rational the second part; and the latter into $\frac{XX'dx\sqrt{(a + bx + cx^2)}}{X'^2(a + bx + cx^2) - X''^2(a' + b'x + c'x^2)} - \frac{XX''dx\sqrt{(a' + b'x + c'x^2)}}{X'^2(a + bx + cx^2) - X''^2(a' + b'x + c'x^2)}$, of which each member admits of being rationalized separately.*

XIII. The Differential

$$x^m dx F : [x^n, \sqrt{(a + bx^n + cx^{2n})}]$$

by putting $x^n = y$, may be transformed into

$$\frac{1}{n} y^{\frac{m+1}{n} - 1} dy F : [y, \sqrt{(a + by + cy^2)}],$$

and in this form, may be rationalized by the method in VI, when

* The square roots of quantities may be generally erased in a denominator, and then we have only to integrate a *monomial*.

$\frac{m+1}{n}$ is a whole number. Also the differential

$x^m dx(a+bx^n+cx^{2n})^{\frac{p}{2}}$

XIV. On the same condition, that $\frac{m+1}{n}$ be a whole number, the differential

$$x^m dx F : [x^n, hx^n + \sqrt{(a+h^2x^{2n})}]$$

and the yet more general one

$$x^m dx F : [x^n, \sqrt{(a+h^2x^{2n})}, hx^n+\sqrt{(a+h^2x^{2n})}]$$

may be rationalized; for putting $hx^n+\sqrt{(a+h^2x^{2n})}=y$, we have

$$x^n = \frac{y^2-a}{2hy}, \sqrt{(a+h^2x^{2n})} = \frac{y^2+a}{2y},$$

$$x^m dx = \frac{1}{n(2h)^{\frac{m+1}{n}}}\left(\frac{y^2+a}{y}\right)\left(\frac{y^2-a}{y}\right)^{\frac{m+1}{n}-1} dy.$$

To this form belongs the less general one

$$dx F : [x, \sqrt{(a+h^2x^2)}, hx+\sqrt{(a+h^2x^2)}]$$

of which the differentials $[x+\sqrt{(1+x^2)}]^n dx$, $[x+\sqrt{(1+x^2)}]^n X dx$, $[ax+b\sqrt{(1+x^2)}][x+\sqrt{(1+x^2)}]^n dx$, which *Euler* has integrated in The Appendix to § 125 of the first volume of his Institut. are particular cases.

INTEGRAL TABLES

OF

TRANSCENDENTAL DIFFERENTIALS.

TABLE

of Formulæ of reduction for the Integral

$$\int d\varphi \sin^m\varphi \cos^n\varphi$$

I.

$$\int d\varphi \sin^m\varphi \cos^n\varphi = \frac{\sin^{m+1}\varphi \cos^{n-1}\varphi}{m+1} + \frac{n-1}{m+1}\int d\varphi \sin^{m+2}\varphi \cos^{n-2}\varphi$$

II.

$$\int d\varphi \sin^m\varphi \cos^n\varphi = -\frac{\sin^{m-1}\varphi \cos^{n+1}\varphi}{n+1} + \frac{m-1}{n+1}\int d\varphi \sin^{m-2}\varphi \cos^{n+2}\varphi$$

III.

$$\int d\varphi \sin^m\varphi \cos^n\varphi = -\frac{\sin^{m-1}\varphi \cos^{n+1}\varphi}{m+n} + \frac{m-1}{m+n}\int d\varphi \sin^{m-2}\varphi \cos \varphi^n$$

IV.

$$\int d\varphi \sin^m\varphi \cos^n\varphi = \frac{\sin^{m+1}\varphi \cos^{n-1}\varphi}{m+n} + \frac{n-1}{m+n}\int d\varphi \sin^m\varphi \cos^{n-2}\varphi$$

V.

$$\int d\varphi \sin^m\varphi \cos^n\varphi = \frac{\sin^{m+1}\varphi \cos^{n+1}\varphi}{m+1} + \frac{m+n+2}{m+1}\int d\varphi \sin^{m+2}\varphi \cos^n\varphi$$

VI.

$$\int d\varphi \sin^m\varphi \cos^n\varphi = -\frac{\sin^{m+1}\varphi \cos^{n+1}\varphi}{n+1} + \frac{m+n+2}{n+1}\int d\varphi \sin^m\varphi \cos^{n+2}\varphi$$

These formulæ obtain, whether m and n be positive or negative, whole or fractional or 0.

TAB. I.

$$\int d\varphi \sin^m \varphi$$

$$\int d\varphi \sin \varphi = -\cos \varphi$$

$$\int d\varphi \sin^2 \varphi = -\frac{1}{2}\sin \varphi \cos \varphi + \frac{1}{2}\varphi$$

$$\int d\varphi \sin^3 \varphi = \left(-\frac{1}{3}\sin^2 \varphi - \frac{2}{3}\right)\cos \varphi$$

$$\int d\varphi \sin^4 \varphi = \left(-\frac{1}{4}\sin^3 \varphi - \frac{3}{8}\sin \varphi\right)\cos \varphi + \frac{3}{8}\varphi$$

$$\int d\varphi \sin^5 \varphi = \left(-\frac{1}{5}\sin^4 \varphi - \frac{4}{15}\sin^2 \varphi - \frac{8}{15}\right)\cos \varphi$$

$$\int d\varphi \sin^6 \varphi = \left(-\frac{1}{6}\sin^5 \varphi - \frac{5}{24}\sin^3 \varphi - \frac{5}{16}\sin. \varphi\right)\cos \varphi + \frac{5}{16}\varphi$$

$$\int d\varphi \sin^7 \varphi = \left(-\frac{1}{7}\sin^6 \varphi - \frac{6}{35}\sin^4 \varphi - \frac{8}{35}\sin^2 \varphi - \frac{16}{35}\right)\cos \varphi$$

$$\int d\varphi \sin^8 \varphi = \left(-\frac{1}{8}\sin^7 \varphi - \frac{7}{48}\sin^5 \varphi - \frac{35}{192}\sin^3 \varphi - \frac{35}{128}\sin \varphi\right)\cos \varphi + \frac{35}{128}\varphi$$

$$\int d\varphi \sin^9 \varphi = \left(-\frac{1}{9}\sin^8 \varphi - \frac{8}{63}\sin^6 \varphi - \frac{16}{105}\sin^4 \varphi - \frac{64}{315}\sin^2 \varphi - \frac{128}{315}\right)\cos \varphi$$

$$\int d\varphi \sin \varphi = -\cos \varphi$$

$$\int d\varphi \sin^2 \varphi = -\frac{1}{4}\sin 2\varphi + \frac{1}{2}\varphi$$

$$\int d\varphi \sin^3 \varphi = \frac{1}{12}\cos 3\varphi - \frac{3}{4}\cos \varphi$$

$$\int d\varphi \sin^4 \varphi = \frac{1}{32}\sin 4\varphi - \frac{1}{4}\sin 2\varphi + \frac{3}{8}\varphi$$

$$\int d\varphi \sin^5 \varphi = -\frac{1}{80}\cos 5\varphi + \frac{5}{48}\cos 3\varphi - \frac{5}{8}\cos \varphi$$

$$\int d\varphi \sin^6 \varphi = -\frac{1}{192}\sin 6\varphi + \frac{3}{64}\sin 4\varphi - \frac{15}{64}\sin 2\varphi + \frac{5}{16}\varphi$$

$$\int d\varphi \sin^7 \varphi = \frac{1}{448}\cos 7\varphi - \frac{7}{320}\cos 5\varphi + \frac{7}{64}\cos 3\varphi - \frac{35}{64}\cos \varphi$$

$$\int d\varphi \sin^8 \varphi = \frac{1}{1024}\sin 8\varphi - \frac{1}{96}\sin 6\varphi + \frac{7}{128}\sin 4\varphi - \frac{7}{32}\sin 2\varphi + \frac{35}{128}\varphi$$

$$\int d\varphi \sin^9 \varphi = -\frac{1}{2304}\cos 9\varphi + \frac{9}{1792}\cos 7\varphi - \frac{9}{320}\cos 5\varphi + \frac{7}{64}\cos 3\varphi - \frac{63}{128}\cos \varphi$$

TAB. II.

$$\int d\varphi \cos^m \varphi$$

$$\int d\varphi \cos \varphi = \sin \varphi$$

$$\int d\varphi \cos^2 \varphi = \frac{1}{2} \sin \varphi \cos \varphi + \frac{1}{2} \varphi$$

$$\int d\varphi \cos^3 \varphi = \left(\frac{1}{3} \cos^2 \varphi + \frac{2}{3}\right) \sin \varphi$$

$$\int d\varphi \cos^4 \varphi = \left(\frac{1}{4} \cos^3 \varphi + \frac{3}{8} \cos \varphi\right) \sin \varphi + \frac{3}{8} \varphi$$

$$\int d\varphi \cos^5 \varphi = \left(\frac{1}{5} \cos^4 \varphi + \frac{4}{15} \cos^2 \varphi + \frac{8}{15}\right) \sin \varphi$$

$$\int d\varphi \cos^6 \varphi = \left(\frac{1}{6} \cos^5 \varphi + \frac{5}{24} \cos^3 \varphi + \frac{5}{16} \cos \varphi\right) \sin \varphi + \frac{5}{16} \varphi$$

$$\int d\varphi \cos^7 \varphi = \left(\frac{1}{7} \cos^6 \varphi + \frac{6}{35} \cos^4 \varphi + \frac{8}{35} \cos^2 \varphi + \frac{16}{35}\right) \sin \varphi$$

$$\int d\varphi \cos^8 \varphi = \left(\frac{1}{8} \cos^7 \varphi + \frac{7}{48} \cos^5 \varphi + \frac{35}{192} \cos^3 \varphi + \frac{35}{128} \cos \varphi\right) \sin \varphi + \frac{35}{128} \varphi$$

$$\int d\varphi \cos^9 \varphi = \left(\frac{1}{9} \cos^8 \varphi + \frac{8}{63} \cos^6 \varphi + \frac{16}{105} \cos^4 \varphi + \frac{64}{315} \cos^2 \varphi + \frac{128}{315}\right) \sin \varphi$$

$$\int d\varphi \cos \varphi = \sin \varphi$$

$$\int d\varphi \cos^2 \varphi = \frac{1}{4} \sin 2\varphi + \frac{1}{2} \varphi$$

$$\int d\varphi \cos^3 \varphi = \frac{1}{12} \sin 3\varphi + \frac{3}{4} \sin \varphi$$

$$\int d\varphi \cos^4 \varphi = \frac{1}{32} \sin 4\varphi + \frac{1}{4} \sin 2\varphi + \frac{3}{8} \varphi$$

$$\int d\varphi \cos^5 \varphi = \frac{1}{80} \sin 5\varphi + \frac{5}{48} \sin 3\varphi + \frac{5}{8} \sin \varphi$$

$$\int d\varphi \cos^6 \varphi = \frac{1}{192} \sin 6\varphi + \frac{3}{64} \sin 4\varphi + \frac{15}{64} \sin 2\varphi + \frac{5}{16} \varphi$$

$$\int d\varphi \cos^7 \varphi = \frac{1}{448} \sin 7\varphi + \frac{7}{320} \sin 5\varphi + \frac{7}{64} \sin 3\varphi + \frac{35}{64} \sin \varphi$$

$$\int d\varphi \cos^8 \varphi = \frac{1}{1024} \sin 8\varphi + \frac{1}{96} \sin 6\varphi + \frac{7}{128} \sin 4\varphi + \frac{7}{32} \sin 2\varphi + \frac{35}{128} \varphi$$

$$\int d\varphi \cos^9 \varphi = \frac{1}{2304} \sin 9\varphi + \frac{9}{1792} \sin 7\varphi + \frac{9}{320} \sin 5\varphi + \frac{7}{64} \sin 3\varphi + \frac{63}{128} \sin \varphi$$

TAB. III.

$$\int d\varphi \sin\varphi \cos^n\varphi$$

$$\int d\varphi \sin\varphi \cos^n\varphi = -\frac{1}{n+1}\cos^{n+1}\varphi$$

$$\text{Cos}\,\varphi = \cos\varphi$$

$$\text{Cos}^2\varphi = \frac{1}{2}\cos 2\varphi + \frac{1}{2}$$

$$\text{Cos}^3\varphi = \frac{1}{4}\cos 3\varphi + \frac{3}{4}\cos\varphi$$

$$\text{Cos}^4\varphi = \frac{1}{8}\cos 4\varphi + \frac{1}{2}\cos 2\varphi + \frac{3}{8}$$

$$\text{Cos}^5\varphi = \frac{1}{16}\cos 5\varphi + \frac{5}{16}\cos 3\varphi + \frac{5}{8}\cos\varphi$$

$$\text{Cos}^6\varphi = \frac{1}{32}\cos 6\varphi + \frac{3}{16}\cos 4\varphi + \frac{15}{32}\cos 2\varphi + \frac{5}{16}$$

$$\text{Cos}^7\varphi = \frac{1}{64}\cos 7\varphi + \frac{7}{64}\cos 5\varphi + \frac{21}{64}\cos 3\varphi + \frac{35}{64}\cos\varphi$$

$$\text{Cos}^8\varphi = \frac{1}{128}\cos 8\varphi + \frac{1}{16}\cos 6\varphi + \frac{7}{32}\cos 4\varphi + \frac{7}{16}\cos 2\varphi + \frac{35}{128}$$

$$\text{Cos}^9\varphi = \frac{1}{256}\cos 9\varphi + \frac{9}{256}\cos 7\varphi + \frac{9}{64}\cos 5\varphi + \frac{21}{64}\cos 3\varphi + \frac{63}{128}\cos\varphi$$

$$\text{Cos}^{10}\varphi = \frac{1}{512}\cos 10\varphi + \frac{5}{256}\cos 8\varphi + \frac{45}{512}\cos 6\varphi + \frac{15}{64}\cos 4\varphi + \frac{105}{256}\cos 2\varphi + \frac{63}{256}$$

$$\text{Cos}^n\varphi = \frac{1}{2^{n-1}}\Big[\cos n\varphi + {}^nA\cos(n-2)\varphi + {}^nB\cos(n-4)\varphi$$
$$+ {}^nC\cos(n-6)\varphi + {}^nD\cos(n-8)\varphi$$
$$+ {}^nC\cos(n-10)\varphi + \text{etc.}\Big]$$

The series in brackets is to be continued until we arrive at negative angles, and $\frac{1}{2} = \frac{1}{2}\cos 0\varphi$ is to be put for $\cos 0\varphi$.

TAB. IV.

$$\int d\varphi \cos\varphi \sin^n\varphi$$

$$\int d\varphi \cos\varphi \sin^n\varphi = \frac{1}{n+1}\sin^{n+1}\varphi$$

$\text{Sin}\ \varphi = \sin\varphi$

$\text{Sin}^2\varphi = -\frac{1}{2}\cos 2\varphi + \frac{1}{2}$

$\text{Sin}^3\varphi = -\frac{1}{4}\sin 3\varphi + \frac{3}{4}\sin\varphi$

$\text{Sin}^4\varphi = \frac{1}{8}\cos 4\varphi - \frac{1}{2}\cos 2\varphi + \frac{3}{8}$

$\text{Sin}^5\varphi = \frac{1}{16}\sin 5\varphi - \frac{5}{16}\sin 3\varphi + \frac{5}{8}\sin\varphi$

$\text{Sin}^6\varphi = -\frac{1}{32}\cos 6\varphi + \frac{3}{16}\cos 4\varphi - \frac{15}{32}\cos 2\varphi + \frac{5}{16}$

$\text{Sin}^7\varphi = -\frac{1}{64}\sin 7\varphi + \frac{7}{64}\sin 5\varphi - \frac{21}{64}\sin 3\varphi + \frac{35}{64}\sin\varphi$

$\text{Sin}^8\varphi = \frac{1}{128}\cos 8\varphi - \frac{1}{16}\cos 6\varphi + \frac{7}{32}\cos 4\varphi - \frac{7}{16}\cos 2\varphi + \frac{35}{128}$

$\text{Sin}^9\varphi = \frac{1}{256}\sin 9\varphi - \frac{9}{256}\sin 7\varphi + \frac{9}{64}\sin 5\varphi - \frac{21}{64}\sin 3\varphi + \frac{63}{128}\sin\varphi$

$\text{Sin}^{10}\varphi = -\frac{1}{512}\cos 10\varphi + \frac{5}{256}\cos 8\varphi - \frac{45}{512}\cos 6\varphi + \frac{15}{64}\cos 4\varphi - \frac{105}{256}\cos 2\varphi + \frac{63}{256}$

....

$\text{Sin}^n\varphi = \pm\frac{1}{2^{n-1}}\left[\cos n\varphi - {}^nA\cos(n-2)\varphi + {}^nB\cos(n-4)\varphi - {}^nC\cos(n-6)\varphi + \&c.\right]$

$\text{Sin}^n\varphi = \pm\frac{1}{2^{n-1}}\left[\sin n\varphi - {}^nA\sin(n-2)\varphi + {}^nB\sin(n-4)\varphi - {}^nC\sin(n-6)\varphi + \text{etc.}\right]$

The first series for $\sin^n\varphi$ has $+$ or $-$ according as n is of the form $4k$ or $4k+2$; the second has $+$ or $-$ according as n is of the form $4k+1$ or $4k+3$. Both series are to be continued to negative angles, and $\frac{1}{2} = \frac{1}{2}\cos 0\varphi$ is to be substituted for $\cos 0\varphi$.

TAB. V.

$$\int d\varphi \sin^2\varphi \cos^n\varphi$$

$$\int d\varphi \sin^2\varphi \cos\varphi = \frac{1}{3}\sin^3\varphi$$

$$\int d\varphi \sin^2\varphi \cos^2\varphi = \frac{1}{4}\sin^3\varphi\cos\varphi - \frac{1}{8}\sin\varphi\cos\varphi + \frac{1}{8}\varphi$$

$$\int d\varphi \sin^2\varphi \cos^3\varphi = \left(\frac{1}{5}\cos^2\varphi + \frac{2}{15}\right)\sin^3\varphi$$

$$\int d\varphi \sin^2\varphi \cos^4\varphi = \frac{1}{6}\sin^3\varphi\cos^3\varphi + \frac{1}{2}\int d\varphi \sin^2\varphi \cos^2\varphi$$

$$\int d\varphi \sin^2\varphi \cos^5\varphi = \left(\frac{1}{7}\cos^4\varphi + \frac{4}{35}\cos^2\varphi + \frac{8}{105}\right)\sin^3\varphi$$

$$\int d\varphi \sin^2\varphi \cos\varphi = -\frac{1}{4}\left(\frac{1}{3}\sin 3\varphi - \sin\varphi\right)$$

$$\int d\varphi \sin^2\varphi \cos^2\varphi = -\frac{1}{8}\left(\frac{1}{4}\sin 4\varphi - \varphi\right)$$

$$\int d\varphi \sin^2\varphi \cos^3\varphi = -\frac{1}{16}\left(\frac{1}{5}\sin 5\varphi + \frac{1}{3}\sin 3\varphi - 2\sin\varphi\right)$$

$$\int d\varphi \sin^2\varphi \cos^4\varphi = -\frac{1}{32}\left(\frac{1}{6}\sin 6\varphi + \frac{1}{2}\sin 4\varphi - \frac{1}{2}\sin 2\varphi - 2\varphi\right)$$

$$\int d\varphi \sin^2\varphi \cos^5\varphi = -\frac{1}{64}\left(\frac{1}{7}\sin 7\varphi + \frac{3}{5}\sin 5\varphi + \frac{1}{3}\sin 3\varphi - 5\sin\varphi\right)$$

$$\int d\varphi \sin^2\varphi \cos^6\varphi = -\frac{1}{128}\left(\frac{1}{8}\sin 8\varphi + \frac{2}{3}\sin 6\varphi + \sin 4\varphi - 2\sin 2\varphi - 5\varphi\right)$$

$$\int d\varphi \sin^2\varphi \cos^7\varphi = -\frac{1}{256}\left(\frac{1}{9}\sin 9\varphi + \frac{5}{7}\sin 7\varphi + \frac{8}{5}\sin 5\varphi - 14\sin\varphi\right)$$

$$\int d\varphi \sin^2\varphi \cos^8\varphi = -\frac{1}{512}\left(\frac{1}{10}\sin 10\varphi + \frac{3}{4}\sin 8\varphi + \frac{13}{6}\sin 6\varphi + 2\sin 4\varphi - 7\sin 2\varphi - 14\varphi\right)$$

$$\int d\varphi \sin^2\varphi \cos^9\varphi = -\frac{1}{1024}\left(\frac{1}{11}\sin 11\varphi + \frac{7}{9}\sin 9\varphi + \frac{19}{7}\sin 7\varphi + \frac{21}{5}\sin 5\varphi - 2\sin 3\varphi - 42\sin\varphi\right)$$

$$\int d\varphi \sin^2\varphi \cos^{10}\varphi = -\frac{1}{2048}\left(\frac{1}{12}\sin 12\varphi + \frac{4}{5}\sin 10\varphi + \frac{13}{4}\sin 8\varphi + \frac{20}{3}\sin 6\varphi + \frac{15}{4}\sin 4\varphi - 24\sin 2\varphi - 42\varphi\right)$$

TAB. VI.

$$\int d\varphi \sin^3\varphi \cos^n\varphi$$

$$\int d\varphi \sin^3\varphi \cos\varphi = \frac{1}{4}\sin^4\varphi$$

$$\int d\varphi \sin^3\varphi \cos^2\varphi = \left(\frac{1}{5}\sin^4\varphi - \frac{1}{15}\sin^2\varphi - \frac{2}{15}\right)\cos\varphi$$

$$\int d\varphi \sin^3\varphi \cos^3\varphi = \left(\frac{1}{6}\cos^2\varphi + \frac{1}{12}\right)\sin^4\varphi$$

$$\int d\varphi \sin^3\varphi \cos^4\varphi = \frac{1}{7}\sin^4\varphi\cos^3\varphi - \frac{3}{7}\int d\varphi \sin^3\varphi \cos^2\varphi$$

$$\int d\varphi \sin^3\varphi \cos\varphi = \frac{1}{8}\left(\frac{1}{4}\cos 4\varphi - \cos 2\varphi\right)$$

$$\int d\varphi \sin^3\varphi \cos^2\varphi = \frac{1}{16}\left(\frac{1}{5}\cos 5\varphi - \frac{1}{3}\cos 3\varphi - 2\cos\varphi\right)$$

$$\int d\varphi \sin^3\varphi \cos^3\varphi = \frac{1}{32}\left(\frac{1}{6}\cos 6\varphi - \frac{3}{2}\cos 2\varphi\right)$$

$$\int d\varphi \sin^3\varphi \cos^4\varphi = \frac{1}{64}\left(\frac{1}{7}\cos 7\varphi + \frac{1}{5}\cos 5\varphi - \cos 3\varphi - 3\cos\varphi\right)$$

$$\int d\varphi \sin^3\varphi \cos^5\varphi = \frac{1}{128}\left(\frac{1}{8}\cos 8\varphi + \frac{1}{3}\cos 6\varphi - \frac{1}{2}\cos 4\varphi - 3\cos 2\varphi\right)$$

$$\int d\varphi \sin^3\varphi \cos^6\varphi = \frac{1}{256}\left(\frac{1}{9}\cos 9\varphi + \frac{3}{7}\cos 7\varphi - \frac{8}{3}\cos 3\varphi - 6\cos\varphi\right)$$

$$\int d\varphi \sin^3\varphi \cos^7\varphi = \frac{1}{512}\left(\frac{1}{10}\cos 10\varphi + \frac{1}{2}\cos 8\varphi + \frac{1}{2}\cos 6\varphi - 2\cos 4\varphi - 7\cos 2\varphi\right)$$

$$\int d\varphi \sin^3\varphi \cos^8\varphi = \frac{1}{1024}\left(\frac{1}{11}\cos 11\varphi + \frac{5}{9}\cos 9\varphi + \cos 7\varphi - \cos 5\varphi - \frac{22}{3}\cos 3\varphi - 14\cos\varphi\right)$$

$$\int d\varphi \sin^3\varphi \cos^9\varphi = \frac{1}{2048}\left(\frac{1}{12}\cos 12\varphi + \frac{3}{5}\cos 10\varphi + \frac{3}{2}\cos 8\varphi + \frac{1}{3}\cos 6\varphi - \frac{27}{4}\cos 4\varphi - 18\cos 2\varphi\right)$$

$$\int d\varphi \sin^3\varphi \cos^{10}\varphi = \frac{1}{4096}\left(\frac{1}{13}\cos 13\varphi + \frac{7}{11}\cos 11\varphi + 2\cos 9\varphi + 2\cos 7\varphi - 5\cos 5\varphi - 21\cos 3\varphi - 36\cos\varphi\right)$$

TAB. VII.

$$\int d\varphi \sin^4\varphi \cos^n\varphi$$

$$\int d\varphi \sin^4\varphi \cos\varphi = \frac{1}{5}\sin^5\varphi$$

$$\int d\varphi \sin^4\varphi \cos^2\varphi = \left(\frac{1}{6}\sin^5\varphi - \frac{1}{24}\sin^3\varphi - \frac{1}{16}\sin\varphi\right)\cos\varphi + \frac{1}{16}\varphi$$

$$\int d\varphi \sin^4\varphi \cos^3\varphi = \left(\frac{1}{7}\cos^2\varphi + \frac{2}{35}\right)\sin^5\varphi$$

$$\int d\varphi \sin^4\varphi \cos\varphi = \frac{1}{16}\left(\frac{1}{5}\sin 5\varphi - \sin 3\varphi + 2\sin\varphi\right)$$

$$\int d\varphi \sin^4\varphi \cos^2\varphi = \frac{1}{32}\left(\frac{1}{6}\sin 6\varphi - \frac{1}{2}\sin 4\varphi - \frac{1}{2}\sin 2\varphi + 2\varphi\right)$$

$$\int d\varphi \sin^4\varphi \cos^3\varphi = \frac{1}{64}\left(\frac{1}{7}\sin 7\varphi - \frac{1}{5}\sin 5\varphi - \sin 3\varphi + 3\sin\varphi\right)$$

$$\int d\varphi \sin^4\varphi \cos^4\varphi = \frac{1}{128}\left(\frac{1}{8}\sin 8\varphi - \sin 4\varphi + 3\varphi\right)$$

$$\int d\varphi \sin^4\varphi \cos^5\varphi = \frac{1}{256}\left(\frac{1}{9}\sin 9\varphi + \frac{1}{7}\sin 7\varphi - \frac{4}{5}\sin 5\varphi - \frac{4}{3}\sin 3\varphi + 6\sin\varphi\right)$$

$$\int d\varphi \sin^4\varphi \cos^6\varphi = \frac{1}{512}\left(\frac{1}{10}\sin 10\varphi + \frac{1}{4}\sin 8\varphi - \frac{1}{2}\sin 6\varphi - 2\sin 4\varphi + \sin 2\varphi + 6\varphi\right)$$

$$\int d\varphi \sin^4\varphi \cos^7\varphi = \frac{1}{1024}\left(\frac{1}{11}\sin 11\varphi + \frac{1}{3}\sin 9\varphi - \frac{1}{7}\sin 7\varphi - \frac{11}{5}\sin 5\varphi - 2\sin 3\varphi + 14\sin\varphi\right)$$

$$\int d\varphi \sin^4\varphi \cos^8\varphi = \frac{1}{2048}\left(\frac{1}{12}\sin 12\varphi + \frac{2}{5}\sin 10\varphi + \frac{1}{4}\sin 8\varphi - 2\sin 6\varphi - \frac{17}{4}\sin 4\varphi + 4\sin 2\varphi + 14\varphi\right)$$

$$\int d\varphi \sin^4\varphi \cos^9\varphi = \frac{1}{4096}\left(\frac{1}{13}\sin 13\varphi + \frac{5}{11}\sin 11\varphi + \frac{2}{3}\sin 9\varphi - \frac{10}{7}\sin 7\varphi - \frac{29}{5}\sin 5\varphi - 3\sin 3\varphi + 36\sin\varphi\right)$$

$$\int d\varphi \sin^4\varphi \cos^{10}\varphi = \frac{1}{8192}\left(\frac{1}{14}\sin 14\varphi + \frac{1}{2}\sin 12\varphi + \frac{11}{10}\sin 10\varphi - \frac{1}{2}\sin 8\varphi - \frac{13}{2}\sin 6\varphi - \frac{19}{2}\sin 4\varphi + \frac{27}{2}\sin 2\varphi + 36\varphi\right)$$

TAB. VIII.

$$\int d\varphi \sin^5\varphi \cos^n\varphi$$

$$\int d\varphi \sin^5\varphi \cos\varphi = \frac{1}{6}\sin^6\varphi$$

$$\int d\varphi \sin^5\varphi \cos^2\varphi = \frac{1}{7}\sin^6\varphi \cos\varphi + \frac{1}{7}\int d\varphi \sin^6\varphi$$

$$\int d\varphi \sin^5\varphi \cos^3\varphi = \left(\frac{1}{8}\cos^2\varphi + \frac{1}{24}\right)\sin^6\varphi$$

$$\int d\varphi \sin^5\varphi \cos\varphi = -\frac{1}{32}\left(\frac{1}{6}\cos 6\varphi - \cos 4\varphi + \frac{5}{2}\cos 2\varphi\right)$$

$$\int d\varphi \sin^5\varphi \cos^2\varphi = -\frac{1}{64}\left(\frac{1}{7}\cos 7\varphi - \frac{3}{5}\cos 5\varphi + \frac{1}{3}\cos 3\varphi + 5\cos\varphi\right)$$

$$\int d\varphi \sin^5\varphi \cos^3\varphi = -\frac{1}{128}\left(\frac{1}{8}\cos 8\varphi - \frac{1}{3}\cos 6\varphi - \frac{1}{2}\cos 4\varphi + 3\cos 2\varphi\right)$$

$$\int d\varphi \sin^5\varphi \cos^4\varphi = -\frac{1}{256}\left(\frac{1}{9}\cos 9\varphi - \frac{1}{7}\cos 7\varphi - \frac{4}{5}\cos 5\varphi + \frac{4}{3}\cos 3\varphi + 6\cos\varphi\right)$$

$$\int d\varphi \sin^5\varphi \cos^5\varphi = -\frac{1}{512}\left(\frac{1}{10}\cos 10\varphi - \frac{5}{6}\cos 6\varphi + 5\cos 2\varphi\right)$$

$$\int d\varphi \sin^5\varphi \cos^6\varphi = -\frac{1}{1024}\left(\frac{1}{11}\cos 11\varphi + \frac{1}{9}\cos 9\varphi - \frac{5}{7}\cos 7\varphi - \cos 5\varphi + \frac{10}{3}\cos 3\varphi + 10\cos\varphi\right)$$

$$\int d\varphi \sin^5\varphi \cos^7\varphi = -\frac{1}{2048}\left(\frac{1}{12}\cos 12\varphi + \frac{1}{5}\cos 10\varphi - \frac{1}{2}\cos 8\varphi - \frac{5}{3}\cos 6\varphi + \frac{5}{4}\cos 4\varphi + 10\cos 2\varphi\right)$$

$$\int d\varphi \sin^5\varphi \cos^8\varphi = -\frac{1}{4096}\left(\frac{1}{13}\cos 13\varphi + \frac{3}{11}\cos 11\varphi - \frac{2}{9}\cos 9\varphi - 2\cos 7\varphi - \cos 5\varphi + \frac{25}{3}\cos 3\varphi + 20\cos\varphi\right)$$

$$\int d\varphi \sin^5\varphi \cos^9\varphi = -\frac{1}{8192}\left(\frac{1}{14}\cos 14\varphi + \frac{1}{3}\cos 12\varphi + \frac{1}{10}\cos 10\varphi - 2\cos 8\varphi - \frac{19}{6}\cos 6\varphi + 5\cos 4\varphi + \frac{45}{2}\cos 2\varphi\right)$$

$$\int d\varphi \sin^5\varphi \cos^{10}\varphi = -\frac{1}{16384}\left(\frac{1}{15}\cos 15\varphi + \frac{5}{13}\cos 13\varphi + \frac{5}{11}\cos 11\varphi - \frac{5}{3}\cos 9\varphi - 5\cos 7\varphi + \frac{1}{5}\cos 5\varphi + \frac{65}{3}\cos 3\varphi + 45\cos\varphi\right)$$

TAB. IX.

$$\int d\varphi \sin^{6}\varphi \cos^{n}\varphi$$

$$\int d\varphi \sin^{6}\varphi \cos\varphi = \frac{1}{7}\sin^{7}\varphi$$

$$\int d\varphi \sin^{6}\varphi \cos^{2}\varphi = \left(\frac{1}{8}\sin^{7}\varphi - \frac{1}{48}\sin^{5}\varphi - \frac{5}{192}\sin^{3}\varphi - \frac{5}{128}\sin\varphi\right)\cos\varphi + \frac{5}{128}\varphi$$

$$\int d\varphi \sin^{6}\varphi \cos^{3}\varphi = \left(\frac{1}{9}\cos^{2}\varphi + \frac{2}{63}\right)\sin^{7}\varphi$$

$$\int d\varphi \sin^{6}\varphi \cos\varphi = -\frac{1}{64}\left(\frac{1}{7}\sin 7\varphi - \sin 5\varphi + 3\sin 3\varphi - 5\sin\varphi\right)$$

$$\int d\varphi \sin^{6}\varphi \cos^{2}\varphi = -\frac{1}{128}\left(\frac{1}{8}\sin 8\varphi - \frac{2}{3}\sin 6\varphi + \sin 4\varphi + 2\sin 2\varphi - 5\varphi\right)$$

$$\int d\varphi \sin^{6}\varphi \cos^{3}\varphi = -\frac{1}{256}\left(\frac{1}{9}\sin 9\varphi - \frac{3}{7}\sin 7\varphi + \frac{8}{3}\sin 3\varphi - 6\sin\varphi\right)$$

$$\int d\varphi \sin^{6}\varphi \cos^{4}\varphi = -\frac{1}{512}\left(\frac{1}{10}\sin 10\varphi - \frac{1}{4}\sin 8\varphi - \frac{1}{2}\sin 6\varphi + 2\sin 4\varphi + \sin 2\varphi - 6\varphi\right)$$

$$\int d\varphi \sin^{6}\varphi \cos^{5}\varphi = -\frac{1}{1024}\left(\frac{1}{11}\sin 11\varphi - \frac{1}{9}\sin 9\varphi - \frac{5}{7}\sin 7\varphi + \sin 5\varphi + \frac{10}{3}\sin 3\varphi - 10\sin\varphi\right)$$

$$\int d\varphi \sin^{6}\varphi \cos^{6}\varphi = -\frac{1}{2048}\left(\frac{1}{12}\sin 12\varphi - \frac{3}{4}\sin 8\varphi + \frac{15}{4}\sin 4\varphi - 10\varphi\right)$$

$$\int d\varphi \sin^{6}\varphi \cos^{7}\varphi = -\frac{1}{4096}\left(\frac{1}{13}\sin 13\varphi + \frac{1}{11}\sin 11\varphi - \frac{2}{3}\sin 9\varphi - \frac{6}{7}\sin 7\varphi + 3\sin 5\varphi + 5\sin 3\varphi - 20\sin\varphi\right)$$

$$\int d\varphi \sin^{6}\varphi \cos^{8}\varphi = -\frac{1}{8192}\left(\frac{1}{14}\sin 14\varphi + \frac{1}{6}\sin 12\varphi - \frac{1}{2}\sin 10\varphi - \frac{3}{2}\sin 8\varphi + \frac{3}{2}\sin 6\varphi + \frac{15}{2}\sin 4\varphi - \frac{5}{2}\sin 2\varphi - 20\varphi\right)$$

$$\int d\varphi \sin^{6}\varphi \cos^{9}\varphi = -\frac{1}{16384}\left(\frac{1}{15}\sin 15\varphi + \frac{3}{13}\sin 13\varphi - \frac{3}{11}\sin 11\varphi - \frac{17}{9}\sin 9\varphi - \frac{3}{7}\sin 7\varphi + \frac{39}{5}\sin 5\varphi + \frac{25}{3}\sin 3\varphi - 45\sin\varphi\right)$$

$$\int d\varphi \sin^{6}\varphi \cos^{10}\varphi = -\frac{1}{32768}\left(\frac{1}{16}\sin 16\varphi + \frac{2}{7}\sin 14\varphi - 2\sin 10\varphi - \frac{5}{2}\sin 8\varphi + 6\sin 6\varphi + 16\sin 4\varphi - 10\sin 2\varphi - 45\varphi\right)$$

TAB. X.

$$\int d\varphi \sin^7\varphi \cos^n\varphi$$

$$\int d\varphi \sin^7\varphi \cos\varphi = \frac{1}{8}\sin^8\varphi$$

$$\int d\varphi \sin^7\varphi \cos^2\varphi = \frac{1}{9}\sin^8\varphi\cos\varphi + \frac{1}{9}\int d\varphi \sin^7\varphi$$

$$\int d\varphi \sin^7\varphi \cos^3\varphi = \left(\frac{1}{10}\cos^2\varphi + \frac{1}{40}\right)\sin^8\varphi$$

$$\int d\varphi \sin^7\varphi \cos^4\varphi = \left(\frac{1}{11}\cos^3\varphi + \frac{1}{33}\cos\varphi\right)\sin^8\varphi + \frac{1}{33}\int d\varphi \sin^7\varphi$$

$$\int d\varphi \sin^7\varphi \cos\varphi = \frac{1}{128}\left(\frac{1}{8}\cos 8\varphi - \cos 6\varphi + \frac{7}{2}\cos 4\varphi - 7\cos 2\varphi\right)$$

$$\int d\varphi \sin^7\varphi \cos^2\varphi = \frac{1}{256}\left(\frac{1}{9}\cos 9\varphi - \frac{5}{7}\cos 7\varphi + \frac{8}{5}\cos 5\varphi - 14\cos\varphi\right)$$

$$\int d\varphi \sin^7\varphi \cos^3\varphi = \frac{1}{512}\left(\frac{1}{10}\cos 10\varphi - \frac{1}{2}\cos 8\varphi + \frac{1}{2}\cos 6\varphi + 2\cos 4\varphi - 7\cos 2\varphi\right)$$

$$\int d\varphi \sin^7\varphi \cos^4\varphi = \frac{1}{1024}\left(\frac{1}{11}\cos 11\varphi - \frac{1}{3}\cos 9\varphi - \frac{1}{7}\cos 7\varphi + \frac{11}{5}\cos 5\varphi - 2\cos 3\varphi - 14\cos\varphi\right)$$

$$\int d\varphi \sin^7\varphi \cos^5\varphi = \frac{1}{2048}\left(\frac{1}{12}\cos 12\varphi - \frac{1}{5}\cos 10\varphi - \frac{1}{2}\cos 8\varphi + \frac{5}{3}\cos 6\varphi + \frac{5}{4}\cos 4\varphi - 10\cos 2\varphi\right)$$

$$\int d\varphi \sin^7\varphi \cos^6\varphi = \frac{1}{4096}\left(\frac{1}{13}\cos 13\varphi - \frac{1}{11}\cos 11\varphi - \frac{2}{3}\cos 9\varphi + \frac{6}{7}\cos 7\varphi + 3\cos 5\varphi - 5\cos 3\varphi - 20\cos\varphi\right)$$

$$\int d\varphi \sin^7\varphi \cos^7\varphi = \frac{1}{8192}\left(\frac{1}{14}\cos 14\varphi - \frac{7}{10}\cos 10\varphi + \frac{7}{2}\cos 6\varphi - \frac{35}{2}\cos 2\varphi\right)$$

$$\int d\varphi \sin^7\varphi \cos^8\varphi = \frac{1}{16384}\left(\frac{1}{15}\cos 15\varphi + \frac{1}{13}\cos 13\varphi - \frac{7}{11}\cos 11\varphi - \frac{7}{9}\cos 9\varphi + 3\cos 7\varphi + \frac{21}{5}\cos 5\varphi - \frac{35}{3}\cos 3\varphi - 35\cos\varphi\right)$$

$$\int d\varphi \sin^7\varphi \cos^9\varphi = \frac{1}{32768}\left(\frac{1}{16}\cos 16\varphi + \frac{1}{7}\cos 14\varphi - \frac{1}{2}\cos 12\varphi - \frac{7}{5}\cos 10\varphi + \frac{7}{4}\cos 8\varphi + 7\cos 6\varphi - \frac{7}{2}\cos 4\varphi - 35\cos 2\varphi\right)$$

TAB. XI.

$$\int d\varphi \sin^8\varphi \cos^n\varphi$$

$$\int d\varphi \sin^8\varphi \cos\varphi = \frac{1}{9}\sin^9\varphi$$

$$\int d\varphi \sin^8\varphi \cos^2\varphi = \frac{1}{10}\sin^9\varphi\cos\varphi + \frac{1}{10}\int d\varphi \sin^8\varphi$$

$$\int d\varphi \sin^8\varphi \cos^3\varphi = \left(\frac{1}{11}\cos^2\varphi + \frac{2}{99}\right)\sin^9\varphi$$

$$\int d\varphi \sin^8\varphi \cos\varphi = \frac{1}{256}\left(\frac{1}{9}\sin 9\varphi - \sin 7\varphi + 4\sin 5\varphi - \frac{28}{3}\sin 3\varphi + 14\sin\varphi\right)$$

$$\int d\varphi \sin^8\varphi \cos^2\varphi = \frac{1}{512}\left(\frac{1}{10}\sin 10\varphi - \frac{3}{4}\sin 8\varphi + \frac{13}{6}\sin 6\varphi - 2\sin 4\varphi - 7\sin 2\varphi + 14\varphi\right)$$

$$\int d\varphi \sin^8\varphi \cos^3\varphi = \frac{1}{1024}\left(\frac{1}{11}\sin 11\varphi - \frac{5}{9}\sin 9\varphi + \sin 7\varphi + \sin 5\varphi - \frac{22}{3}\sin 3\varphi + 14\sin\varphi\right)$$

$$\int d\varphi \sin^8\varphi \cos^4\varphi = \frac{1}{2048}\left(\frac{1}{12}\sin 12\varphi - \frac{2}{5}\sin 10\varphi + \frac{1}{4}\sin 8\varphi + 2\sin 6\varphi - \frac{17}{4}\sin 4\varphi - 4\sin 2\varphi + 14\varphi\right)$$

$$\int d\varphi \sin^8\varphi \cos^5\varphi = \frac{1}{4096}\left(\frac{1}{13}\sin 13\varphi - \frac{3}{11}\sin 11\varphi - \frac{2}{9}\sin 9\varphi + 2\sin 7\varphi - \sin 5\varphi - \frac{25}{3}\sin 3\varphi + 20\sin\varphi\right)$$

$$\int d\varphi \sin^8\varphi \cos^6\varphi = \frac{1}{8192}\left(\frac{1}{14}\sin 14\varphi - \frac{1}{6}\sin 12\varphi - \frac{1}{2}\sin 10\varphi + \frac{3}{2}\sin 8\varphi + \frac{3}{2}\sin 6\varphi + \frac{15}{2}\sin 4\varphi - \frac{5}{2}\sin 2\varphi + 20\varphi\right)$$

$$\int d\varphi \sin^8\varphi \cos^7\varphi = \frac{1}{16384}\left(\frac{1}{15}\sin 15\varphi - \frac{1}{13}\sin 13\varphi - \frac{7}{11}\sin 11\varphi + \frac{7}{9}\sin 9\varphi + 3\sin 7\varphi - \frac{21}{5}\sin 5\varphi - \frac{35}{3}\sin 3\varphi + 35\sin\varphi\right)$$

$$\int d\varphi \sin^8\varphi \cos\varphi^8 = \frac{1}{32768}\left(\frac{1}{16}\sin 16\varphi - \frac{2}{3}\sin 12\varphi + \frac{7}{2}\sin 8\varphi - 14\sin 4\varphi + 35\varphi\right)$$

TAB. XII.

$$\int d\varphi \sin^9\varphi \cos^n\varphi$$

$$\int d\varphi \sin^9\varphi \cos\varphi = \frac{1}{10}\sin^{10}\varphi$$

$$\int d\varphi \sin^9\varphi \cos^2\varphi = \frac{1}{11}\sin^{10}\varphi\cos\varphi + \frac{1}{11}\int d\varphi \sin^9\varphi$$

$$\int d\varphi \sin^9\varphi \cos^3\varphi = \left(\frac{1}{12}\cos^2\varphi + \frac{1}{60}\right)\sin^{10}\varphi$$

$$\int d\varphi \sin^9\varphi \cos\varphi = -\frac{1}{512}\left(\frac{1}{10}\cos 10\varphi - \cos 8\varphi + \frac{9}{2}\cos 6\varphi - 1\right.$$
$$+ 21\, \mathrm{c}$$

$$\int d\varphi \sin^9\varphi \cos^2\varphi = -\frac{1}{1024}\left(\frac{1}{11}\cos 11\varphi - \frac{7}{9}\cos 9\varphi + \frac{19}{7}\cos 7\varphi - \frac{21}{5}\right.$$
$$- 2\cos 3\varphi + 42\, \mathrm{c}$$

$$\int d\varphi \sin^9\varphi \cos^3\varphi = -\frac{1}{2048}\left(\frac{1}{12}\cos 12\varphi - \frac{3}{5}\cos 10\varphi + \frac{3}{2}\cos 8\varphi - \frac{1}{3}\right.$$
$$- \frac{27}{4}\cos 4\varphi + 18\cos$$

$$\int d\varphi \sin^9\varphi \cos^4\varphi = -\frac{1}{4096}\left(\frac{1}{13}\cos 13\varphi - \frac{6}{11}\cos 11\varphi + \frac{2}{3}\cos 9\varphi + \frac{10}{7}\mathrm{co}\right.$$
$$- \frac{29}{5}\cos 5\varphi + 3\cos 3\varphi + 36\cos$$

$$\int d\varphi \sin^9\varphi \cos^5\varphi = -\frac{1}{8192}\left(\frac{1}{14}\cos 14\varphi - \frac{1}{3}\cos 12\varphi + \frac{1}{10}\cos 10\varphi + 2\cos\right.$$
$$- \frac{19}{6}\cos 6\varphi - 5\cos 4\varphi + \frac{45}{2}\cos 2$$

$$\int d\varphi \sin^9\varphi \cos^6\varphi = -\frac{1}{16384}\left(\frac{1}{15}\cos 15\varphi - \frac{3}{13}\cos 13\varphi - \frac{3}{11}\cos 11\varphi + \frac{17}{9}\cos\right.$$
$$- \frac{3}{7}\cos 7\varphi - \frac{39}{5}\cos 5\varphi + \frac{25}{3}\cos 3\varphi + 45\cos\varphi$$

$$\int d\varphi \sin^9\varphi \cos^7\varphi = -\frac{1}{32768}\left(\frac{1}{16}\cos 16\varphi - \frac{1}{7}\cos 14\varphi - \frac{1}{2}\cos 12\varphi + \frac{7}{5}\cos 10\right.$$
$$\left. + \frac{7}{4}\cos 8\varphi - 7\cos 6\varphi - \frac{7}{2}\cos 4\varphi + 35\cos 2\varphi\right)$$

$$\int d\varphi \sin^9\varphi \cos^8\varphi = -\frac{1}{65536}\left(\frac{1}{17}\cos 17\varphi - \frac{1}{15}\cos 15\varphi - \frac{8}{13}\cos 13\varphi + \frac{8}{11}\cos 11\right.$$
$$\left. + \frac{28}{9}\cos 9\varphi - 4\cos 7\varphi - \frac{56}{5}\cos 5\varphi + \frac{56}{3}\cos 3\varphi + 70\cos\varphi\right)$$

TAB. XIII.

$$\int \frac{d\varphi}{\sin^n\varphi}, \int \frac{d\varphi}{\cos^n\varphi}$$

$$= \log \operatorname{tang} \frac{\varphi}{2}$$

$$= -\frac{\cos\varphi}{\sin\varphi} = -\cot\varphi$$

$$= -\frac{\cos\varphi}{2\sin^2\varphi} + \frac{1}{2}\int\frac{d\varphi}{\sin\varphi}$$

$$= \left(-\frac{1}{3\sin^3\varphi} - \frac{2}{3\sin\varphi}\right)\cos\varphi = \cot\varphi - \frac{1}{3}\cot^3\varphi$$

$$= \left(-\frac{1}{4\sin^4\varphi} - \frac{3}{8\sin^2\varphi}\right)\cos\varphi + \frac{3}{8}\int\frac{d\varphi}{\sin\varphi}$$

$$= \left(-\frac{1}{5\sin^5\varphi} - \frac{4}{15\sin^3\varphi} - \frac{8}{15\sin\varphi}\right)\cos\varphi$$

$$= \left(-\frac{1}{6\sin^6\varphi} - \frac{5}{24\sin^4\varphi} - \frac{5}{16\sin^2\varphi}\right)\cos\varphi + \frac{5}{16}\int\frac{d\varphi}{\sin\varphi}$$

$$= \left(-\frac{1}{7\sin^7\varphi} - \frac{6}{35\sin^5\varphi} - \frac{8}{35\sin^3\varphi} - \frac{16}{35\sin\varphi}\right)\cos\varphi$$

$$\int\frac{d\varphi}{\cos\varphi} = \log \operatorname{tang}\left(45^\circ + \frac{\varphi}{2}\right)$$

$$\int\frac{d\varphi}{\cos^2\varphi} = \frac{\sin\varphi}{\cos\varphi} = \operatorname{tang}\varphi$$

$$\int\frac{d\varphi}{\cos^3\varphi} = \frac{\sin\varphi}{2\cos^2\varphi} + \frac{1}{2}\int\frac{d\varphi}{\cos\varphi}$$

$$\int\frac{d\varphi}{\cos^4\varphi} = \left(\frac{1}{3\cos^3\varphi} + \frac{2}{3\cos\varphi}\right)\sin\varphi = \operatorname{tang}\varphi + \frac{1}{3}\operatorname{tang}^3\varphi$$

$$\int\frac{d\varphi}{\cos^5\varphi} = \left(\frac{1}{4\cos^4\varphi} + \frac{3}{8\cos^2\varphi}\right)\sin\varphi + \frac{3}{8}\int\frac{d\varphi}{\cos\varphi}$$

$$\int\frac{d\varphi}{\cos^6\varphi} = \left(\frac{1}{5\cos^5\varphi} + \frac{4}{15\cos^3\varphi} + \frac{8}{15\cos\varphi}\right)\sin\varphi$$

$$\int\frac{d\varphi}{\cos^7\varphi} = \left(\frac{1}{6\cos^6\varphi} + \frac{5}{24\cos^4\varphi} + \frac{5}{16\cos^2\varphi}\right)\sin\varphi + \frac{5}{16}\int\frac{d\varphi}{\cos\varphi}$$

$$\int\frac{d\varphi}{\cos^8\varphi} = \left(\frac{1}{7\cos^7\varphi} + \frac{6}{35\cos^5\varphi} + \frac{8}{35\cos^3\varphi} + \frac{16}{35\cos\varphi}\right)\sin\varphi$$

TAB. XII.

$$\int d\varphi \sin^9\varphi \cos^n\varphi$$

$$\int d\varphi \sin^9\varphi \cos\varphi = \frac{1}{10}\sin^{10}\varphi$$

$$\int d\varphi \sin^9\varphi \cos^2\varphi = \frac{1}{11}\sin^{10}\varphi \cos\varphi + \frac{1}{11}\int d\varphi \sin^9\varphi$$

$$\int d\varphi \sin^9\varphi \cos^3\varphi = \left(\frac{1}{12}\cos^2\varphi + \frac{1}{60}\right)\sin^{10}\varphi$$

$$\int d\varphi \sin^9\varphi \cos\varphi = -\frac{1}{512}\left(\frac{1}{10}\cos 10\varphi - \cos 8\varphi + \frac{9}{2}\cos 6\varphi - 12\cos 4\varphi + 21\cos 2\varphi\right)$$

$$\int d\varphi \sin^9\varphi \cos^2\varphi = -\frac{1}{1024}\left(\frac{1}{11}\cos 11\varphi - \frac{7}{9}\cos 9\varphi + \frac{19}{7}\cos 7\varphi - \frac{21}{5}\cos 5\varphi - 2\cos 3\varphi + 42\cos\varphi\right)$$

$$\int d\varphi \sin^9\varphi \cos^3\varphi = -\frac{1}{2048}\left(\frac{1}{12}\cos 12\varphi - \frac{3}{5}\cos 10\varphi + \frac{3}{2}\cos 8\varphi - \frac{1}{3}\cos 6\varphi - \frac{27}{4}\cos 4\varphi + 18\cos 2\varphi\right)$$

$$\int d\varphi \sin^9\varphi \cos^4\varphi = -\frac{1}{4096}\left(\frac{1}{13}\cos 13\varphi - \frac{6}{11}\cos 11\varphi + \frac{2}{3}\cos 9\varphi + \frac{10}{7}\cos 7\varphi - \frac{29}{5}\cos 5\varphi + 3\cos 3\varphi + 36\cos\ \right)$$

$$\int d\varphi \sin^9\varphi \cos^5\varphi = -\frac{1}{8192}\left(\frac{1}{14}\cos 14\varphi - \frac{1}{3}\cos 12\varphi + \frac{1}{10}\cos 10\varphi + 2\cos 8\varphi - \frac{19}{6}\cos 6\varphi - 5\cos 4\varphi + \frac{45}{2}\cos 2\varphi\right)$$

$$\int d\varphi \sin^9\varphi \cos^6\varphi = -\frac{1}{16384}\left(\frac{1}{15}\cos 15\varphi - \frac{3}{13}\cos 13\varphi - \frac{3}{11}\cos 11\varphi + \frac{17}{9}\cos 9\varphi - \frac{3}{7}\cos 7\varphi - \frac{39}{5}\cos 5\varphi + \frac{25}{3}\cos 3\varphi + 45\cos\varphi\right)$$

$$\int d\varphi \sin^9\varphi \cos^7\varphi = -\frac{1}{32768}\left(\frac{1}{16}\cos 16\varphi - \frac{1}{7}\cos 14\varphi - \frac{1}{2}\cos 12\varphi + \frac{7}{5}\cos 10\varphi + \frac{7}{4}\cos 8\varphi - 7\cos 6\varphi - \frac{7}{2}\cos 4\varphi + 35\cos 2\varphi\right)$$

$$\int d\varphi \sin^9\varphi \cos^8\varphi = -\frac{1}{65536}\left(\frac{1}{17}\cos 17\varphi - \frac{1}{15}\cos 15\varphi - \frac{8}{13}\cos 13\varphi + \frac{8}{11}\cos 11\varphi + \frac{28}{9}\cos 9\varphi - 4\cos 7\varphi - \frac{56}{5}\cos 5\varphi + \frac{56}{3}\cos 3\varphi + 70\cos\varphi\right)$$

TAB. XIII.

$$\int \frac{d\varphi}{\sin^n\varphi}, \int \frac{d\varphi}{\cos^n\varphi}$$

$$\int \frac{d\varphi}{\sin\varphi} = \log \operatorname{tang} \frac{\varphi}{2}$$

$$\int \frac{d\varphi}{\sin^2\varphi} = -\frac{\cos\varphi}{\sin\varphi} = -\cot\varphi$$

$$\int \frac{d\varphi}{\sin^3\varphi} = -\frac{\cos\varphi}{2\sin^2\varphi} + \frac{1}{2}\int \frac{d\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi}{\sin^4\varphi} = \left(-\frac{1}{3\sin^3\varphi} - \frac{2}{3\sin\varphi}\right)\cos\varphi = \cot\varphi - \frac{1}{3}\cot^3\varphi$$

$$\int \frac{d\varphi}{\sin^5\varphi} = \left(-\frac{1}{4\sin^4\varphi} - \frac{3}{8\sin^2\varphi}\right)\cos\varphi + \frac{3}{8}\int \frac{d\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi}{\sin^6\varphi} = \left(-\frac{1}{5\sin^5\varphi} - \frac{4}{15\sin^3\varphi} - \frac{8}{15\sin\varphi}\right)\cos\varphi$$

$$\int \frac{d\varphi}{\sin^7\varphi} = \left(-\frac{1}{6\sin^6\varphi} - \frac{5}{24\sin^4\varphi} - \frac{5}{16\sin^2\varphi}\right)\cos\varphi + \frac{5}{16}\int \frac{d\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi}{\sin^8\varphi} = \left(-\frac{1}{7\sin^7\varphi} - \frac{6}{35\sin^5\varphi} - \frac{8}{35\sin^3\varphi} - \frac{16}{35\sin\varphi}\right)\cos\varphi$$

$$\int \frac{d\varphi}{\cos\varphi} = \log \operatorname{tang}\left(45^\circ + \frac{\varphi}{2}\right)$$

$$\int \frac{d\varphi}{\cos^2\varphi} = \frac{\sin\varphi}{\cos\varphi} = \operatorname{tang}\varphi$$

$$\int \frac{d\varphi}{\cos^3\varphi} = \frac{\sin\varphi}{2\cos^2\varphi} + \frac{1}{2}\int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi}{\cos^4\varphi} = \left(\frac{1}{3\cos^3\varphi} + \frac{2}{3\cos\varphi}\right)\sin\varphi = \operatorname{tang}\varphi + \frac{1}{3}\operatorname{tang}^3\varphi$$

$$\int \frac{d\varphi}{\cos^5\varphi} = \left(\frac{1}{4\cos^4\varphi} + \frac{3}{8\cos^2\varphi}\right)\sin\varphi + \frac{3}{8}\int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi}{\cos^6\varphi} = \left(\frac{1}{5\cos^5\varphi} + \frac{4}{15\cos^3\varphi} + \frac{8}{15\cos\varphi}\right)\sin\varphi$$

$$\int \frac{d\varphi}{\cos^7\varphi} = \left(\frac{1}{6\cos^6\varphi} + \frac{5}{24\cos^4\varphi} + \frac{5}{16\cos^2\varphi}\right)\sin\varphi + \frac{5}{16}\int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi}{\cos^8\varphi} = \left(\frac{1}{7\cos^7\varphi} + \frac{6}{35\cos^5\varphi} + \frac{8}{35\cos^3\varphi} + \frac{16}{35\cos\varphi}\right)\sin\varphi$$

TAB. XIV.

$$\int \frac{d\varphi \sin^n\varphi}{\cos\varphi}, \quad \int \frac{d\varphi \cos^n\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi \sin\varphi}{\cos\varphi} = -\log\cos\varphi = \log\sec\varphi$$

$$\int \frac{d\varphi \sin^2\varphi}{\cos\varphi} = -\sin\varphi + \int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi \sin^3\varphi}{\cos\varphi} = -\frac{\sin^2\varphi}{2} + \int \frac{d\varphi \sin\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi \sin^4\varphi}{\cos\varphi} = -\frac{\sin^3\varphi}{3} - \sin\varphi + \int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi \sin^5\varphi}{\cos\varphi} = -\frac{\sin^4\varphi}{4} - \frac{\sin^2\varphi}{2} + \int \frac{d\varphi \sin\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi \sin^6\varphi}{\cos\varphi} = -\frac{\sin^5\varphi}{5} - \frac{\sin^3\varphi}{3} - \sin\varphi + \int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi \sin^7\varphi}{\cos\varphi} = -\frac{\sin^6\varphi}{6} - \frac{\sin^4\varphi}{4} - \frac{\sin^2\varphi}{2} + \int \frac{d\varphi \sin\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi \sin^8\varphi}{\cos\varphi} = -\frac{\sin^7\varphi}{7} - \frac{\sin^5\varphi}{5} - \frac{\sin^3\varphi}{3} - \sin\varphi + \int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi \cos\varphi}{\sin\varphi} = \log\sin\varphi$$

$$\int \frac{d\varphi \cos^2\varphi}{\sin\varphi} = \cos\varphi + \int \frac{d\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi \cos^3\varphi}{\sin\varphi} = \frac{\cos^2\varphi}{2} + \int \frac{d\varphi \cos\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi \cos^4\varphi}{\sin\varphi} = \frac{\cos^3\varphi}{3} + \cos\varphi + \int \frac{d\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi \cos^5\varphi}{\sin\varphi} = \frac{\cos^4\varphi}{4} + \frac{\cos^2\varphi}{2} + \int \frac{d\varphi \cos\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi \cos^6\varphi}{\sin\varphi} = \frac{\cos^5\varphi}{5} + \frac{\cos^3\varphi}{3} + \cos\varphi + \int \frac{d\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi \cos^7\varphi}{\sin\varphi} = \frac{\cos^6\varphi}{6} + \frac{\cos^4\varphi}{4} + \frac{\cos^2\varphi}{2} + \int \frac{d\varphi \cos\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi \cos^8\varphi}{\sin\varphi} = \frac{\cos^7\varphi}{7} + \frac{\cos^5\varphi}{5} + \frac{\cos^3\varphi}{3} + \cos\varphi + \int \frac{d\varphi}{\sin\varphi}$$

TAB. XV.

$$\int \frac{d\varphi \sin^n\varphi}{\cos^2\varphi}, \quad \int \frac{d\varphi \cos^n\varphi}{\sin^2\varphi}.$$

$$\int \frac{d\varphi \sin\varphi}{\cos^2\varphi} = \frac{1}{\cos\varphi} = \sec\varphi$$

$$\int \frac{d\varphi \sin^2\varphi}{\cos^2\varphi} = \frac{\sin\varphi}{\cos\varphi} - \varphi = \operatorname{tang}\varphi - \varphi$$

$$\int \frac{d\varphi \sin^3\varphi}{\cos^2\varphi} = \left(-\sin^2\varphi + 2\right)\frac{1}{\cos\varphi} = \cos\varphi + \sec\varphi$$

$$\int \frac{d\varphi \sin^4\varphi}{\cos^2\varphi} = \left(-\frac{1}{2}\sin^3\varphi + \frac{3}{2}\sin\varphi\right)\frac{1}{\cos\varphi} - \frac{3}{2}\varphi$$

$$\int \frac{d\varphi \sin^5\varphi}{\cos^2\varphi} = \left(-\frac{1}{3}\sin^4\varphi - \frac{4}{3}\sin^2\varphi + \frac{8}{3}\right)\frac{1}{\cos\varphi}$$

$$\int \frac{d\varphi \sin^6\varphi}{\cos^2\varphi} = \left(-\frac{1}{4}\sin^5\varphi - \frac{5}{8}\sin^3\varphi + \frac{15}{8}\sin\varphi\right)\frac{1}{\cos\varphi} - \frac{15}{8}\varphi$$

$$\int \frac{d\varphi \sin^7\varphi}{\cos^2\varphi} = \left(-\frac{1}{5}\sin^6\varphi - \frac{2}{5}\sin^4\varphi - \frac{8}{5}\sin^2\varphi + \frac{16}{5}\right)\frac{1}{\cos\varphi}$$

$$\int \frac{d\varphi \sin^8\varphi}{\cos^2\varphi} = \left(-\frac{1}{6}\sin^7\varphi - \frac{7}{24}\sin^5\varphi - \frac{35}{48}\sin^3\varphi + \frac{35}{16}\sin\varphi\right)\frac{1}{\cos\varphi} - \frac{35}{16}\varphi$$

$$\int \frac{d\varphi \cos\varphi}{\sin^2\varphi} = -\frac{1}{\sin\varphi} = -\operatorname{cosec}\varphi$$

$$\int \frac{d\varphi \cos^2\varphi}{\sin^2\varphi} = -\frac{\cos\varphi}{\sin\varphi} - \varphi = -\cot\varphi - \varphi$$

$$\int \frac{d\varphi \cos^3\varphi}{\sin^2\varphi} = \left(\cos^2\varphi - 2\right)\frac{1}{\sin\varphi} = -\sin\varphi - \operatorname{cosec}\varphi$$

$$\int \frac{d\varphi \cos^4\varphi}{\sin^2\varphi} = \left(\frac{1}{2}\cos^3\varphi - \frac{3}{2}\cos\varphi\right)\frac{1}{\sin\varphi} - \frac{3}{2}\varphi$$

$$\int \frac{d\varphi \cos^5\varphi}{\sin^2\varphi} = \left(\frac{1}{3}\cos^4\varphi + \frac{4}{3}\cos^2\varphi - \frac{8}{3}\right)\frac{1}{\sin\varphi}$$

$$\int \frac{d\varphi \cos^6\varphi}{\sin^2\varphi} = \left(\frac{1}{4}\cos^5\varphi + \frac{5}{8}\cos^3\varphi - \frac{15}{8}\cos\varphi\right)\frac{1}{\sin\varphi} - \frac{15}{8}\varphi$$

$$\int \frac{d\varphi \cos^7\varphi}{\sin^2\varphi} = \left(\frac{1}{5}\cos^6\varphi + \frac{2}{5}\cos^4\varphi + \frac{8}{5}\cos^2\varphi - \frac{16}{5}\right)\frac{1}{\sin\varphi}$$

$$\int \frac{d\varphi \cos^8\varphi}{\sin\varphi} = \left(\frac{1}{6}\cos^7\varphi + \frac{7}{24}\cos^5\varphi + \frac{35}{48}\cos^3\varphi - \frac{36}{16}\cos\varphi\right)\frac{1}{\sin\varphi} - \frac{35}{16}\varphi$$

TAB. XVI.

$$\int \frac{d\varphi \sin^n\varphi}{\cos^3\varphi}, \quad \int \frac{d\varphi \cos^n\varphi}{\sin^3\varphi}$$

$$\int \frac{d\varphi \sin\varphi}{\cos^3\varphi} = \frac{1}{2\cos^2\varphi}$$

$$\int \frac{d\varphi \sin^2\varphi}{\cos^3\varphi} = \frac{\sin\varphi}{2\cos^2\varphi} - \frac{1}{2}\int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi \sin^3\varphi}{\cos^3\varphi} = \frac{1}{2\cos^2\varphi} + \log\cos\varphi$$

$$\int \frac{d\varphi \sin^4\varphi}{\cos^3\varphi} = \left(-\sin^3\varphi + \frac{3}{2}\sin\varphi\right)\frac{1}{\cos^2\varphi} - \frac{3}{2}\int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi \sin^5\varphi}{\cos^3\varphi} = \left(-\frac{1}{2}\sin^4\varphi + 1\right)\frac{1}{\cos^2\varphi} + 2\log\cos\varphi$$

$$\int \frac{d\varphi \sin^6\varphi}{\cos^3\varphi} = \left(-\frac{1}{3}\sin^5\varphi - \frac{5}{3}\sin^3\varphi + \frac{5}{2}\sin\varphi\right)\frac{1}{\cos^2\varphi} - \frac{5}{2}\int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi \sin^7\varphi}{\cos^3\varphi} = \left(-\frac{1}{4}\sin^6\varphi - \frac{3}{4}\sin^4\varphi + \frac{3}{2}\right)\frac{1}{\cos^2\varphi} + 3\log\cos\varphi$$

$$\int \frac{d\varphi \sin^8\varphi}{\cos^3\varphi} = \left(-\frac{1}{5}\sin^7\varphi - \frac{7}{15}\sin^5\varphi - \frac{7}{3}\sin^3\varphi + \frac{7}{2}\sin\varphi\right)\frac{1}{\cos^2\varphi} - \frac{7}{2}\int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi \cos\varphi}{\sin^3\varphi} = -\frac{1}{2\sin^2\varphi}$$

$$\int \frac{d\varphi \cos^2\varphi}{\sin^3\varphi} = -\frac{\cos\varphi}{2\sin^2\varphi} - \frac{1}{2}\int \frac{d\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi \cos^3\varphi}{\sin^3\varphi} = -\frac{1}{2\sin^2\varphi} - \log\sin\varphi$$

$$\int \frac{d\varphi \cos^4\varphi}{\sin^3\varphi} = \left(\cos^3\varphi - \frac{3}{2}\cos\varphi\right)\frac{1}{\sin^2\varphi} - \frac{3}{2}\int \frac{d\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi \cos^5\varphi}{\sin^3\varphi} = \left(\frac{1}{2}\cos^4\varphi - 1\right)\frac{1}{\sin^2\varphi} - 2\log\sin\varphi$$

$$\int \frac{d\varphi \cos^6\varphi}{\sin^3\varphi} = \left(\frac{1}{3}\cos^5\varphi + \frac{5}{3}\cos^3\varphi - \frac{5}{2}\cos\varphi\right)\frac{1}{\sin^2\varphi} - \frac{5}{2}\int \frac{d\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi \cos^7\varphi}{\sin^3\varphi} = \left(\frac{1}{4}\cos^6\varphi + \frac{3}{4}\cos^4\varphi - \frac{3}{2}\right)\frac{1}{\sin^2\varphi} - 3\log\sin\varphi$$

$$\int \frac{d\varphi \cos^8\varphi}{\sin^3\varphi} = \left(\frac{1}{5}\cos^7\varphi + \frac{7}{15}\cos^5\varphi + \frac{7}{3}\cos^3\varphi - \frac{7}{2}\cos\varphi\right)\frac{1}{\sin^2\varphi} - \frac{7}{2}\int \frac{d\varphi}{\sin\varphi}$$

TAB. XVII

$$\int \frac{d\varphi \sin^n\varphi}{\cos^4\varphi}, \quad \int \frac{d\varphi \cos^n\varphi}{\sin^4\varphi}.$$

$$\int \frac{d\varphi \sin\varphi}{\cos^4\varphi} = \frac{1}{3\cos^3\varphi}$$

$$\int \frac{d\varphi \sin^2\varphi}{\cos^4\varphi} = \frac{\sin^3\varphi}{3\cos^3\varphi} = \frac{1}{3}\operatorname{tang}^3\varphi$$

$$\int \frac{d\varphi \sin^3\varphi}{\cos^4\varphi} = \left(\sin^2\varphi - \frac{2}{3}\right)\frac{1}{\cos^3\varphi}$$

$$\int \frac{d\varphi \sin^4\varphi}{\cos^4\varphi} = \left(\frac{4}{3}\sin^3\varphi - \sin\varphi\right)\frac{1}{\cos^3\varphi} + \varphi = \frac{1}{3}\operatorname{tang}^3\varphi - \operatorname{tang}\varphi + \varphi$$

$$\int \frac{d\varphi \sin^5\varphi}{\cos^4\varphi} = \left(-\sin^4\varphi + 4\sin^2\varphi - \frac{8}{3}\right)\frac{1}{\cos^3\varphi}$$

$$\int \frac{d\varphi \sin^6\varphi}{\cos^4\varphi} = \left(-\frac{1}{2}\sin^5\varphi + \frac{10}{3}\sin^3\varphi - \frac{5}{2}\sin\varphi\right)\frac{1}{\cos^3\varphi} + \frac{5}{2}\varphi$$

$$\int \frac{d\varphi \sin^7\varphi}{\cos^4\varphi} = \left(-\frac{1}{3}\sin^6\varphi - 2\sin^4\varphi + 8\sin^2\varphi - \frac{16}{3}\right)\frac{1}{\cos^3\varphi}$$

$$\int \frac{d\varphi \sin^8\varphi}{\cos^4\varphi} = \left(-\frac{1}{4}\sin^7\varphi - \frac{7}{8}\sin^5\varphi + \frac{35}{6}\sin^3\varphi - \frac{35}{8}\sin\varphi\right)\frac{1}{\cos^3\varphi} + \frac{35}{8}\varphi$$

$$\int \frac{d\varphi \cos\varphi}{\sin^4\varphi} = -\frac{1}{3\sin^3\varphi}$$

$$\int \frac{d\varphi \cos^2\varphi}{\sin^4\varphi} = -\frac{\cos^3\varphi}{3\sin^3\varphi} = -\frac{1}{3}\cot^3\varphi$$

$$\int \frac{d\varphi \cos^3\varphi}{\sin^4\varphi} = \left(-\cos^2\varphi + \frac{2}{3}\right)\frac{1}{\sin^3\varphi}$$

$$\int \frac{d\varphi \cos^4\varphi}{\sin^4\varphi} = \left(-\frac{4}{3}\cos^3\varphi + \cos\varphi\right)\frac{1}{\sin^3\varphi} + \varphi = -\frac{1}{3}\cot^3\varphi + \cot\varphi + \varphi$$

$$\int \frac{d\varphi \cos^5\varphi}{\sin^4\varphi} = \left(\cos^4\varphi - 4\cos^2\varphi + \frac{8}{3}\right)\frac{1}{\sin^3\varphi}$$

$$\int \frac{d\varphi \cos^6\varphi}{\sin^4\varphi} = \left(\frac{1}{2}\cos^5\varphi - \frac{10}{3}\cos^3\varphi + \frac{5}{2}\cos\varphi\right)\frac{1}{\sin^3\varphi} + \frac{5}{2}\varphi$$

$$\int \frac{d\varphi \cos^7\varphi}{\sin^4\varphi} = \left(\frac{1}{3}\cos^6\varphi + 2\cos^4\varphi - 8\cos^2\varphi + \frac{16}{3}\right)\frac{1}{\sin^3\varphi}$$

$$\int \frac{d\varphi \cos^8\varphi}{\sin^4\varphi} = \left(\frac{1}{4}\cos^7\varphi + \frac{7}{8}\cos^5\varphi - \frac{35}{6}\cos^3\varphi + \frac{35}{8}\cos\varphi\right)\frac{1}{\sin^3\varphi} + \frac{35}{8}\varphi$$

TAB. XVIII.

$$\int \frac{d\varphi \sin^n\varphi}{\cos^5\varphi}, \quad \int \frac{d\varphi \cos^n\varphi}{\sin^5\varphi}$$

$$\int \frac{d\varphi \sin\varphi}{\cos^5\varphi} = \frac{1}{4\cos^4\varphi}$$

$$\int \frac{d\varphi \sin^2\varphi}{\cos^5\varphi} = \left(\frac{1}{8}\sin^3\varphi + \frac{1}{8}\sin\varphi\right)\frac{1}{\cos.^4\varphi} - \frac{1}{8}\int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi \sin^3\varphi}{\cos^5\varphi} = \frac{\sin^4\varphi}{4\cos^4\varphi} = \frac{1}{4}\operatorname{tang}^4\varphi$$

$$\int \frac{d\varphi \sin^4\varphi}{\cos^5\varphi} = \left(\frac{5}{8}\sin^3\varphi - \frac{3}{8}\sin\varphi\right)\frac{1}{\cos^4\varphi} + \frac{3}{8}\int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi \sin^5\varphi}{\cos^5\varphi} = \left(\frac{3}{4}\sin^4\varphi - \frac{1}{2}\sin^2\varphi\right)\frac{1}{\cos^4\varphi} - \log\cos\varphi$$

$$= \frac{1}{4}\operatorname{tang}^4\varphi - \frac{1}{2}\operatorname{tang}^2\varphi - \log\cos\varphi$$

$$\int \frac{d\varphi \sin^6\varphi}{\cos^5\varphi} = \left(-\sin^5\varphi + \frac{25}{8}\sin^3\varphi - \frac{15}{8}\sin\varphi\right)\frac{1}{\cos^4\varphi} + \frac{15}{8}\int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi \sin^7\varphi}{\cos^5\varphi} = \left(-\frac{1}{2}\sin^6\varphi + \frac{9}{4}\sin^4\varphi - \frac{3}{2}\sin^2\varphi\right)\frac{1}{\cos^4\varphi} - 3\log\cos\varphi$$

$$\int \frac{d\varphi \cos\varphi}{\sin^5\varphi} = -\frac{1}{4\sin^4\varphi}$$

$$\int \frac{d\varphi \cos^2\varphi}{\sin^5\varphi} = \left(-\frac{1}{8}\cos^3\varphi - \frac{1}{8}\cos\varphi\right)\frac{1}{\sin^4\varphi} - \frac{1}{8}\int \frac{d\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi \cos^3\varphi}{\sin^5\varphi} = -\frac{\cos^4\varphi}{4\sin^4\varphi} = -\frac{1}{4}\cot^4\varphi$$

$$\int \frac{d\varphi \cos^4\varphi}{\sin^5\varphi} = \left(-\frac{5}{8}\cos^3\varphi + \frac{3}{8}\cos\varphi\right)\frac{1}{\sin^4\varphi} + \frac{3}{8}\int \frac{d\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi \cos^5\varphi}{\sin^5\varphi} = \left(-\frac{3}{4}\cos^4\varphi + \frac{1}{2}\cos^2\varphi\right)\frac{1}{\sin^4\varphi} + \log\sin\varphi$$

$$= -\frac{1}{4}\cot^4\varphi + \frac{1}{2}\cot^2\varphi + \log\sin\varphi$$

$$\int \frac{d\varphi \cos^6\varphi}{\sin^5\varphi} = \left(\cos^5\varphi - \frac{25}{8}\cos^3\varphi + \frac{15}{8}\cos\varphi\right)\frac{1}{\sin^4\varphi} + \frac{15}{8}\int \frac{d\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi \cos^7\varphi}{\sin^5\varphi} = \left(\frac{1}{2}\cos^6\varphi - \frac{9}{4}\cos^4\varphi + \frac{3}{2}\cos^2\varphi\right)\frac{1}{\sin^4\varphi} + 3\log\sin\varphi$$

TAB. XIX.

$$\int \frac{d\varphi \sin^n\varphi}{\cos^6\varphi}, \int \frac{d\varphi \cos^n\varphi}{\sin^6\varphi}$$

$$\int \frac{d\varphi \sin\varphi}{\cos^6\varphi} = \frac{1}{5\cos^5\varphi}$$

$$\int \frac{d\varphi \sin^2\varphi}{\cos^6\varphi} = \left(-\frac{2}{15}\sin^5\varphi + \frac{1}{3}\sin^3\varphi\right)\frac{1}{\cos^5\varphi}$$

$$\int \frac{d\varphi \sin^3\varphi}{\cos^6\varphi} = \left(\frac{1}{3}\sin^2\varphi - \frac{2}{15}\right)\frac{1}{\cos^5\varphi}$$

$$\int \frac{d\varphi \sin^4\varphi}{\cos^6\varphi} = \frac{1}{5}\operatorname{tang}^5\varphi$$

$$\int \frac{d\varphi \sin^5\varphi}{\cos^6\varphi} = \left(\sin^4\varphi - \frac{4}{3}\sin^2\varphi + \frac{8}{15}\right)\frac{1}{\cos^5\varphi}$$

$$\int \frac{d\varphi \sin^6\varphi}{\cos^6\varphi} = \frac{1}{5}\operatorname{tang}^5\varphi - \frac{1}{3}\operatorname{tang}^3\varphi + \operatorname{tang}\varphi - \varphi$$

$$\int \frac{d\varphi \sin^7\varphi}{\cos^6\varphi} = \left(-\sin^6\varphi + 6\sin^4\varphi - 8\sin^2\varphi + \frac{16}{5}\right)\frac{1}{\cos^5\varphi}$$

$$\int \frac{d\varphi \sin^8\varphi}{\cos^6\varphi} = -\frac{\sin^7\varphi}{2\cos^5\varphi} + \frac{7}{2}\int \frac{d\varphi \sin^6\varphi}{\cos^6\varphi}$$

$$\int \frac{d\varphi \cos\varphi}{\sin^6\varphi} = -\frac{1}{5\sin^5\varphi}$$

$$\int \frac{d\varphi \cos^2\varphi}{\sin^6\varphi} = \left(\frac{2}{15}\cos^5\varphi - \frac{1}{3}\cos^3\varphi\right)\frac{1}{\sin^5\varphi}$$

$$\int \frac{d\varphi \cos^3\varphi}{\sin^6\varphi} = \left(-\frac{1}{3}\cos^2\varphi + \frac{2}{15}\right)\frac{1}{\sin^5\varphi}$$

$$\int \frac{d\varphi \cos^4\varphi}{\sin^6\varphi} = -\frac{1}{5}\cot^5\varphi$$

$$\int \frac{d\varphi \cos^5\varphi}{\sin^6\varphi} = \left(-\cos^4\varphi + \frac{4}{3}\cos^2\varphi - \frac{8}{15}\right)\frac{1}{\sin^5\varphi}$$

$$\int \frac{d\varphi \cos^6\varphi}{\sin^6\varphi} = -\frac{1}{5}\cot^5\varphi + \frac{1}{3}\cot^3\varphi - \cot\varphi - \varphi$$

$$\int \frac{d\varphi \cos^7\varphi}{\sin^6\varphi} = \left(\cos^6\varphi - 6\cos^4\varphi + 8\cos^2\varphi - \frac{16}{5}\right)\frac{1}{\sin^5\varphi}$$

$$\int \frac{d\varphi \cos^8\varphi}{\sin^6\varphi} = \frac{\cos^7\varphi}{2\sin^5\varphi} + \frac{7}{2}\int \frac{d\varphi \cos^6\varphi}{\sin^6\varphi}$$

TAB. XX.

$$\int \frac{d\varphi \sin^m\varphi}{\cos^7\varphi}, \quad \int \frac{d\varphi \cos^m\varphi}{\sin^7\varphi}$$

$$\int \frac{d\varphi \sin\varphi}{\cos^7\varphi} = \frac{1}{6\cos^6\varphi}$$

$$\int \frac{d\varphi \sin^2\varphi}{\cos^7\varphi} = \left(-\frac{1}{16}\sin^5\varphi + \frac{1}{6}\sin^3\varphi + \frac{1}{16}\sin\varphi\right)\frac{1}{\cos^6\varphi} - \frac{1}{16}\int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi \sin^3\varphi}{\cos^7\varphi} = \left(\frac{1}{4}\sin^2\varphi - \frac{1}{12}\right)\frac{1}{\cos^6\varphi}$$

$$\int \frac{d\varphi \sin^4\varphi}{\cos^7\varphi} = \left(\frac{1}{16}\sin^5\varphi + \frac{1}{6}\sin^3\varphi - \frac{1}{16}\sin\varphi\right)\frac{1}{\cos^6\varphi} + \frac{1}{16}\int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi \sin^5\varphi}{\cos^7\varphi} = \frac{1}{6}\operatorname{tang}^6\varphi$$

$$\int \frac{d\varphi \sin^6\varphi}{\cos^7\varphi} = \left(\frac{11}{16}\sin^5\varphi - \frac{5}{6}\sin^3\varphi + \frac{5}{16}\sin\varphi\right)\frac{1}{\cos^6\varphi} - \frac{5}{16}\int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi \sin^7\varphi}{\cos^7\varphi} = \frac{1}{6}\operatorname{tang}^6\varphi - \frac{1}{4}\operatorname{tang}^4\varphi + \frac{1}{2}\operatorname{tang}^2\varphi + \log\cos\varphi$$

$$\int \frac{d\varphi \sin^8\varphi}{\cos^7\varphi} = \left(-\sin^7\varphi + \frac{77}{16}\sin^5\varphi - \frac{35}{6}\sin^3\varphi + \frac{36}{16}\sin\varphi\right)\frac{1}{\cos^6\varphi} - \frac{35}{16}\int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi \cos\varphi}{\sin^7\varphi} = -\frac{1}{6\sin^6\varphi}$$

$$\int \frac{d\varphi \cos^2\varphi}{\sin^7\varphi} = \left(\frac{1}{16}\cos^5\varphi - \frac{1}{6}\cos^3\varphi - \frac{1}{16}\cos\varphi\right)\frac{1}{\sin^6\varphi} - \frac{1}{16}\int \frac{d\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi \cos^3\varphi}{\sin^7\varphi} = \left(-\frac{1}{4}\cos^2\varphi + \frac{1}{12}\right)\frac{1}{\sin^6\varphi}$$

$$\int \frac{d\varphi \cos^4\varphi}{\sin^7\varphi} = \left(-\frac{1}{16}\cos^5\varphi - \frac{1}{6}\cos^3\varphi + \frac{1}{16}\cos\varphi\right)\frac{1}{\sin^6\varphi} + \frac{1}{16}\int \frac{d\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi \cos^5\varphi}{\sin^7\varphi} = -\frac{1}{6}\cot^6\varphi$$

$$\int \frac{d\varphi \cos^6\varphi}{\sin^7\varphi} = \left(-\frac{11}{16}\cos^5\varphi + \frac{5}{6}\cos^3\varphi - \frac{5}{16}\cos\varphi\right)\frac{1}{\sin^6\varphi} - \frac{5}{16}\int \frac{d\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi \cos^7\varphi}{\sin^7\varphi} = -\frac{1}{6}\cot^6\varphi + \frac{1}{4}\cot^4\varphi - \frac{1}{2}\cot^2\varphi - \log\sin\varphi$$

$$\int \frac{d\varphi \cos^8\varphi}{\sin^7\varphi} = \left(\cos^7\varphi - \frac{77}{16}\cos^5\varphi + \frac{35}{6}\cos^3\varphi - \frac{35}{16}\cos\varphi\right)\frac{1}{\sin^6\varphi} - \frac{35}{16}\int \frac{d\varphi}{\sin\varphi}$$

TAB. XXI.

$$\int \frac{d\varphi \sin^n\varphi}{\cos^8\varphi}, \quad \int \frac{d\varphi \cos^n\varphi}{\sin^8\varphi}$$

$$\int \frac{d\varphi \sin\varphi}{\cos^8\varphi} = \frac{1}{7\cos^7\varphi}$$

$$\int \frac{d\varphi \sin^2\varphi}{\cos^8\varphi} = \left(\frac{8}{105}\sin^7\varphi - \frac{4}{15}\sin^5\varphi + \frac{1}{3}\sin^3\varphi\right)\frac{1}{\cos^7\varphi}$$

$$\int \frac{d\varphi \sin^3\varphi}{\cos^8\varphi} = \left(\frac{1}{5}\sin\varphi - \frac{2}{35}\right)\frac{1}{\cos^7\varphi}$$

$$\int \frac{d\varphi \sin^4\varphi}{\cos^8\varphi} = \left(-\frac{2}{35}\sin^7\varphi + \frac{1}{5}\sin^5\varphi\right)\frac{1}{\cos^7\varphi}$$

$$\int \frac{d\varphi \sin^5\varphi}{\cos^8\varphi} = \left(\frac{1}{3}\sin^4\varphi - \frac{4}{15}\sin^2\varphi + \frac{8}{105}\right)\frac{1}{\cos^7\varphi}$$

$$\int \frac{d\varphi \sin^6\varphi}{\cos^8\varphi} = \frac{1}{7}\operatorname{tang}^7\varphi$$

$$\int \frac{d\varphi \sin^7\varphi}{\cos^8\varphi} = \left(\sin^6\varphi - 2\sin^4\varphi + \frac{8}{5}\sin^2\varphi - \frac{16}{35}\right)\frac{1}{\cos^7\varphi}$$

$$\int \frac{d\varphi \sin^8\varphi}{\cos^8\varphi} = \frac{1}{7}\operatorname{tang}^7\varphi - \frac{1}{5}\operatorname{tang}^5\varphi + \frac{1}{3}\operatorname{tang}^3\varphi - \operatorname{tang}\varphi + \varphi$$

$$\int \frac{d\varphi \cos\varphi}{\sin^8\varphi} = -\frac{1}{7\sin^7\varphi}$$

$$\int \frac{d\varphi \cos^2\varphi}{\sin^8\varphi} = \left(-\frac{8}{105}\cos^7\varphi + \frac{4}{15}\cos^5\varphi - \frac{1}{3}\cos^3\varphi\right)\frac{1}{\sin^7\varphi}$$

$$\int \frac{d\varphi \cos^3\varphi}{\sin^8\varphi} = \left(-\frac{1}{5}\cos^2\varphi + \frac{2}{35}\right)\frac{1}{\sin^7\varphi}$$

$$\int \frac{d\varphi \cos^4\varphi}{\sin^8\varphi} = \left(\frac{2}{35}\cos^7\varphi - \frac{1}{5}\cos^5\varphi\right)\frac{1}{\sin^7\varphi}$$

$$\int \frac{d\varphi \cos^5\varphi}{\sin^8\varphi} = \left(-\frac{1}{3}\cos^4\varphi + \frac{4}{15}\cos^2\varphi - \frac{8}{105}\right)\frac{1}{\sin^7\varphi}$$

$$\int \frac{d\varphi \cos^6\varphi}{\sin^8\varphi} = -\frac{1}{7}\cot^7\varphi$$

$$\int \frac{d\varphi \cos^7\varphi}{\sin^8\varphi} = \left(-\cos^6\varphi + 2\cos^4\varphi - \frac{8}{5}\cos^2\varphi + \frac{16}{35}\right)\frac{1}{\sin^7\varphi}$$

$$\int \frac{d\varphi \cos^8\varphi}{\sin^8\varphi} = -\frac{1}{7}\cot^7\varphi + \frac{1}{5}\cot^5\varphi - \frac{1}{3}\cot^3\varphi + \cot\varphi + \varphi$$

TAB. XXII.

$$\int \frac{d\varphi}{\sin\varphi \cos^n\varphi},\quad \int \frac{d\varphi}{\sin^2\varphi \cos^n\varphi}$$

$$\int \frac{d\varphi}{\sin\varphi \cos\varphi} = \log \operatorname{tang} \varphi$$

$$\int \frac{d\varphi}{\sin\varphi \cos^2\varphi} = \frac{1}{\cos\varphi} + \int \frac{d\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi}{\sin\varphi \cos^3\varphi} = \frac{1}{2\cos^2\varphi} + \log \operatorname{tang} \varphi$$

$$\int \frac{d\varphi}{\sin\varphi \cos^4\varphi} = \frac{1}{3\cos^3\varphi} + \frac{1}{\cos\varphi} + \int \frac{d\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi}{\sin\varphi \cos^5\varphi} = \frac{1}{4\cos^4\varphi} + \frac{1}{2\cos^2\varphi} + \log \operatorname{tang} \varphi$$

$$\int \frac{d\varphi}{\sin\varphi \cos^6\varphi} = \frac{1}{5\cos^5\varphi} + \frac{1}{3\cos^3\varphi} + \frac{1}{\cos\varphi} + \int \frac{d\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi}{\sin\varphi \cos^7\varphi} = \frac{1}{6\cos^6\varphi} + \frac{1}{4\cos^4\varphi} + \frac{1}{2\cos^2\varphi} + \log \operatorname{tang} \varphi$$

$$\int \frac{d\varphi}{\sin\varphi \cos^8\varphi} = \frac{1}{7\cos^7\varphi} + \frac{1}{5\cos^5\varphi} + \frac{1}{3\cos^3\varphi} + \frac{1}{\cos\varphi} + \int \frac{d\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi}{\sin^2\varphi \cos\varphi} = -\frac{1}{\sin\varphi} + \int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi}{\sin^2\varphi \cos^2\varphi} = -2 \cot 2\varphi$$

$$\int \frac{d\varphi}{\sin^2\varphi \cos^3\varphi} = \left(\frac{1}{2\cos^2\varphi} - \frac{3}{2}\right)\frac{1}{\sin\varphi} + \frac{3}{2}\int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi}{\sin^2\varphi \cos^4\varphi} = \frac{1}{3\sin\varphi\cos^3\varphi} - \frac{8}{3}\cot 2\varphi$$

$$\int \frac{d\varphi}{\sin^2\varphi \cos^5\varphi} = \left(\frac{1}{4\cos^4\varphi} + \frac{5}{8\cos^2\varphi} - \frac{15}{8}\right)\frac{1}{\sin\varphi} + \frac{15}{8}\int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi}{\sin^2\varphi \cos^6\varphi} = \left(\frac{1}{5\cos^5\varphi} + \frac{2}{5\cos^3\varphi}\right)\frac{1}{\sin\varphi} - \frac{16}{5}\cot 2\varphi$$

$$\int \frac{d\varphi}{\sin^2\varphi \cos^7\varphi} = \left(\frac{1}{6\cos^6\varphi} + \frac{7}{24\cos^4\varphi} + \frac{35}{48\cos^2\varphi} - \frac{35}{16}\right)\frac{1}{\sin\varphi} + \frac{35}{16}\int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi}{\sin^2\varphi \cos^8\varphi} = \left(\frac{1}{7\cos^7\varphi} + \frac{8}{35\cos^5\varphi} + \frac{16}{35\cos^3\varphi}\right)\frac{1}{\sin\varphi} - \frac{128}{35}\cot 2\varphi$$

TAB. XXIII.

$$\int \frac{d\varphi}{\sin^3\varphi \cos^n\varphi}, \quad \int \frac{d\varphi}{\sin^4\varphi \cos^n\varphi}$$

$$\int \frac{d\varphi}{\sin^3\varphi \cos\varphi} = -\frac{1}{2\sin^2\varphi} + \log \operatorname{tang}\varphi$$

$$\int \frac{d\varphi}{\sin^3\varphi \cos^2\varphi} = \frac{1}{\sin^2\varphi \cos\varphi} + 3\int \frac{d\varphi}{\sin^3\varphi}$$

$$\int \frac{d\varphi}{\sin^3\varphi \cos^3\varphi} = -\frac{2\cos 2\varphi}{\sin^2 2\varphi} + 2\log \operatorname{tang}\varphi$$

$$\int \frac{d\varphi}{\sin^3\varphi \cos^4\varphi} = \left(\frac{1}{3\cos^3\varphi} + \frac{5}{3\cos\varphi}\right)\frac{1}{\sin^2\varphi} + 5\int \frac{d\varphi}{\sin^3\varphi}$$

$$\int \frac{d\varphi}{\sin^3\varphi \cos^5\varphi} = \frac{1}{4\sin^2\varphi \cos^4\varphi} + \frac{3}{2}\int \frac{d\varphi}{\sin^3\varphi \cos^3\varphi}$$

$$\int \frac{d\varphi}{\sin^3\varphi \cos^6\varphi} = \left(\frac{1}{5\cos^5\varphi} + \frac{7}{15\cos^3\varphi} + \frac{7}{3\cos\varphi}\right)\frac{1}{\sin^2\varphi} + 7\int \frac{d\varphi}{\sin^3\varphi}$$

$$\int \frac{d\varphi}{\sin^3\varphi \cos^7\varphi} = \left(\frac{1}{6\cos^6\varphi} + \frac{1}{3\cos^4\varphi}\right)\frac{1}{\sin^2\varphi} + 2\int \frac{d\varphi}{\sin^3\varphi \cos^3\varphi}$$

$$\int \frac{d\varphi}{\sin^3\varphi \cos^8\varphi} = \left(\frac{1}{7\cos^7\varphi} + \frac{9}{35\cos^5\varphi} + \frac{3}{5\cos^3\varphi} + \frac{3}{\cos\varphi}\right)\frac{1}{\sin^2\varphi} + 9\int \frac{d\varphi}{\sin^3\varphi}$$

$$\int \frac{d\varphi}{\sin^4\varphi \cos\varphi} = -\frac{1}{3\sin^3\varphi} - \frac{1}{\sin\varphi} + \int \frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi}{\sin^4\varphi \cos^2\varphi} = -\frac{1}{3\cos\varphi \sin^3\varphi} - \frac{8}{3}\cot 2\varphi$$

$$\int \frac{d\varphi}{\sin^4\varphi \cos^3\varphi} = \frac{1}{2\cos^2\varphi \sin^3\varphi} + \frac{5}{2}\int \frac{d\varphi}{\sin^4\varphi \cos\varphi}$$

$$\int \frac{d\varphi}{\sin^4\varphi \cos^4\varphi} = \left(-\frac{8}{3\sin^3 2\varphi} - \frac{16}{3\sin 2\varphi}\right)\cos 2\varphi$$

$$\int \frac{d\varphi}{\sin^4\varphi \cos^5\varphi} = \left(\frac{1}{4\cos^4\varphi} + \frac{7}{8\cos^2\varphi}\right)\frac{1}{\sin^3\varphi} + \frac{35}{8}\int \frac{d\varphi}{\sin^4\varphi \cos\varphi}$$

$$\int \frac{d\varphi}{\sin^4\varphi \cos^6\varphi} = \frac{1}{5\cos^5\varphi \sin^3\varphi} + \frac{8}{5}\int \frac{d\varphi}{\sin^4\varphi \cos^4\varphi}$$

$$\int \frac{d\varphi}{\sin^4\varphi \cos^7\varphi} = \left(\frac{1}{6\cos^6\varphi} + \frac{3}{8\cos^4\varphi} + \frac{21}{16\cos^2\varphi}\right)\frac{1}{\sin^3\varphi} + \frac{105}{16}\int \frac{d\varphi}{\sin^4\varphi \cos\varphi}$$

$$\int \frac{d\varphi}{\sin^4\varphi \cos^8\varphi} = \left(\frac{1}{7\cos^7\varphi} + \frac{2}{7\cos^5\varphi}\right)\frac{1}{\sin^3\varphi} + \frac{16}{7}\int \frac{d\varphi}{\sin^4\varphi \cos^4\varphi}$$

TAB. XXIV.

$$\int \frac{d\varphi}{\sin^5\varphi \cos^n\varphi}, \int \frac{d\varphi}{\sin^6\varphi \cos^n\varphi}$$

$$\int \frac{d\varphi}{\sin^5\varphi \cos\varphi} = -\frac{1}{4\sin^4\varphi} - \frac{1}{2\sin^2\varphi} + \log \operatorname{tang}\varphi$$

$$\int \frac{d\varphi}{\sin^5\varphi \cos^2\varphi} = \left(-\frac{1}{4\sin^4\varphi} - \frac{5}{8\sin^2\varphi} + \frac{15}{8}\right)\frac{1}{\cos\varphi} + \frac{15}{8}\int\frac{d\varphi}{\sin\varphi}$$

$$\int \frac{d\varphi}{\sin^5\varphi \cos^3\varphi} = -\frac{1}{4\cos^2\varphi\sin^4\varphi} + \frac{3}{2}\int\frac{d\varphi}{\sin^3\varphi\cos^3\varphi}$$

$$\int \frac{d\varphi}{\sin^5\varphi \cos^4\varphi} = -\frac{1}{3\sin^4\varphi\cos^3\varphi} + \frac{7}{3}\int\frac{d\varphi}{\sin^5\varphi\cos^2\varphi}$$

$$\int \frac{d\varphi}{\sin^5\varphi \cos^5\varphi} = \left(-\frac{4}{\sin^4 2\varphi} - \frac{6}{\sin^2 2\varphi}\right)\cos 2\varphi + 6\log \operatorname{tang}\varphi$$

$$\int \frac{d\varphi}{\sin^5\varphi \cos^6\varphi} = \left(\frac{1}{5\cos^5\varphi} + \frac{3}{5\cos^3\varphi}\right)\frac{1}{\sin^4\varphi} + \frac{21}{5}\int\frac{d\varphi}{\sin^5\varphi\cos^2\varphi}$$

$$\int \frac{d\varphi}{\sin^5\varphi \cos^7\varphi} = \frac{1}{6\sin^4\varphi\cos^6\varphi} + \frac{5}{3}\int\frac{d\varphi}{\sin^5\varphi\cos^5\varphi}$$

$$\int \frac{d\varphi}{\sin^5\varphi \cos^8\varphi} = \left(\frac{1}{7\cos^7\varphi} + \frac{11}{35\cos^5\varphi} + \frac{33}{35\cos^3\varphi}\right)\frac{1}{\sin^4\varphi} + \frac{33}{5}\int\frac{d\varphi}{\sin^5\varphi\cos^2\varphi}$$

$$\int \frac{d\varphi}{\sin^6\varphi \cos\varphi} = -\frac{1}{5\sin^5\varphi} - \frac{1}{3\sin^3\varphi} - \frac{1}{\sin\varphi} + \int\frac{d\varphi}{\cos\varphi}$$

$$\int \frac{d\varphi}{\sin^6\varphi \cos^2\varphi} = \left(-\frac{1}{5\sin^5\varphi} - \frac{2}{5\sin^3\varphi}\right)\frac{1}{\cos\varphi} - \frac{16}{5}\cot 2\varphi$$

$$\int \frac{d\varphi}{\sin^6\varphi \cos^3\varphi} = \left(-\frac{1}{5\sin^5\varphi} - \frac{7}{15\sin^3\varphi} - \frac{7}{3\sin\varphi}\right)\frac{1}{\cos^2\varphi} + 7\int\frac{d\varphi}{\cos^3\varphi}$$

$$\int \frac{d\varphi}{\sin^6\varphi \cos^4\varphi} = -\frac{1}{5\sin^5\varphi\cos^3\varphi} + \frac{8}{5}\int\frac{d\varphi}{\sin^4\varphi\cos^4\varphi}$$

$$\int \frac{d\varphi}{\sin^6\varphi \cos^5\varphi} = \left(-\frac{1}{5\sin^5\varphi} - \frac{3}{5\sin^3\varphi}\right)\frac{1}{\cos^4\varphi} + \frac{21}{5}\int\frac{d\varphi}{\sin^2\varphi\cos^5\varphi}$$

$$\int \frac{d\varphi}{\sin^6\varphi \cos^6\varphi} = \left(-\frac{32}{5\sin^5 2\varphi} - \frac{128}{15\sin^3 2\varphi} - \frac{256}{15\sin 2\varphi}\right)\cos 2\varphi$$

$$\int \frac{d\varphi}{\sin^6\varphi \cos^7\varphi} = \frac{1}{6\sin^5\varphi\cos^6\varphi} - \left(\frac{11}{30\sin^5\varphi} + \frac{11}{10\sin^3\varphi}\right)\frac{1}{\cos^4\varphi} - \frac{77}{10}\int\frac{d\varphi}{\sin^2\varphi\cos^5\varphi}$$

TAB. XXV.

$$\int\frac{d\varphi}{\sin^7\varphi\cos^n\varphi},\ \int\frac{d\varphi}{\sin^8\varphi\cos^n\varphi}$$

$$\int\frac{d\varphi}{\sin^7\varphi\cos\varphi}=-\frac{1}{6\sin^6\varphi}-\frac{1}{4\sin^4\varphi}-\frac{1}{2\sin^2\varphi}+\log\operatorname{tang}\varphi$$

$$\int\frac{d\varphi}{\sin^7\varphi\cos^2\varphi}=\left(-\frac{1}{6\sin^6\varphi}-\frac{7}{24\sin^4\varphi}-\frac{35}{48\sin^2\varphi}+\frac{35}{16}\right)\frac{1}{\cos\varphi}+\frac{35}{16}\int\frac{d\varphi}{\sin\varphi}$$

$$\int\frac{d\varphi}{\sin^7\varphi\cos^3\varphi}=\left(-\frac{6}{6\sin^6\varphi}-\frac{1}{3\sin^4\varphi}\right)\frac{1}{\cos^2\varphi}+2\int\frac{d\varphi}{\sin^3\varphi\cos^3\varphi}$$

$$\int\frac{d\varphi}{\sin^7\varphi\cos^4\varphi}=\left(-\frac{1}{6\sin^6\varphi}-\frac{3}{8\sin^4\varphi}-\frac{21}{16\sin^2\varphi}\right)\frac{1}{\cos^3\varphi}+\frac{105}{16}\int\frac{d\varphi}{\sin\varphi\cos^4\varphi}$$

$$\int\frac{d\varphi}{\sin^7\varphi\cos^5\varphi}=-\frac{1}{6\cos^4\varphi\sin^6\varphi}+\frac{5}{3}\int\frac{d\varphi}{\sin^5\varphi\cos^5\varphi}$$

$$\int\frac{d\varphi}{\sin^7\varphi\cos^6\varphi}=-\frac{1}{6\cos^5\varphi\sin^6\varphi}+\left(\frac{11}{30\cos^5\varphi}+\frac{11}{10\cos^3\varphi}\right)\frac{1}{\sin^4\varphi}$$
$$+\frac{77}{10}\int\frac{d\varphi}{\sin^5\varphi\cos^2\varphi}$$

$$\int\frac{d\varphi}{\sin^7\varphi\cos^7\varphi}=\left(-\frac{32}{3\sin^6 2\varphi}-\frac{40}{3\sin^4 2\varphi}-\frac{20}{\sin^2 2\varphi}\right)\cos 2\varphi+20\log\operatorname{tang}\varphi$$

$$\int\frac{d\varphi}{\sin^8\varphi\cos\varphi}=-\frac{1}{7\sin^7\varphi}-\frac{1}{5\sin^5\varphi}-\frac{3}{3\sin^3\varphi}-\frac{1}{\sin\varphi}+\int\frac{d\varphi}{\cos\varphi}$$

$$\int\frac{d\varphi}{\sin^8\varphi\cos^2\varphi}=\left(-\frac{1}{7\sin^7\varphi}-\frac{8}{35\sin^5\varphi}-\frac{16}{35\sin^3\varphi}\right)\frac{1}{\cos\varphi}-\frac{128}{35}\cot 2\varphi$$

$$\int\frac{d\varphi}{\sin^8\varphi\cos^3\varphi}=\left(-\frac{1}{7\sin^7\varphi}-\frac{9}{35\sin^5\varphi}-\frac{3}{5\sin^3\varphi}-\frac{3}{\sin\varphi}\right)\frac{1}{\cos^2\varphi}+9\int\frac{d\varphi}{\cos^3\varphi}$$

$$\int\frac{d\varphi}{\sin^8\varphi\cos^4\varphi}=\left(-\frac{7}{7\sin^7\varphi}-\frac{2}{7\sin^5\varphi}\right)\frac{1}{\cos^3\varphi}+\frac{16}{7}\int\frac{d\varphi}{\sin^4\varphi\cos^4\varphi}$$

$$\int\frac{d\varphi}{\sin^8\varphi\cos^5\varphi}=\left(-\frac{1}{7\sin^7\varphi}-\frac{11}{35\sin^5\varphi}-\frac{33}{35\sin^3\varphi}\right)\frac{1}{\cos^4\varphi}$$
$$+\frac{33}{5}\int\frac{d\varphi}{\sin^2\varphi\cos^5\varphi}$$

$$\int\frac{d\varphi}{\sin^8\varphi\cos^6\varphi}=-\frac{1}{7\sin^7\varphi\cos^5\varphi}+\frac{12}{7}\int\frac{d\varphi}{\sin^6\varphi\cos^6\varphi}$$

$$\int\frac{d\varphi}{\sin^8\varphi\cos^7\varphi}=-\frac{1}{7\sin^7\varphi\cos^6\varphi}+\frac{13}{6}\int\frac{d\varphi}{\sin^6\varphi\cos^7\varphi}$$

Notes on the preceding Tables.

1. The formulæ from page 238 to page 263 for the integral $\int d\varphi \sin^m \varphi \cos^n \varphi$, may also be applied to the integral $\int d\varphi . \sin.^m (k\varphi+l) . \cos.^n (k\varphi+l)$, k and l being constants. In those formulæ we have only to substitute $k\varphi+l$ for φ, and to multiply the whole by $\frac{1}{k}$. Thus we find

$$\int dx \cos(k\varphi+l) = \frac{1}{k} \sin(k\varphi+l)$$

$$\int d\varphi \sin(k\varphi+l) = -\frac{1}{k} \cos(k\varphi+l)$$

$$\int d\varphi \cos(k\varphi+l) \sin^n(k\varphi+l) = \frac{\sin^{n+1}(k\varphi+l)}{k(n+1)}$$

$$\int d\varphi \sin(k\varphi+l) \cos^n(k\varphi+l) = -\frac{\cos^{n+1}(k\varphi+l)}{k(n+1)}$$

$$\int \frac{d\varphi}{\sin^3(k\varphi+l)\cos^2(k\varphi+l)} = \frac{1}{k\sin^2(k\varphi+l)\cos(k\varphi+l)}$$
$$-\frac{3\cos(k\varphi+l)}{2k\sin^2(k\varphi+l)} + \frac{3}{2k} \log \text{tang} \tfrac{1}{2}(k\varphi+l).$$

2. Differentials of the forms $d\varphi \text{ tang}^m \varphi$, $d\varphi \sec^m \varphi \cot^n \varphi$, $d\varphi \sec^m \varphi \text{ tang}^n \varphi \text{ cosec}^p \varphi$, &c., may be reduced to the forms $d\varphi \sin^m \varphi \cos^n \varphi$, by substituting for tang φ, cot φ, sec φ, cosec φ, their values $\frac{\sin\varphi}{\cos\varphi}$, $\frac{\cos\varphi}{\sin\varphi}$, $\frac{1}{\cos\varphi}$, $\frac{1}{\sin\varphi}$

3. The following formulæ being of frequent use, are worthy of remark,

$$\int d\varphi \sin(k\varphi+l)\cos(k'\varphi+l') = -\frac{\cos[(k+k')\varphi+l+l']}{2(k+k')}$$
$$-\frac{\cos[(k-k')\varphi+l-l']}{2(k-k')}$$

$$\int d\varphi \sin(k\varphi+l)\sin(k'\varphi+l') = \frac{\sin[(k-k')\varphi+l-l']}{2(k-k')}$$
$$-\frac{\sin[(k+k')\varphi+l+l']}{2(k+k')}$$

$$\int d\varphi \cos(k\varphi+l)\cos(k'\varphi+l') = \frac{\sin[(k+k')\varphi+l+l']}{2(k+k')}$$
$$+\frac{\sin[(k-k')\varphi+l-l']}{2(k-k')}$$

TAB. XXVI.

$$\int \varphi^n d\varphi \sin\varphi$$

General Formulæ

$$\int \varphi^n d\varphi \sin\varphi = -\varphi^n \cos\varphi + n\varphi^{n-1}\sin\varphi + n(n-1)\varphi^{n-2}\cos\varphi$$
$$-n(n-1)(n-2)\varphi^{n-3}\sin\varphi$$
$$-n(n-1)(n-2)(n-3)\varphi^{n-4}\cos\varphi + + - -$$

Particular Cases.

$$\int \varphi\, d\varphi \sin\varphi = -\varphi\cos\varphi + \sin\varphi$$

$$\int \varphi^2 d\varphi \sin\varphi = -\varphi^2\cos\varphi + 2\varphi\sin\varphi + 2\cos\varphi$$

$$\int \varphi^3 d\varphi \sin\varphi = -\varphi^3\cos\varphi + 3\varphi^2\sin\varphi + 6\varphi\cos\varphi - 6\sin\varphi$$

$$\int \varphi^4 d\varphi \sin\varphi = -\varphi^4\cos\varphi + 4\varphi^3\sin\varphi + 12\varphi^2\cos\varphi - 24\varphi\sin\varphi$$
$$-24\cos\varphi$$

$$\int \varphi^5 d\varphi \sin\varphi = -\varphi^5\cos\varphi + 5\varphi^4\sin\varphi + 20\varphi^3\cos\varphi - 60\varphi^2\sin\varphi$$
$$-120\varphi\cos\varphi + 120\sin\varphi$$

$$\int \varphi^n d\varphi \cos\varphi$$

General Formulæ.

$$\int \varphi^n d\varphi \cos\varphi = \varphi^n\sin\varphi + n\varphi^{n-1}\cos\varphi - n(n-1)\varphi^{n-2}\sin\varphi$$
$$-n(n-1)(n-2)\varphi^{n-3}\cos\varphi + + - -$$

Particular Cases.

$$\int \varphi\, d\varphi \cos\varphi = \varphi\sin\varphi + \cos\varphi$$

$$\int \varphi^2 d\varphi \cos\varphi = \varphi^2\sin\varphi + 2\varphi\cos\varphi - 2\sin\varphi$$

$$\int \varphi^3 d\varphi \cos\varphi = \varphi^3\sin\varphi + 3\varphi^2\cos\varphi - 6\varphi\sin\varphi - 6\cos\varphi$$

$$\int \varphi^4 d\varphi \cos\varphi = \varphi^4\sin\varphi + 4\varphi^3\cos\varphi - 12\varphi^2\sin\varphi - 24\varphi\cos\varphi$$
$$+24\sin\varphi$$

$$\int \varphi^5 d\varphi \cos\varphi = \varphi^5\sin\varphi + 5\varphi^4\cos\varphi - 20\varphi^3\sin\varphi - 60\varphi^2\cos\varphi$$
$$+120\varphi\cos\varphi + 120\sin\varphi$$

TAB. XXVII. $\int X\phi dx$.

[X an algebraic function of x; $\phi =$ arc sin x, arc cos x, arc tang x, &c.]

General Formulæ.

$$\int X dx \text{ arc sin } x = \text{arc sin } x \cdot \int X dx - \int \frac{dx \int X dx}{\sqrt{(1-x^2)}}$$

$$\int X dx \text{ arc cos } x = \text{arc cos } x \cdot \int X dx + \int \frac{dx \int X dx}{\sqrt{(1-x^2)}}$$

$$\int X dx \text{ arc tang } x = \text{arc tang } x \cdot \int X dx - \int \frac{dx \int X dx}{1+x^2}$$

$$\int X dx \text{ arc cot } x = \text{arc cot } x \cdot \int X dx + \int \frac{dx \int X dx}{1+x^2}$$

$$\int X dx \text{ arc sec } x = \text{arc sec } x \cdot \int X dx - \int \frac{dx \int X dx}{x\sqrt{(x^2-1)}}$$

$$\int X dx \text{ arc cosec } x = \text{arc cosec } x \cdot \int X dx + \int \frac{dx \int X dx}{x\sqrt{(x^2-1)}}$$

$$\int X dx \text{ arc sin vers } x = \text{arc sin v.} x \cdot \int X dx - \int \frac{dx \int X dx}{\sqrt{(2x-x^2)}}$$

Particular Cases.

$$\int dx \text{ arc sin } x = x \text{ arc sin } x - \int \frac{x dx}{\sqrt{(1-x^2)}}$$

$$\int x^m dx \text{ arc sin } x = \frac{x^{m+1}}{m+1} \text{ arc sin } x - \frac{1}{m+1} \int \frac{x^{m+1} dx}{\sqrt{(1-x^2)}}$$

$$\int \frac{dx}{\sqrt{(1-x^2)}} \text{ arc sin } x = \frac{1}{2}\left(\text{arc sin } x\right)^2$$

$$\int \frac{x dx}{\sqrt{(1-x^2)}} \text{ arc sin } x = -\text{ arc sin } x \cdot \sqrt{(1-x^2)} + x$$

$$\int \frac{x^2 dx}{\sqrt{(1-x^2)}} \text{arc sin } x = \left(-\frac{1}{2} x\sqrt{(1-x^2)} + \frac{1}{4} \text{arc sin } x\right) \text{arc sin } x + \frac{1}{4} x^2$$

$$\int \frac{x^3 dx}{\sqrt{(1-x^2)}} \text{arc sin } x = -\left(\frac{1}{3}x^2 + \frac{2}{3}\right)\sqrt{(1-x^2)} \cdot \text{arc sin } x + \frac{1}{9} x^3 + \frac{2}{3} x$$

$$\int \frac{x^4 dx}{\sqrt{(1-x^2)}} \text{arc sin } x = \left[-\left(\frac{1}{4}x^3 + \frac{3}{8}x\right)\sqrt{(1-x^2)} + \frac{3}{16} \text{ arc sin } x\right] \times \text{arc sin } x + \frac{1}{16}x^4 + \frac{3}{16}x^2$$

$$\int \frac{x^5 dx}{\sqrt{(1-x^2)}} \text{ arc sin } x = -\left(\frac{1}{5}x^4 + \frac{4}{15}x^2 + \frac{8}{15}\right)\sqrt{X} \cdot \text{arc sin } x + \frac{1}{25}x^5 + \frac{4}{45}x^3 + \frac{8}{15}x$$

$$\int \frac{dx}{(1-x^2)^{\frac{3}{2}}} \text{ arc sin } x = \frac{x \text{ arc sin } x}{\sqrt{(1-x^2)}} + \frac{1}{2}\log(1-x^2)$$

$$\int \frac{x dx}{(1-x^2)^{\frac{3}{2}}} \text{ arc sin } x = \frac{\text{arc sin } x}{\sqrt{(1-x^2)}} + \frac{1}{2}\log\frac{1-x}{1+x}$$

$$\int x^m dx \text{ arc cos } x = \frac{x^{m+1}}{m+1} \text{ arc cos } x + \frac{1}{m+1}\int \frac{x^{m+1}dx}{\sqrt{(1-x^2)}}$$

$$\int x^m dx \text{ arc tang } x = \frac{x^{m+1}}{m+1} \text{ arc tang } x - \frac{1}{m+1}\int \frac{x^{m+1}dx}{1+x^2}$$

$$\int x^m dx \text{ arc cot } x = \frac{x^{m+1}}{m+1} \text{ arc cot } x + \frac{1}{m+1}\int \frac{x^{m+1}dx}{1+x^2}$$

$$\int x^m dx \text{ arc sec } x = \frac{x^{m+1}}{m+1} \text{ arc sec } x - \frac{1}{m+1}\int \frac{x^m dx}{\sqrt{(x^2-1)}}$$

$$\int x^m dx \text{ arc cosec } x = \frac{x^{m+1}}{m+1} \text{ arc cosec } x + \frac{1}{m+1}\int \frac{x^m dx}{\sqrt{(x^2-1)}}$$

$$\int x^m dx \text{ arc sin vers } x = \frac{x^{m+1}}{m+1} \text{ arc sin v. } x - \frac{1}{m+1}\int \frac{x^{m+1}dx}{\sqrt{(2x-x^2)}}$$

$$\int \frac{dx}{1+x^2} \text{ arc tang } x = \frac{1}{2}\left(\text{arc tang } x\right)^2$$

$$\int \frac{x^2 dx}{1+x^2} \text{ arc tang } x = \left(x - \frac{1}{2}\text{ arc tang } x\right) \text{ arc tang } x - \frac{1}{2}\log(1+x^2)$$

$$\int \frac{dx}{(1+x^2)^2} \text{ arc tang } x = \left(\frac{x}{2(1+x^2)} + \frac{1}{4}\text{ arc tang } x\right) \text{ arc tang } x + \frac{1}{4(1+x^2)}$$

$$\int \frac{dx}{\sqrt{(1-x^2)}} \text{ arc cos } x = -\frac{1}{2}\left(\text{arc cos } x\right)^2$$

$$\int \frac{dx}{1+x^2} \text{ arc cot } x = -\frac{1}{2}\left(\text{arc cot } x\right)^2$$

$$\int \frac{dx}{\sqrt{(2x-x^2)}} \text{ arc sin vers } x = \frac{1}{2}\left(\text{arc sin v. } x\right)^2$$

TAB. XXVIII.

$$\int X\mathrm{d}x \log Z$$

(X, Z, algebraical Functions of x)

A general Formula.

$$\int X\mathrm{d}x \log Z = \log Z \,.\, \int X\mathrm{d}x - \int \frac{\mathrm{d}Z \int X\mathrm{d}x}{Z}$$

Particular Cases.

$$\int X\mathrm{d}x \log x = \log x \,.\, \int X\mathrm{d}x - \int \frac{\mathrm{d}x \int X\mathrm{d}x}{x}$$

$$\int x^m \mathrm{d}x \log x = \frac{x^{m+1}}{m+1}\left(\log x - \frac{1}{m+1}\right)$$

$$\int (a+bx)^m \mathrm{d}x \log x = \frac{(a+bx)^{m+1}}{(m+1)\,b} \log x - \frac{1}{(m+1)\,b} \int \frac{\mathrm{d}x (a+bx)^{m+1}}{x}$$

$$\int x^{-1} \mathrm{d}x \log x = \int \frac{\mathrm{d}x}{x} \log x = \frac{1}{2} \log^2 x$$

$$\int \frac{\mathrm{d}x}{a+bx} \log x = \frac{1}{b} \log x \,.\, \log (a+bx) - \frac{1}{b} \int \frac{\mathrm{d}x}{x} \log (a+bx)$$

Hence we obtain either*

$$\int \frac{\mathrm{d}x}{a+bx} \log x = \frac{1}{b} \log x \,.\, \log \frac{a+bx}{a} - \frac{x}{a} + \frac{bx^2}{2^2a^2} - \frac{b^2x^3}{3^2a^3} + \text{\&c.}$$

or

$$\int \frac{\mathrm{d}x}{a+bx} \log x = \frac{1}{b} \log x \,.\, \log (a+bx) - \frac{1}{2b} (\log bx)^2 + \frac{a}{b^2x} - \frac{a^2}{2^2b^3x^2} + \frac{a^3}{3^2b^4x^3} - \frac{a^4}{4^2b^5x^4} + \text{\&c.}$$

$$\int x^m \mathrm{d}x \log (a+bx) = \frac{x^{m+1}}{m+1} \log (a+bx) - \frac{b}{m+1} \int \frac{x^{m+1}\mathrm{d}x}{a+bx}$$

$$\int \frac{\mathrm{d}x}{x} \log (a+bx) = \log a \,.\, \log x + \frac{bx}{a} - \frac{b^2x^2}{2^2a^2} + \frac{b^3x^3}{3^2a^3} - \text{\&c.}$$

$$\int \frac{\mathrm{d}x}{x} \log (a+bx) = \frac{1}{2} (\log bx)^2 - \frac{a}{bx} + \frac{a^2}{2^2b^2x^2} - \frac{a^3}{3^2b^3x^3} + \text{\&c.}$$

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* See the two last formulæ in this page; where $\log (a+bx)$ is expanded according to the increasing or decreasing powers of x, multiplied by $\frac{\mathrm{d}x}{x}$, and afterwards integrated.

TAB. XXIX.

$$\int X dx \log^n x$$

A General Formula.

$$\int X dx \log^n x = X' \log^n x - nX'' \log^{n-1} x + n(n-1) X''' \log^{n-2} x + n(n-1)(n-2) X'''' \log^{n-3} x + \&c.$$

$$X = \int X dx,\ X'' = \int \frac{X' dx}{x},\ X''' = \int \frac{X'' dx}{x},\ \&c.$$

Particular Cases.

$$\int x^m dx \log^n x = \frac{x^{m+1}}{m+1}\left(\log^n x - \frac{n}{m+1}\log^{n-1} x + \frac{n(n-1)}{(m+1)^2}\log^{n-2} x - \frac{n(n-1)(n-2)}{(m+1)^3}\log^{n-3} x + \&c.\ ^*\right)$$

$$\int x^{-1} dx \log^n x = \int \frac{dx}{x}\log^n x = \frac{1}{n+1}\log^{n+1} x$$

$$\int x^m dx \log x = \frac{x^{m+1}}{m+1}\left(\log x - \frac{1}{m+1}\right)$$

$$\int x^m dx \log^2 x = \frac{x^{m+1}}{m+1}\left(\log^2 x - \frac{2}{m+1}\log x + \frac{2.1}{(m+1)^2}\right)$$

$$\int x^m dx \log^3 x = \frac{x^{m+1}}{m+1}\left(\log^3 x - \frac{3}{m+1}\log^2 x + \frac{3.2}{(m+1)^2}\log x - \frac{3.2.1}{(m+1)^3}\right)$$

$$\int \frac{x^m dx}{\sqrt{\log x}} = \frac{x^{m+1}}{(m+1)\sqrt{\log x}}\left(1 + \frac{1}{(2m+2)\log x} + \frac{1.3}{[(2m+2)\log x]^2} + \frac{1.3.5}{[(2m+2)\log x]^3} + \frac{1.3.5.7}{[(2m+2)\log x]^4} + \textit{in infinit.}\right)$$

$$\int \frac{x^m dx}{\sqrt{\log \frac{1}{x}}} = \frac{x^{m+1}}{(m+1)\sqrt{\log \frac{1}{x}}}\left(1 + \frac{1}{(2m+2)\log x} + \frac{1.3}{[(2m+2)\log x]^2} + \frac{1.3.5}{[(2m+2)\log x]^3} + \frac{1.3.5.7}{[(2m+2)\log x]^4} + \textit{in infinit.}\right)$$

The former of the two formulæ $\int \frac{x^m dx}{\sqrt{\log x}}$, $\int \frac{x^m dx}{\sqrt{\log \frac{1}{x}}}$ is imaginary when x lies between 0 and 1; the latter is so, when x is > 1.

* The series will terminate when n is a positive integer. When n is a negative integer, a finite series may also be found. See the following page.

TAB. XXX.

$$\int \frac{X dx}{\log^n x}$$

A general Formula.

$$\int \frac{X dx}{\log^n x} = -\frac{Xx}{(n-1)\log^{n-1} x} - \frac{X'x}{(n-1)(n-2)\log^{n-2} x} - \frac{X''x}{(n-1)(n-2)(n-3)\log^{n-3} x} - \&c.$$

$$X' = \frac{d(Xx)}{dx},\ X'' = \frac{d(X'x)}{dx},\ X''' = \frac{d(X''x)}{dx},\ \&c.$$

Particular Cases.

$$\int \frac{x^m dx}{\log^n x} = -\frac{x^{m+1}}{(n-1)\log^{n-1} x} - \frac{(m+1)x^{m+1}}{(n-1)(n-2)\log^{n-2} x} - \frac{(m+1)^2 x^{m+1}}{(n-1)(n-2)(n-3)\log^{n-3} x} - \dots\dots - \frac{(m+1)^{n-2} x^{m+1}}{(n-1)(n-2)(n-3)\dots 2.1 \log x} + \frac{(m+1)^{n-1}}{(n-1)(n-2)\dots 2.1} \int \frac{x^m dx}{\log x}$$

$$\int \frac{dx}{x} \log^n x = \frac{1}{n+1} \log^{n+1} x$$

$$\int \frac{dx}{\log x} = \log \log x + \frac{\log x}{1} + \frac{1}{2} \cdot \frac{\log^2 x}{1.2} + \frac{1}{3} \cdot \frac{\log^3 x}{1.2.3} + \frac{1}{4} \cdot \frac{\log^4 x}{1.2.3.4} + \frac{1}{5} \cdot \frac{\log^5 x}{1.2.3.4.5} + \textit{in infinit.}$$

$$\int \frac{x^m dx}{\log x} = \int \frac{dy}{\log y}\ (y = x^{m+1})$$

$$\int \frac{x^m dx}{\log^2 x} = -\frac{x^{m+1}}{\log x} + \frac{m+1}{1} \int \frac{x^m dx}{\log x}$$

$$\int \frac{x^m dx}{\log^3 x} = -\frac{x^{m+1}}{2 \log^2 x} - \frac{(m+1)x^{m+1}}{2.1 \log x} + \frac{(m+1)^2}{2.1} \int \frac{x^m dx}{\log x}$$

$$\int \frac{dx}{\log \frac{1}{x}} = \log \log x - \frac{\log x}{1} + \frac{1}{2} \cdot \frac{\log^2 x}{1.2} - \frac{1}{3} \cdot \frac{\log^3 x}{1.2.3} + \frac{1}{4} \cdot \frac{\log^4 x}{1.2.3.4} - \frac{1}{5} \cdot \frac{\log^5 x}{1.2.3.4.5} + - + - \textit{in infinit.}$$

$$\int' \frac{dx}{\sqrt{\log \frac{1}{x}}} = \sqrt{\pi}$$ [The integral being taken from $x = 0$ to $x = 1$. (Euler. *Com. Acad. Petrop. tom. XVI. p.* 111.)]

$$\int' dx \left(\log \frac{1}{x}\right)^{\frac{2n+1}{2}} = \frac{1.3.5.7\dots\dots(2n+1)}{2^{n+1}} \sqrt{\pi}$$ (Idem.)

TAB. XXXI.

$$\int a^x X dx$$

A general Formula.

$$\int a^x X dx = \frac{a^x X}{\log a} - \frac{a^x X'}{\log^2 a} + \frac{a^x X''}{\log^3 a} - \frac{a^x X'''}{\log^4 a} + - + - \ldots\ldots$$

$$X' = \frac{dX}{dx},\ X'' = \frac{dX'}{dx},\ X''' = \frac{dX''}{dx},\ \&c.$$

$$\int a^x X dx = a^x X_1 - a^x X_2 \log a + a^x X_3 \log^2 a - a^x X_4 \log^3 a$$
$$+ a^x X_5 \log^4 a - + - + \ldots\ldots\ldots\ldots$$

$$X_1 = \int X dx,\ X_2 = \int X_1 dx,\ X_3 = \int X_2 dx,\ \&c.$$

Particular Cases.

$$\int a^x x^n dx = \frac{a^x x^n}{\log a} = \frac{n a^x x^{n-1}}{\log^2 a} + \frac{n(n-1) a^x x^{n-2}}{\log^3 a}$$
$$- \frac{n(n-1)(n-2) a^x x^{n-3}}{\log^4 a} + \ldots\ldots \pm \frac{n(n-1)(n-2)\ldots 2 \,.\, 1\, a^x}{\log^{n+1} a}$$

$$\int \frac{a^x dx}{x^n} = - \frac{a^x}{(n-1)x^{n-1}} - \frac{a^x \log a}{(n-1)(n-2)x^{n-2}} - \frac{a^x \log^2 a}{(n-1)(n-2)(n-3)x^{n-3}}$$
$$- \frac{a^x \log^3 a}{(n-1)(n-2)(n-3)(n-4)x^{n-4}} - \ldots \frac{a^x \log^{n-2} a}{(n-1)(n-2)\ldots 2 \,.\, 1 x}$$
$$+ \frac{\log^{n-1} a}{(n-1)(n-2)\ldots\ldots 2 \,.\, 1} \int \frac{a^x dx}{x}$$

$$\int a^x dx = \frac{a^x}{\log a},\quad \int a^{mx} dx = \frac{a^{mx}}{m \log a},\quad \int e^{mx} dx = \frac{e^{mx}}{m}$$

$$\int a^x x dx = \frac{a^x x}{\log a} - \frac{a^x}{\log^2 a}$$

$$\int a^x x^2 dx = \frac{a^x x^2}{\log a} - \frac{2 a^x x}{\log^2 a} + \frac{2 \,.\, 1\, a^x}{\log^3 a}$$

$$\int a^x x^3 dx = \frac{a^x x^3}{\log a} - \frac{3 a^x x^2}{\log^2 a} + \frac{3 \,.\, 2\, a^x x}{\log^3 a} - \frac{3 \,.\, 2 \,.\, 1\, a^x}{\log^4 a}$$

$$\int \frac{a^x dx}{x} = \log x + \frac{x \log a}{1} + \frac{x^2 \log^2 a}{1 \,.\, 2 \,.\, 2} + \frac{x^3 \log^3 a}{1 \,.\, 2 \,.\, 3 \,.\, 3} + \frac{x^4 \log^4 a}{1 \,.\, 2 \,.\, 3 \,.\, 4 \,.\, 4}$$
$$+ \frac{x^5 \log^5 a}{1 \,.\, 2 \,.\, 3 \,.\, 4 \,.\, 5 \,.\, 5} + \textit{in infinit.}$$

$$\int \frac{a^x dx}{x^2} = - \frac{a^x}{x} + \log a \int \frac{a^x dx}{x}$$

$$\int\frac{a^x dx}{x^3} = -\frac{a^x}{2x^2} - \frac{a^x \log a}{2.1x} + \frac{\log^2 a}{2.1}\int\frac{a^x dx}{x}$$

$$\int\frac{a^x dx}{x^4} = -\frac{a^x}{3x^3} - \frac{a^x \log a}{3.2\,x^2} - \frac{a^x \log^2 a}{3.2.1x} + \frac{\log^3 a}{3.2.1}\int\frac{a^x dx}{x}$$

$$\int\frac{a^x dx}{\sqrt{x}} = \frac{a^x}{\sqrt{x}}\left(\frac{1}{\log a} + \frac{1}{2x\log^2 a} + \frac{1.3}{2^2x^2\log^3 a} + \frac{1.3.5}{2^3x^3\log^4 a} + \&c.\right)$$

$$\int\frac{a^x dx}{\sqrt{x}} = \frac{a^x}{\sqrt{x}}\left(\frac{2x}{1} - \frac{2^2x^2\log a}{1.3} + \frac{2^3x^3\log^2 a}{1.3.5} - \frac{2^4x^4\log^3 a}{1.3.5.7} + \&c.\right)$$

$$\int\frac{a^x dx}{1-x} = a^x\left[\frac{1}{(1-x)\log a} - \frac{1}{(1-x)^2\log^2 a} + \frac{1.2}{(1-x)^3\log^3 a}\right.$$
$$\left. - \frac{1.2.3}{(1-x)^4\log^4 a} + \frac{1.2.3.4}{(1-x)^5\log^5 a} - \&c.\right]$$

$$\int a^{mx}x^n dx = \frac{1}{m^{n+1}}\int a^y y^n dy \text{ for } y = mx$$

$$\int x^{nx}x^m dx = \int\left(1 + \frac{nx\log x}{1} + \frac{n^2x^2\log^2 x}{1.2} + \frac{n^3x^3\log^3 x}{1.2.3} + \&c.\right)x^m dx$$

$$= {}^{*}\, x^{m+1}\left(\frac{1}{m+1} - \frac{nx}{(m+2)^2} + \frac{n^2x^2}{(m+3)^3} - \frac{n^3x^3}{(m+4)^4} + \&c.\right)$$

$$+ \frac{nx^{m+2}\log x}{1}\left(\frac{1}{m+2} - \frac{nx}{(m+3)^2} + \frac{n^2x^2}{(m+4)^3} - \frac{n^3x^3}{(m+5)^4} + \&c.\right)$$

$$+ \frac{n^2x^{m+3}\log^2 x}{1.2}\left(\frac{1}{m+3} - \frac{nx}{(m+4)^2} + \frac{n^2x^2}{(m+5)^3} - \frac{n^3x^3}{(m+6)^4} + \&c.\right)$$

$$+ \frac{n^3x^{m+4}\log^3 x}{1.2.3}\left(\frac{1}{m+4} - \frac{nx}{(m+5)^2} + \frac{n^2x^2}{(m+6)^3} - \frac{n^3x^3}{(m+7)^4} + \&c.\right)$$

$$\&c. \qquad \&c. \qquad \&c.$$

$$\int e^{-x^2}dx = \sqrt{\pi}$$ [The integral being taken from $x = -\infty$ to $x = +\infty$. Laplace, *Mécan. cél. livre X, No. 5.*]

* By the integration of $x^m dx$, $x^{m+1}dx\log x$, $x^{m+2}dx\log^2 x$, $x^{m+3}dx\log^3 x$, &c. The value of the integral $\int x^{nx}x^m dx$, taken between $x = 0$ and 1, on the supposition that $m+1$ is positive is

$$= \frac{1}{m+1} - \frac{n}{(m+2)^2} + \frac{n^2}{(m+3)^3} - \frac{n^3}{(m+4)^4} + \frac{n^4}{(m+5)^5} - \&c.$$

TAB. XXXII.

$$\int e^{ax}dx \sin^n x, \quad \int e^{ax}dx \cos^n x$$

Formulæ of reduction.

$$\int e^{ax}dx \sin^n x = \frac{e^{ax}\sin^{n-1}x(a\sin x - n\cos x)}{a^2+n^2} + \frac{n(n-1)}{a^2+n^2}\int e^{ax}dx \sin^{n-2}x$$

$$\int e^{ax}dx \cos^n x = \frac{e^{ax}\cos^{n-1}x(a\cos x + n\sin x)}{a^2+n^2} + \frac{n(n-1)}{a^2+n^2}\int e^{ax}dx \cos^{n-2}x$$

Particular Formulæ

$$\int e^{ax}dx \sin x = \frac{e^{ax}(a\sin x - \cos x)}{a^2+1}$$

$$\int e^{ax}dx \sin^2 x = \frac{e^{ax}\sin x(a\sin x - 2\cos x)}{a^2+4} + \frac{1.2}{a(a^2+4)}e^{ax}$$

$$\int e^{ax}dx \sin^3 x = \frac{e^{ax}\sin^2 x(a\sin x - 3\cos x)}{a^2+9} + \frac{2.3e^{ax}(a\sin x - \cos x)}{(a^2+1)(a^2+9)}$$

....

$$\int e^{ax}dx \cos x = \frac{e^{ax}(a\cos x + \sin x)}{a^2+1}$$

$$\int e^{ax}dx \cos^2 x = \frac{e^{ax}\cos x(a\cos x + 2\sin x)}{a^2+4} + \frac{1.2}{a(a^2+4)}e^{ax}$$

$$\int e^{ax}dx \cos^3 x = \frac{e^{ax}\cos^2 x(a\cos x + 3\sin x)}{a^2+9} + \frac{2.3e^{ax}(a\cos x + \sin x)}{(a^2+1)(a^2+9)}$$

....

$$\int e^{ax}dx \sin kx = \frac{e^{ax}(a\sin kx - k\cos kx)}{a^2+k^2}$$

$$\int e^{ax}dx \cos kx = \frac{e^{ax}(a\cos kx + k\sin kx)}{a^2+k^2}$$

By means of the two last formulæ, we may also integrate $\int e^{ax}dx \sin^m x \cos^n x$. If $\sin^m x \cos^n x$ be expressed in terms of the sines and cosines of multiple angles, we shall obtain pure monomials of the forms $e^{ax}dx \sin kx$, $e^{ax}dx \cos kx$, $e^{ax}dx$.

TAB. XXXIII.

$$\int\frac{(f+g\cos\varphi)\,d\varphi}{(a+b\cos\varphi)^n}$$

Formulæ of Reduction.

$$\int\frac{(f+g\cos\varphi)\,d\varphi}{(a+b\cos\varphi)^n}=\frac{(ag-bf)\sin\varphi}{(n-1)(a^2-b^2)(a+b\cos\varphi)^{n-1}}$$
$$+\frac{1}{(n-1)(a^2-b^2)}\int\frac{[(n-1)(af-bg)+(n-2)(ag-bf)\cos\varphi]\,d\varphi}{(a+b\cos\varphi)^{n-1}}$$

Particular Cases.

$$\int\frac{d\varphi}{a+b\cos\varphi}=\frac{2}{\sqrt{(a^2-b^2)}}\text{ arc tang }\frac{(a-b)\operatorname{tang}\frac{1}{2}\varphi}{\sqrt{(a^2-b^2)}}$$
$$=\frac{1}{\sqrt{(a^2-b^2)}}\text{ arc tang }\frac{\sin\varphi\sqrt{(a^2-b^2)}}{b+a\cos\varphi}$$
$$=\frac{1}{\sqrt{(a^2-b^2)}}\text{ arc sin }\frac{\sin\varphi\sqrt{(a^2-b^2)}}{a+b\cos\varphi}$$
$$=\frac{1}{\sqrt{(a^2-b^2)}}\text{ arc cos }\frac{b+a\cos\varphi}{a+b\cos\varphi}$$

$$\int\frac{d\varphi}{a+b\cos\varphi}=\frac{1}{\sqrt{(b^2-a^2)}}\log\frac{b+a\cos\varphi+\sin\varphi\sqrt{(b^2-a^2)}}{a+b\cos\varphi}$$

The first of these values are for $b<a$, the other value for $b>a$; for $b=a$, we have

$$\int\frac{d\varphi}{a+a\cos\varphi}=\frac{1}{a}\int\frac{d\varphi}{1+\cos\varphi}=\frac{1}{a}\operatorname{tang}\frac{1}{2}\varphi$$

$$\int\frac{d\varphi\sin\varphi}{a+b\cos\varphi}=-\frac{1}{b}\log(a+b\cos\varphi)$$

$$\int\frac{d\varphi\cos\varphi}{a+b\cos\varphi}=\frac{\varphi}{b}-\frac{a}{b}\int\frac{d\varphi}{a+b\cos\varphi}$$

$$\int\frac{d\varphi}{(a+b\cos\varphi)^2}=\frac{1}{a^2-b^2}\left(\frac{-b\sin\varphi}{a+b\cos\varphi}+a\int\frac{d\varphi}{a+b\cos\varphi}\right)$$

$$\int\frac{d\varphi\cos\varphi}{(a+b\cos\varphi)^2}=\frac{1}{a^2-b^2}\left(\frac{a\sin\varphi}{a+b\cos\varphi}-b\int\frac{d\varphi}{a+b\cos\varphi}\right)$$

TAB. XXXIV.

The Integral

$$\int d\varphi\,(1+n\cos\varphi)^p$$

resolved by multiple angles.

I. *p positive.* $p=+m$

A general Formula.*

$$\int d\varphi\,(1+n\cos\varphi)^m = A\varphi + B\sin\varphi + \frac{1}{2}C\sin 2\varphi + \frac{1}{3}D\sin 3\varphi$$
$$+\frac{1}{4}E\sin 4\varphi + \frac{1}{5}F\sin 5\varphi + \&c.$$

$$A = 1 + \frac{1}{2}\,{}^m\mathrm{B}n^2 + \frac{1.3}{2.4}\,{}^m\mathrm{D}n^4 + \frac{1.3.5}{2.4.6}\,{}^m\mathrm{F}n^6$$
$$+\frac{1.3.5.7}{2.4.6.8}\,{}^m\mathrm{H}n^8 + \&c.$$

$$B = 2n\left(\frac{1}{2}\,{}^m\mathrm{A} + \frac{1.3}{2.4}\,{}^m\mathrm{C}n^2 + \frac{1.3.5}{2.4.6}\,{}^m\mathrm{E}n^4\right.$$
$$\left.+\frac{1.3.5.7}{2.4.6.8}\,{}^m\mathrm{G}n^6 + \&c.\right)$$

$$C = \frac{2mnA - 2B}{(m+2)n}, \quad D = \frac{(m-1)nB - 4C}{(m+3)n}$$

$$E = \frac{(m-2)nC - 6D}{(m+4)n}, \quad F = \frac{(m-3)nD - 8E}{(m+5)n}$$

$$G = \frac{(m-4)nE - 10F}{(m+6)n}, \quad H = \frac{(m-5)nF - 12G}{(m+7)n}$$

&c.

[The series for A and B terminate when m is an integer.]

---

* The integral $\int d\varphi\,(a+b\cos\varphi)^p$ may be reduced to this form by putting $\frac{b}{a}=n$; for then

$$\int d\varphi\,(a+b\cos\varphi)^p = a^p\int d\varphi\,(1+n\cos\varphi)^p.$$

Particular Cases.

For $m = 1$, we have

$$A = 1,\ B = n,\ (C,\ D,\ E,\ \&c. = 0).$$

For $m = 2$, we have

$$A = 1 + \frac{1}{2}n^2,\ B = 2n,\ C = \frac{1}{2}n^2,\ (D,\ E,\ \&c. = 0).$$

For $m = 3$, we have

$$A = 1 + \frac{3}{2}n^2,\ B = 3n + \frac{3}{4}n^3,\ C = \frac{3}{2}n^2,$$

$$D = \frac{1}{4}n^3,\ (E,\ F,\ \&c. = 0).$$

For $m = 4$, we have

$$A = 1 + 3n^2 + \frac{3}{8}n^4,\ B = 4n + 3n^3,\ C = 3n^2 + \frac{1}{2}n^4,$$

$$D = n^3,\ E = \frac{1}{8}n^4,\ (F,\ G,\ \&c. = 0).$$

For $m = \frac{1}{2}$, we have

$$A = 1 - \frac{1\,.\,1}{4\,.\,4}n^2 - \frac{1\,.\,1\,.\,3\,.\,5}{4\,.\,4\,.\,8\,.\,8}n^4 - \frac{1\,.\,1\,.\,3\,.\,5\,.\,7\,.\,9}{4\,.\,4\,.\,8\,.\,8\,.\,12\,.\,12}n^6 - \&c.$$

$$B = \frac{1}{2}n + \frac{1\,.\,1\,.\,3}{2\,.\,4\,.\,8}n^3 + \frac{1\,.\,1\,.\,3\,.\,5\,.\,7}{2\,.\,4\,.\,8\,.\,8\,.\,12}n^5 + \&c.$$

$$C = \frac{2nA - 4B}{5n},\ D = \frac{-nB - 8C}{7n},\ \&c.$$

II. p *negative*. $p = -m$.

Formulæ of Reduction.

Let

$$\int d\varphi\,(1 + n\cos\varphi)^{-m} = A\varphi + B\sin\varphi + \frac{1}{2}C\sin 2\varphi + \frac{1}{3}D\sin 3\varphi + \frac{1}{4}E\sin 4\varphi + \frac{1}{5}F\sin 5\varphi + \&c.$$

$$\int d\varphi\,(1 + n\cos\varphi)^{-m-1} = A'\varphi + B'\sin\varphi + \frac{1}{2}C'\sin 2\varphi + \frac{1}{3}D'\sin 3\varphi + \frac{1}{4}E'\sin 4\varphi + \frac{1}{5}F'\sin 5\varphi + \&c.$$

Then

$$A' = \frac{2mA - (m-1)nB}{2m(1-n^2)} = A + \frac{ndA}{mdn}$$

$$B' = \frac{2(A-A')}{n} = B + \frac{ndB}{mdn}$$

$$C' = \frac{2(B-B') - 2nA'}{n} = C + \frac{ndC}{mdn}$$

$$D' = \frac{2(C-C') - nB'}{n} = D + \frac{ndD}{mdn}$$

$$E' = \frac{2(D-D') - nC'}{n} = E + \frac{ndE}{mdn}$$

&c. &c.

By aid of these double formulæ of reduction, which serve mutually to prove one another, the coefficients A, B, C, D, &c. corresponding to the values $p = -2$, -3, -4, &c., may be estimated, from their values for $p = -1$. When $p = -1$, we have

$$A = \frac{1}{\sqrt{(1-n^2)}}$$

$$B = \frac{2 - 2\sqrt{(1-n^2)}}{n\sqrt{(1-n^2)}}$$

$$C = \frac{4 - 2n^2 - 4\sqrt{(1-n^2)}}{n^2\sqrt{(1-n^2)}}$$

$$D = \frac{8 - 6n^2 - 2(4-n^2)\sqrt{(1-n^2)}}{n^3\sqrt{(1-n^2)}}$$

$$E = \frac{16 - 16n^2 + 2n^4 - 2(8-4n^2)\sqrt{(1-n^2)}}{n^4\sqrt{(1-n^2)}}$$

....

$$\overset{\mu}{A} = \frac{2}{\sqrt{(1-n^2)}}\left(\frac{1-\sqrt{(1-n^2)}}{n}\right)^{\mu}$$

The values of A, B, C, D, &c. for $p = -\frac{1}{2}$, $-\frac{3}{2}$, $-\frac{5}{2}$, &c. may be found by the same formulæ, from their values for $p = \frac{1}{2}$, (see p. 276); they do not, however, well admit of representation, otherwise than by series.

To the developement of the angular function $(1+n\cos\varphi)^p$ into a series of the form $A+B\cos\varphi+C\cos 2\varphi+D\cos 3\varphi+$ &c. for the purpose of integrating the differential $d\varphi\,(1+n\cos\varphi)^p$, Analysts have frequently directed their attention. This form is particularly useful in astronomy, where it occurs in that of $(r^2+r'^2-rr'\cos\varphi)^p$, or rather in the somewhat more simple one: $(1+a^2-a\cos\varphi)^p$. The most ample details on this subject may be found in Euler's *Instit. Calcul. Integ.*, and in the *Traité du Cal. Diff. et Integ.* of Lacroix. Laplace, in the second book of his *Mécanique Celeste*, has given the following formulæ of reduction:

Adopting his notation, let

$$(1+a^2-a\cos\theta)^{-s}=\frac{1}{2}b_s^{(0)}+b_s^{(1)}\cos\theta+b_s^{(2)}\cos 2\theta+\text{\&c.}$$

$$(1+a^2-a\cos\theta)^{-s-1}=\frac{1}{2}b_{s+1}^{(0)}+b_{s+1}^{(1)}\cos\theta+b_{s+1}^{(2)}\cos 2\theta+\text{\&c.}$$

Then

$$b_s^{(i)}=\frac{(i-1)(1+a^2)b_s^{(i-1)}-(i+s-2)ab_s^{(i-2)}}{(i-s)a}$$

$$b_{s+1}^{(i)}=\frac{(s+i)(1+a^2)b_s^{(i)}-2(i-s+1)ab_s^{(i+1)}}{s(1-a^2)^2}$$

$$b_{s+1}^{(i)}=\frac{(s-i)(1+a^2)b_s^{(i)}+2(i+s-1)ab_s^{(i-1)}}{s(1-a^2)^2}$$

For the values $b_s^{(0)}$, $b_s^{(1)}$, he gives the following series:

$$b_s^{(0)}=2\left[1+s^2.a^2+\left(\frac{s(s+1)}{1.2}\right)^2.a^4+\left(\frac{s(s+1)(s+2)}{1.2.3}\right)^2.a^6+\text{\&c.}\right]$$

$$b_s^{(1)}=2a\left[s+s.\frac{s(s+1)}{1.2}a^2+\frac{s(s+1)}{1.2}.\frac{s(s+1)(s+2)}{1.2.3}a^4+\text{\&c.}\right]$$

Hence we obtain, when $s=-\frac{1}{2}$, and therefore $p=\frac{1}{2}$ the following series:

$$\frac{1}{2}b^{(0)}_{-\frac{1}{2}} = 1 + \left(\frac{1}{2}\right)^2 \alpha^2 + \left(\frac{1.1}{2.4}\right)^2 \alpha^4 + \left(\frac{1.1.3}{2.4.6}\right)^2 \alpha^6 + \&c.$$

$$b^{(1)}_{-\frac{1}{2}} = -\alpha\left(1 - \frac{1.1}{2.4}\alpha^2 - \frac{1}{4}\cdot\frac{1.1.3}{2.4.6}\alpha^4 - \frac{1.3}{4.6}\cdot\frac{1.1.3.5}{2.4.6.8}\alpha^6 - \&c.\right)$$

These series converge with great rapidity when α is a small fraction. By aid of these and the preceding formulæ, the values of $b^{(2)}_{-\frac{1}{2}}$, $b^{(3)}_{-\frac{1}{2}}$, &c., $b^{(2)}_{-\frac{1}{2}}$, $b^{(3)}_{-\frac{1}{2}}$, &c., as also of their differentials relative to α, when required, as in the work quoted, may very easily be found.

FINIS.

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